A General Theory of Holdouts: Online Appendix

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OA.1 Full Model and Simplification

Now we formulate a more general set up where the principal can allow the agent to accept a fraction $1 - h_i$ of the exchange offers, or equivalently, let the agent choose $h_i \in H_i \subset [0, 1]$. In particular, every agent can fully hold out, $1 \in H_i$ and the principal doesn't ration $0 \in H_i$.[1] Moreover, the principal needs to state how the old contracts R^O would be affected by the new contracts R, which means a new function $R^O(\cdot, \cdot|R)$ to replace R^O. To summarize, in the spirit of the revelation principle, I formalize the notion of exchange offers as follows:

Definition OA.1 (Direct Exchange Offer)**.** *A direct exchange offer is a tuple* $(H, h, R, R^{O|R})$ *where*

- $H = \prod_{i=1}^{N} H_i$ *is the product space of* A_i*'s action space* H_i *such that* $\{0, 1\} \subset H_i \subset [0, 1]$*;*
- $h = (h_1, h_2, \dots, h_N) \in H$ *is the (recommended) holdout profile of the agents;*
- R *is a mapping from* $\mathbb{R}_+ \times H$ *to* $\mathbb{R}^N$ *where the* i*th element* $R_i(v, h)$ *determines the unit payoff of* A_i*'s new contract given the asset value is* v *and the holdout profile* h*;*
- $R^{O|R}(\cdot, \cdot) \equiv R^O(\cdot, \cdot|R)$ *is a mapping from* $\mathbb{R}_+ \times H$ *to* $\mathbb{R}_+^{N+1}$ *where the* i*th element* $R_i^O(v, h|R)$ *determines the unit payoff of* A_i*'s old contract (or principal's if* $i = 0$*) given the asset value is* v *and the holdout profile* h

such that

- *the allocation is feasible:*
$$\sum_{i=0}^{N} h_i \, R_i^O(v, h|R) + \sum_{i=1}^{N} (1 - h_i) \, R_i(v, h) = v \tag{OA.1}$$
- *the action* h_i *is incentive compatible:*
$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i'|h_{-i}, R, R^{O|R}) \tag{OA.2}$$
  where
$$u_i(h_i|h_{-i}, R, R^{O|R}) := (1 - h_i) \, R_i(v, h) + h_i \, R_i^O(v, h|R) \tag{OA.3}$$

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[1] If the principal wants to exclude an agent, they could offer the same contract the agent has. So disallowing rationing doesn't reduce the generality.

is A_i's payoff given the action profile $h = (h_{-i}, h_i)$ and the corresponding project value v.

The word "recommended" might surprise the reader: Why would the principal recommend a holdout profile as part of the exchange offer? As others before in the mechanism design literature (e.g., Myerson, 1983), I allow the principal to provide a public coordination device by recommending an action profile to overcome the concern of multiple equilibria outside those proposed by the principal.

Principal's original problem (OP) The principal aims to design an exchange offer $(H, h, R, R^{O|R})$ in exchange for the old contracts. In the case of full commitment, the constrained optimization problem of the principal is

$$\max_{H,h,R,R^{O|R}} v(h) - \sum_{i=1}^{N}(1-h_i)\cdot R_i(v(h),h) - \sum_{i=1}^{N} h_i \cdot R_i^O(v(h),h|R) \tag{OP}$$

such that the action is incentive-compatible

$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i|h_{-i}, R, R^{O|R}) \quad \forall i \in \mathcal{N}. \tag{OA.4}$$

To understand the principal's payoff, it is helpful to consider the situation where some agents tender fully, whereas others hold out. The principal's payoff is the value of the asset given the profile h, minus the payoff that accrues to the tendering agents, $R_i\,(v(h),h)$, minus what accrues to holdouts $R_i(v(h),h|R)$, which is a function of the old contracts.

OA.1.1 Simplication due to Weak Consistency

This weak consistency assumption allows us to write the contractual design problem in a separable form.

Proposition OA.1 (Separation)**.** *Under Weak Consistency (Definition 1), we can rewrite the original design problem as*

$$\max_{h,R} R_0^O\,(v(h) - x(h), h) \tag{OA.5}$$

where $x(h) = \sum_{j=1}^{N}(1-h_j)\,R_j(v(h),h)$ under the agents' IC condition

$$h_i \in \arg\max_{h_i' \in H_i} (1-h_i')R_i(v(h_{-i},h_i'),(h_{-i},h_i')) \tag{OA.6}$$

$$+h_i' R_i^O\,(v(h_{-i},h_i') - x(h_{-i},h_i'),(h_{-i},h_i')) \;\; \forall i \tag{OA.7}$$

where $x(h_{-i},h_i') = \sum_{j=1}^{N}(1-h_j)\,R_j(v(h_{-i},h_i'),(h_{-i},h_i'))$

One thing to clarify is that the formulation does not mean the new contracts R have priorities over the existing contracts R^O since they could be zero for the realized value v. But this is also more than a notational asymmetry between old and new contracts: The new contract could determine the division of asset value between old and new. This amounts to assuming all old contracts are dilutable by new ones, but not necessarily diluted. I relax this implicit assumption in Section 5.

This proposition allows us to define a simpler concept of exchange offers

Definition OA.2 (Consistent Exchange Offer)**.** *A consistent exchange offer is a tuple(H, h, R) where*

- *$H = \prod_{i=1}^{N} H_i$ is the product space of A_i's action space H_i such that $\{0,1\} \subset H_i \subset [0,1]$;*

- *$h = (h_1, h_2, \ldots, h_N) \in H$ is the (recommended) action profile of the agents;*
- *R is a mapping from $\mathbb{R}_+ \times H$ to $\mathbb{R}_+^N$ where the ith element $R_i(v, h)$ determines the unit payoff of A_i's new contract given the asset value is v and the holdout profile h;*

such that

- *the allocation is feasible:*

$$\sum_{i=0}^{N} h_i\, R_i^O(v - x, h) + \sum_{i=1}^{N} (1 - h_i)\, R_i(v, h) = v \tag{OA.8}$$

  where $x = \sum_{i=1}^{N}(1 - h_i)R(v, h)$;

- *the action h_i is incentive compatible:*

$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i' | h_{-i}, R) \tag{OA.9}$$

  where

$$u_i(h_i | h_{-i}, R) := (1 - h_i)\, R_i(v, h) + h_i\, R_i^O(v - x, h) \tag{OA.10}$$

  is A_i's payoff given the action profile $h = (h_{-i}, h_i)$ and the corresponding project value v.

It's clear that from the assumption that $v(h)$ is decreasing in h and that $c < v(\mathbf{0}) - v(\mathbf{1})$, the first best is to implement $h = \mathbf{0}$. It is implementable if all agents can coordinate: As per the Coase Theorem, the positive surplus can be split by bilateral bargaining.

OA.1.2 Simplication from Equivalence Exchange Offers

The next result further simplifies the analysis, saying that it is without loss of generality to look at the implementation of $h = \mathbf{0}$, i.e., the equilibrium where everyone tenders. One may argue that it might not be ideal to implement $\mathbf{0}$ when it's too costly to hold in an additional agent, which only slightly improves the asset value. This will not be the case here as the principal can offer the exact same contract as what the agent initially has, and the agent would weakly prefer to exchange. Indeed, the principal would never find it optimal to do so because there are cheaper ways of implementing an exchange offer, as we will show below.

Proposition OA.2 (Equivalence)**.** *For any consistent exchange offer (H, h^*, R) such that $h^* \neq \mathbf{0}$ is implementable for the principal, there exists an alternative consistent exchange offer, with the same action space H, in which $h = \mathbf{0}$ is implementable, and the principal obtains the same payoff as under the original exchange offer.*

The proof builds on a simple idea: If it is optimal for an agent to retain some of its original shares, then the principal could simply offer the existing contracts through the new contracts. However, the complication comes from our setting that the asset value depends on the actions but not the form of the new contracts. Thus, the asset value is artificially inflated[2] when offering the existing contracts through new contracts. This formulation makes the holdout problem more acute: If the principal offers the same value to each tendering

[2] This formulation also encompasses the more realistic case when the asset value is not enhanced when the exact same contract is offered.

agent, the value of the outside option is higher due to the inflated asset value, so the IC may no longer hold. We solve this issue by giving away the artificially inflated asset value to the agents through the new contracts. Moreover, even if the value that can be distributed to the holders of initial contracts is made the same, the holdout might obtain a higher value through the initial contracts, as fewer agents hold initial contracts. This problem can also be handled by allocating more value to the contracts.

OA.2 Non-contingent Exchange Offers with Financing

In this section, we augment the baseline model with financial constraints and the possibility of borrowing when the cash offer has to be paid ex-ante. These cash transfers can only come from the principal's equilibrium allocation plus her initial wealth W, if any. Notice that this implicitly assumes perfect capital markets. For instance, if the exchange offer includes some cash transfers, then the principal is able to borrow F from an outside lender via safe debt.[3] I will first analyze the case when the existing agents have no recourse to the borrowed cash, then the case when agents have recourse to the borrowed cash, and lastly when the debt can be imposed on the existing agents.

OA.2.1 Optimal non-contingent offers without recourse to externally raised funds

In this case, if $h = \mathbf{0}$ is to be implemented via a cash transfer, the following conditions must be met.

1. Each A$_i$ is paid at least as much as what he would otherwise get by holding out

$$t_i(0) \geq R_i^O\left(v(e_i), e_i\right), \forall i \in \mathcal{N} \tag{OA.11}$$

2. The total payment can be financed through internal cash W and borrowing F from an external financier

$$\sum_{j=1}^{N} t_j(0) \leq F + W, \tag{OA.12}$$

   where the debt can be repaid from the residual claim and any additional cash that is left over after paying off the agents

$$F \leq R_0^O\left(v(\mathbf{0}), \mathbf{0}\right) + \left(F + W - \sum_{j=1}^{N} t_j(0)\right). \tag{OA.13}$$

   That is, F is a safe debt. Notice then that the principal's payments are only restricted by his initial wealth, W, and the value of the asset under the exchange offer and not by any financial friction.

Armed with these, the next proposition shows the condition under which the holdout problem arises.

Proposition OA.3. *The necessary and sufficient condition for the existence of a cash exchange offer that implements $h = \mathbf{0}$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{OA.14}$$

[3] A more serious discussion of how financing friction impedes takeovers can be found in Burkart et al. (2014)

Moreover, the principal is willing to implement the exchange offer if and only if

$$v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) \geq c. \tag{OA.15}$$

OA.2.2 Optimal non-contingent offers with recourse to externally raised funds

A few things changes when agents have recourse to the borrowed cash. To implement the outcome $h = \mathbf{0}$, the set of necessary conditions becomes

- Tendering is better off than holding out for A_i

$$t_i(0) \geq R_i^O\left(v(e_i) + F + W - \sum_{j=1, j\neq i}^{N} t_j(0), e_i\right), \forall i \in \mathcal{N} \tag{OA.16}$$

- The total payment can be financed by new borrowing and internal wealth

$$\sum_{j=1}^{N} t_j(0) \leq F + W \tag{OA.17}$$

- The principal has enough residual claims to pay off the debt

$$F \leq R_0^O\left(v(\mathbf{0}) + F + W - \sum_{j=1}^{N} t_j(0), \mathbf{0}\right) \tag{OA.18}$$

The main difference is that inside $R_i^O(\cdot, e_i)$, the total amount of assets that can be distributed to A_i has an additional non-negative term $F + W - \sum_{j=1,j\neq i}^{N} t_j(0)$ which strengthen the incentive to holdout.

Proposition OA.4. *Suppose all bilateral contracts are non-decreasing, i.e., the function $R_i^O(\cdot, e_i)$ is non-decreasing for all i. Let $t_i^* = \inf\{t : t \geq R_i^O(v(e_i) + t, e_i)\}$, then a necessary and sufficient condition for the existence of a cash exchange offer that implements $h = 0$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} t_i^*. \tag{OA.19}$$

Moreover, $\sum_{i=1}^{N} t_i^*$ is the minimum cost of all feasible cash transfers when $W \leq \sum_{i=1}^{N} t_i^*$.

Proof. In what follows, we first prove a lemma describing the property of t_i^* defined; then we show the condition is necessary. After that, we show it's also sufficient in two cases depending on the relative magnitude of W and $\sum_{i=1}^{N} t_i^*$.

Lemma OA.1. *If $f(\cdot)$ is a weakly increasing function, then $\inf\{t : t \geq f(x + t)\}$ is weakly increasing in x.*

Proof. We prove it by contradiction. Suppose that the statement is not true, i.e., there exists $x_1 < x_2$ but

$$t_2 := \inf\{t : t \geq f(x_2 + t)\} < t_1 := \inf\{t : t \geq f(x_1 + t)\} \tag{OA.20}$$

By the definition of t_2, for any $\varepsilon > 0$, there exists $t' \leq t_2 + \varepsilon$ such that $t' \in \{t : t \geq f(x_2 + t)\}$. Thus we have

$$t' \geq f(x_2 + t') \geq f(x_1 + t') \tag{OA.21}$$

where the first inequality comes from that $t' \in \{t : t \geq f(x_2 + t)\}$, and the second from the weak monotonicity of f and that $x_2 > x_1$. This implies that $t' \in \{t : t \geq f(x_1 + t)\}$, i.e., $t' \geq t_1$. Since this holds true for any $\varepsilon > 0$, it must be that $t_2 \geq t_1$, contradicting the assumption that $t_2 < t_1$. Thus, it must be true that f is weakly increasing in x. □

We now show that the condition is necessary. Suppose an exchange offer $\{t_i(\cdot)\}_i$ exists and we let $T = \sum_{j=1}^{N} t_j(0)$. By the definition of R_0^O, the break-even condition (OA.18) could be written as

$$F \leq R_0^O \left(v(\mathbf{0}) + F + W - \sum_{j=1}^{N} t_j(0), 0 \right) = v(\mathbf{0}) + F + W - T - 0 \Leftrightarrow W + v(\mathbf{0}) \geq T \tag{OA.22}$$

which says that financing is possible as long as the principal is solvent. This condition already resembles the condition in our proposition, except that we have to put a bound on T.

We can rewrite the individual IC (OA.16) as

$$t_i(0) \geq R_i^O \left(v(e_i) + F + W - T + t_i(0), e_i \right), \forall i \in \mathcal{N} \tag{OA.23}$$

and from the feasibility condition (OA.17), we know the slack $F + W - T$ is weakly positive. And from Lemma OA.1, the lowest possible $t_i(0)$ is increasing in $F + W - T$. (Note, T includes $t_i(0)$ but it doesn't affect the reasoning below.) Therefore, a lower bound of the minimum transfer needed to hold in each existing contract holder is given by $t_i^* = \inf\{t : t \geq R_i^O(v(e_i) + t, e_i)\}$. I.e., any offer must be satisfied $t_i(0) \geq t_i^*$ and *a fortiori* $T \geq \sum_{i=1}^{N} t_i^*$. Thus, we conclude

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} t_i^*. \tag{OA.24}$$

is a necessary condition of the existence of a cash exchange offer with borrowing.

Now we proceed to prove that this condition is also sufficient when $W \leq \sum_{i=1}^{N} t_i^*$ by constructing a cash exchange offer that satisfies all the conditions (OA.16), (OA.17) and (OA.18). Consider the following transfer

$$t_i(h_i) = \begin{cases} t_i^* & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{OA.25}$$

and the principal borrows the minimum $F := \sum_{i=1}^{N} t_i^* - W$ to finance the cash offer . Clearly, the condition (OA.17) is satisfied by the choice of F, and condition (OA.18) is satisfied given the condition in the proposition. We only need to prove the IC (inequality (OA.16)) is satisfied. Plugging in the definition of $\{t_j^*\}_{j \in \mathcal{N}}$ and F, the player i deviate, he would get $R_i^O \left(v(e_i) + F + W - \sum_{j \neq i} t_j^*, e_i \right) = R_i^O(v(e_i) + t_i^*, e_i) \leq t_i^*$ which confirms the IC.

It is easy to see that this is the minimum cost exchange offer as any offer t_i made to agent i must be higher than t_i^* in equilibrium. Thus $\sum_{i=1}^{N} t_i^*$ achieves the lowest possible cost.

When $W > \sum_{i=1}^N t_i^*$, an exchange offer exists if the following system of inequalities has a solution

$$\begin{cases} t_i \geq R_i^O(v(e) + W - T + t_i, e_i), \forall i \in \mathcal{N} \\ T = \sum_{i=1}^N t_i \end{cases} . \tag{OA.26}$$

Let $\Delta = W - T$ and define $t_i^* := \inf\{t : t \geq R_i^O(v(e) + \Delta + t_i, e_i)\}$, the system of the inequalities has a solution if and only if the equation

$$\sum_{i=1}^N t_i^* = W - \Delta \tag{OA.27}$$

has a solution. Notice that the LHS is weakly increasing in Δ by Lemma OA.1 while the RHS is decreasing in Δ. At $\Delta = 0$ the RHS is smaller than the RHS by the case condition $W > \sum_{i=1}^N t_i^*$, and the RHS is zero at $\Delta = W$; Therefore, there must exist an $\Delta^* \in (0, W)$ that solves the equation. We can similarly verify that the transfer

$$t_i(h_i) = \begin{cases} t_i^*(\Delta^*) & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{OA.28}$$

with borrowing $F = 0$ constitute a cash exchange offer, which satisfies all the conditions (OA.16), (OA.17) and (OA.18). The fact that $\Delta^* \in (0, W)$ indicates that the principal will have to pay more than $\sum_{i=1}^N t_i^*$ but not her entire internal wealth W.

□

When the existing contracts have recourse to the assets, then any payment to other agents through borrowing will have a "dilution" effect: if the principal increases the payment to one agent, the RHS of Equation (OA.16) would be lower, reducing the payoff from holding out. Of course, one might suspect that the principal would also need to borrow more to implement the repayment so that F is also higher. But it is never in the principal's interest to do so. In the proof, we show that the optimal non-contingent offer can be described by a fixed-point equation, with the optimal borrowing being just to borrow enough to implement the exchange offers.

Example OA.1 (Debt)**.** *Suppose the existing contracts are debts. Each agent has an outstanding debt D_i.*

Figure OA.1 shows that when the existing contracts are debts, the only possible situation in which a cash exchange is feasible is to pay off the debt of the existing contracts.

Example OA.2 (Equity)**.** *Now suppose every existing contract holder has an equity claim α_i.*

In contrast, as shown in Figure OA.2, the equity holder would have levered equity: he needs to be compensated by more than his share of the asset when he holds out because the ex-ante borrowing increases the value of assets that he has recourse to.

OA.2.3 Optimal non-contingent offers with access to externally raised funds and that debt is not borne by the principal

In this extension, we consider another extension where the debt is imposed on the firm's assets, which affects the existing contract holders' payoff.

To implement the outcome $h = \mathbf{0}$ when the agent has access to the funds raised, the set of necessary conditions is that

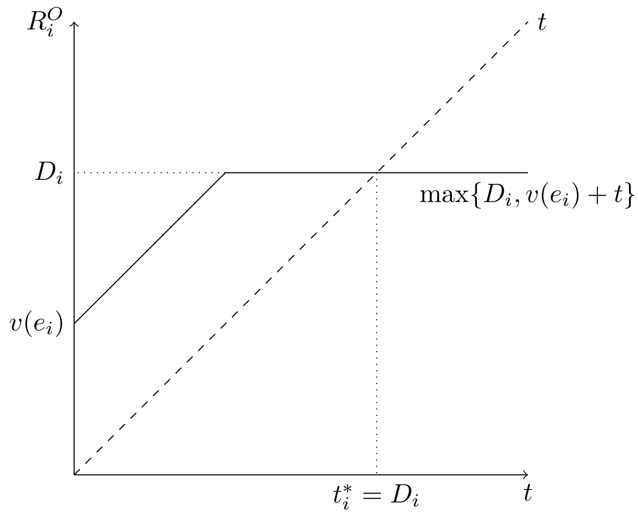

Figure OA.1: Agents with Debts D_i: $R_i^O(v(e_i) + t_i(0), e_i) = \max\{D_i, v(e_i) + t_i(0)\}$

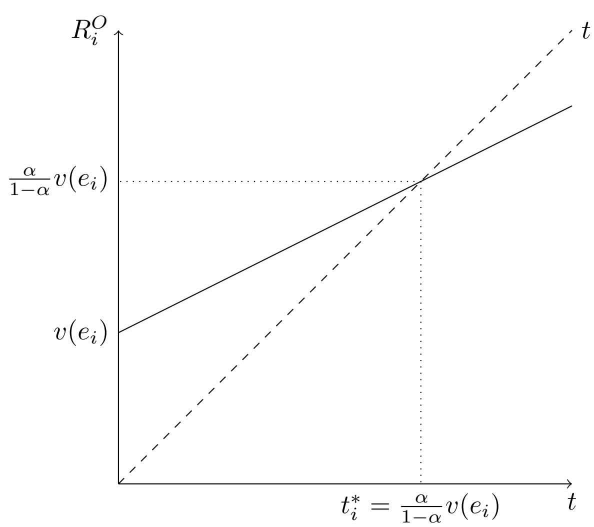

Figure OA.2: Agent with Equity α_i: $R_i^O(v(e_i) + t_i(0), e_i) = \alpha(v(e_i) + t)$

- The total payment can be financed by new borrowing and internal wealth

$$\sum_{j=1}^{N} t_j(0) \leq F + W \tag{OA.29}$$

- Tendering is better off than holding out for A_i

$$t_i(0) \geq R_i^O \left(v(e_i) + W - \sum_{j=1, j \neq i}^{N} t_j(0), e_i \right), \forall i \in \mathcal{N} \tag{OA.30}$$

Since the new debt F is paid ahead of the existing contract holder, it has no effect other than relaxing the intertemporal constraint. So the optimal solution is independent of the choice of F as any excess borrowing would be undone by the repayment.

Proposition OA.5. *The optimal non-contingent offer is given by*

$$t_i(0) = t_i^*(\Delta^*) \tag{OA.31}$$

where $t_i^*(\Delta) := \inf\{t \geq 0 : t \geq R_i^O(v(e_i)+\Delta+t, e_i)\}$ *and* Δ^* *solve the fixed-point equation* $\sum_{i=1}^{N} t_i^*(\Delta) = W - \Delta$.

Proof. Following the same analysis as in the proof of Proposition OA.4, the solution to the optimal non-contingent exchange offers can be described by the system of equations

$$\begin{cases} t_i = R^O(v(e_i) + W - T + t_i, e_i) \quad \forall i \in \mathcal{N} \\ T = \sum_{i=1}^{N} t_i \end{cases} \tag{OA.32}$$

But different from Proposition OA.4, the term $\Delta = W - T$ can be negative here. The solution that minimize T can be described by the fixed-point equation

$$\sum_{i=1}^{N} t_i^*(\Delta) = W - \Delta \tag{OA.33}$$

where $t_i^*(\Delta) = \inf\{t \geq 0 : t \geq R_i^O(v(e_i)+\Delta+t, e_i)\}$. Since the LHS is increasing in Δ and the RHS decreasing, the LHS exceeds the RHS at $\Delta = W$, and a unique solution $\Delta^* < W$ exists. □

OA.3 Proof of the Existence, Uniqueness and Characterization of Credible Contracts

Proof of Propositions 7 and 8. **Overview** To tackle this problem, we decompose the problem into two sub-problems. First, for each h, we assign a number $J(h)$,[4] and define the sets of contracts that are i) IC at each h and ii) allow the principal to guarantee a payoff of at least $J(\hat{h})$ for all $\hat{h}$:

$$\mathcal{C}^\delta(h|J) := \left\{ R \in \mathcal{I}(h) : J(\hat{h}|R) \geq \delta J(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\} \tag{SP1}$$

[4] So, with some abuse of notations, J is both an operator and a vector here.

The set of contract $\mathcal{C}^\delta(\cdot|J)$ is no longer recursively defined, so we can easily see that the set is unique, despite that it might be empty for some values of J. Indeed, we will show in the proof that the $\mathcal{C}^\delta(h|J)$ is non-empty if $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$ for $\delta > 0$.

Second, for any sets of contracts $\mathcal{R}$ available at h, we define an upper bound of the value attainable by the principal using contracts within $\mathcal{R}$ to be

$$J(h|\mathcal{R}) := \sup_{\tilde{R} \in \mathcal{R}(h)} J(h|\tilde{R}) \tag{SP2}$$

We follow the convention and define the supremum to be $-\infty$ if the set $\mathcal{R}(h)$ is empty. The supremum need not be attainable if the contract space $\mathcal{R}$ is not compact or the objective function is not continuous in $\tilde{R}$. But regardless, we have the following

Lemma OA.2 (Fixed Point)**.** *Let J^* be the vector that solves the fixed-point equation*

$$J^*(h) = J(h|\mathcal{C}^\delta(h|J^*)) \quad \forall h \in H, \tag{OA.34}$$

then $\mathcal{C}^\delta(\cdot|J^*)$ satisfies the definition of credible contracts in Definition 6. On the other hand, for any credible contracts $\mathcal{C}^\delta$ defined in Definition 6, whenever it exists, the value function $J(h|\mathcal{C}^\delta)$, as defined in Equation (SP2) *solves the fixed-point equation* (OA.34).

The proof is largely standard: It formalizes the idea that the recursive definition can be characterized by a fixed-point equation. Here, we look at the fixed point of the value function instead of the sets to circumvent technical issues with the mapping between sets of contracts. This approach is very similar to the classic dynamic contracting problems where the dynamic contract problem is reduced to a static one given the continuation value. The main difference is that here, the recursion is over the action space instead of time, so there is no linear order of dependence. Here, the value functions of two different action profiles can mutually depend on each other, which brings up the issue of existence and uniqueness. The next result says it is not a concern.

Proof. I first prove that $\mathcal{C}^\delta(\cdot|J^*)$ satisfies the definition of credible contracts. For any contract $R \in \mathcal{C}^\delta(h|J^*)$, the IC at h is satisfied automatically, so we only need to check that at any deviation node $\hat{h} \in \mathcal{B}(h)$, it dominates any contract $\tilde{R} \in \mathcal{C}^\delta(\hat{h}|J^*)$. From the definition of J^* and thus $\mathcal{C}^\delta(h|J^*)$, we know that for any $R \in \mathcal{C}^\delta(h|J^*)$, we have $J(\hat{h}|R) \geq \delta J^*(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)$ and that

$$J^*(\hat{h}) = J(\hat{h}|\mathcal{C}^\delta(\hat{h}|J^*)) = \sup_{\tilde{R} \in \mathcal{C}^\delta(\hat{h}|J^*)} J(\hat{h}|\tilde{R}). \tag{OA.35}$$

Passing the inequality from the supremum to each contract in $\mathcal{C}^\delta(\hat{h}|J^*)$, we arrive at

$$J(\hat{h}|R) \geq \delta J(\hat{h}|\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}^\delta(\hat{h}|J^*) \quad \forall \hat{h} \in \mathcal{B}(h) \tag{OA.36}$$

which proves that $\mathcal{C}^\delta(\cdot|J^*)$ is a set of credible contracts.

Now I prove the other direction by showing that $J(h|\mathcal{C}^\delta)$ as a vector solves the fixed-point Equation (OA.34), i.e., $J(h|\mathcal{C}^\delta) = J(h|\mathcal{C}^\delta(h|J(h|\mathcal{C}^\delta)))$. For any h, by definition of $\mathcal{C}^\delta$ and $J(\cdot|\cdot)$, we have $J(h|\mathcal{C}^\delta) = \sup_{\tilde{R} \in \mathcal{C}^\delta(h)} J(h|\tilde{R})$ and that $\mathcal{C}^\delta(h) = \left\{ R \in \mathcal{I}(h) : J(\hat{h}|R) \geq \delta J(\hat{h}|\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\}$.

Substitute in the definition of $\mathcal{C}^\delta(\hat{h})$ and passing inequality to the supremum, we can write

$$\mathcal{C}^\delta(h) = \left\{ R \in \mathcal{I}(h) : J(\hat{h}|R) \geq \delta J(\hat{h}|\mathcal{C}^\delta) \quad \forall \hat{h} \in \mathcal{B}(h) \right\} = \mathcal{C}^\delta(h|J(h|\mathcal{C}^\delta)). \tag{OA.37}$$

To see this, suppose instead $\exists \hat{h} : J(\hat{h}|R) < \delta J(\hat{h}|\mathcal{C}^\delta)$, then by definition of sup, there exists a $\tilde{R}$ such that $J(\hat{h}|R) < \delta J(\hat{h}|\tilde{R})$, contradicting the definition of $\mathcal{C}^\delta(h)$. Finally, applying the J operator on the identity $\mathcal{C}^\delta(h) = \mathcal{C}^\delta(h|J(h|\mathcal{C}^\delta))$, we get $J(h|\mathcal{C}^\delta) = J(h|\mathcal{C}^\delta(h|J(h|\mathcal{C}^\delta)))$. Thus, we established the equivalence of the recursive Definition 6 and the fixed-point characterization (OA.34). □

The key step in the proof is that the constraint the credibility puts is asymmetric for agents who deviate from holdout to tendering and for those who deviate from tendering to holdout. In the former case, to deter tendering, we must reduce the payoff from tendering for the deviating agent. This can be easily achieved by reducing his payoff from tendering to 0. Doing so would not affect the credibility constraint as it weakly reduces the total payoffs to all agents under the 1-Lipschitz condition, which is weakly beneficial for the principal. However, to discourage an agent from holding out, the principal must try to minimize his payoff off-path. However, there is a limit to what the principal can achieve by imposing externalities on him. In other words, the principal can only punish deviating agents by granting higher payoff to other tendering agents, but doing so would weakly lower the principal's payoff. There will be no renegotiation as long as it's still below the principal's value function at the deviation profile. So, the maximum punishment the principal can credibly impose on the deviator on the deviation node is the one that makes her payoff equivalent to her value function at the deviation node.

This asymmetry in constraints reveals an asymmetric inter-dependence of the value functions that the value of $J(h|\mathcal{C}^\delta(h|J))$ only depends on the values of $J(\hat{h}|J)$ for the profiles $\hat{h}$ where there are more deviating agents than h, i.e., $\xi(\hat{h}) \subset \xi(h)$. Thus, we can prove the existence by constructing a vector J^* that solves the fixed-point equation (OA.34) in finite steps. We start from an arbitrary vector J^0 in the feasible space (specified in the proof) and calculate the value function on the action profile $\mathbf{1}$ on which everyone holds out. It turns out that, as expected, the value function $J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J^0))$ is independent of the choice of J^0. Then we replace the value of $J(\mathbf{1})$ by $J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J^0))$, and use that vector, renamed J^1, for the next iteration, i.e., calculating the value function $J(h|\mathcal{C}^\delta(h|J^1))$ on the action profiles where exactly one agent tenders. Again, it turns out the value function is independent of the initial choice J^0: it only depends on the value $J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J^0))$. We update the vector and continue the process by calculating the value functions on all the profiles where one more agent tenders. This process ends after we calculate the value function on the node $\mathbf{0}$ on which everyone tenders and set the vector J^{N+1} to be J^*. Finally, we conclude that the vector found J^* is indeed the solution to the fixed-point equation by noticing $J^*(h) = J(h|\mathcal{C}^\delta(h|J^{k+1})) = J(h|\mathcal{C}^\delta(h|J^*))$ for any h such that $|\xi(h)| = k$.

The uniqueness can be obtained by noticing that in the construction above, the fixed point found is independent of the choice of the initial J^0. In the proof, I give a more formal proof by contradiction, showing that there's no other solution than the one found using the procedure above.

Solving for fixed-point To show a fixed point J^* exists and is unique, I first prove that the set $\mathcal{C}^\delta(h|J)$ is non-empty for all $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$; then I display the asymmetry mentioned by solving the problem SP2 over the sets $\mathcal{C}^\delta(h|J)$ for any vector $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, i.e., I want to calculate $J(h|\mathcal{C}^\delta(h|J))$.

Non-emptiness of $\mathcal{C}^\delta(h|J)$ I first show that $\mathcal{C}^\delta(h|J)$ is non-empty for any $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$. To do so, I only need to give one example of a contract, and an natural one would be this "no-additional-punishment contract".

- At h, the principal pays to whoever holds out 0 in the new contracts (note the holdout will not have new contracts), and to whoever tenders what he would otherwise obtain if he were to hold out and no punishement is imposed, i.e., $R_i(v(h), h) = R_i^O(v(h + e_i), h)\mathbb{1}_{i\in\xi(h)}$. The IC, the first constraint in the definition (Equation SP1), is clearly satisfied.

- At $\hat{h} \in \mathcal{B}(h)$, the principal pays nothing to the tendering agents and any arbitrary amount, e.g., 0, to those who hold out in the new contract, which they don't accept. Then the total payout to all agents is $0 + \hat{h} \cdot R^O(v(\hat{h}) - 0, \hat{h})$ which is no larger than $v(\hat{h}) - \delta J(\hat{h})$ by the range of the value of $J(\hat{h})$. So the second constraint in the definition (Equation SP1) is also satisfied.

- It takes any arbitrary values on any other action profiles.

Since at least one contract exists in $\mathcal{C}^\delta(h|J)$ when $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, it's non-empty.

Now, I prove another auxiliary lemma that would be used in the main proof.

Lemma OA.3. *Let $f(\cdot)$ and $g(\cdot)$ be two weakly increasing 1-Lipschitz functions and so is their sum. Given three constants $a \geq b \geq 0$ and $c > 0$, the solution to the problem*

$$\inf_{x\in[b,a]} g(a - x) \quad \textit{subject to} \quad g(a - x) + f(a - x) + x \leq c \tag{OA.38}$$

exists if and only of $f(a - b) + g(a - b) \leq c$ and one such solution is given by

$$\bar{x} := \max\{x \in [b, a] : g(a - x) + f(a - x) + x = c\}. \tag{OA.39}$$

Proof. Invoking Lemma 1, the fact that $f(\cdot) + g(\cdot)$ is 1-Lipschitz implies that $g(a - x) + f(a - x) + x$ is a weakly increasing function and its minimum can always be attained at $x = b$, so the feasible set is non-empty if and only if $f(a - b) + g(a - b) \leq c$. Moreover, the continuity of $f(\cdot)$ and $g(\cdot)$ also implies the feasible set $\{x \in [b, a], g(a - x) + f(a - x) + x \leq b\}$ is compact so the infimum can be attained whenever it is non-empty. Since $g(a - x)$ is a weakly decreasing function of x, its minimum can be achieved at the largest x in which the constraint is satisfied. Since $g(a - x) + f(a - x) + x$ is a weakly increasing, an obvious one is simply $\bar{x} = \max\{x \in [b, a] : g(a - x) + f(a - x) + x = c\}$. □

Asymmetry in ICs We want to show the value of the vector J only affects the credibility constraints at the deviation node $\hat{h}$, which in turn affects the IC constraint through the off-path threat $u_i(1 - h_i|h_{-i}, R)$. To be more specific, let's say, at h, the agent A_j deviates to $\hat{h} = (h_{-j}, 1 - h_j)$, which includes two cases:

- Agent A_j deviates from 1 to 0, i.e., $h_j = 1$ and $\hat{h}_j = 0$: This is the easy case as P only needs to make a zero offer to A_j. The IC to make sure agent A_j holds out is

$$u_j(h_j = 1|h_{-j}, R) \equiv R_j^O\left(v(h) - \sum_{i\in\xi(h)} R_i(v(h), h), h\right) \geq R_j(v(\hat{h}), \hat{h}). \tag{OA.40}$$

We can set the RHS to 0 but we need to check that the credibility constraint at $\hat{h}$ will not be vioalted. The credibility constraint can be written as

$$x(\hat{h}) + \sum_{i \notin \xi(\hat{h})} R_i^O \left(v(\hat{h}) - x(\hat{h}), \hat{h} \right) \leq v(\hat{h}) - \delta J(\hat{h}) \tag{OA.41}$$

where $x(\hat{h}) = \sum_{k \in \xi(h)} R_k(v(\hat{h}), \hat{h}) + R_j(v(\hat{h})$. Note here I used the fact that $\xi(\hat{h}) = \xi(h) \coprod \{j\}$ and consequently $\{j\} \coprod \xi(\hat{h})^c \coprod \xi(h) = \mathcal{N}$, which allows me to write the total payoff on the left-hand side to all agents in three parts. Under the 1-Lipschitz conditon Assumption A2, the minimum of the LHS is achieved by setting $x(\hat{h}) = 0$[5] and the credibility constraint is satisfied as long as $J(\hat{h}) \leq \delta^{-1}(v(\hat{h}) - \delta \hat{h} \cdot R^O(v(\hat{h}), \hat{h})$.

- Agent A_j deviates from 0 to 1, i.e., $h_j = 0$ and $\hat{h}_j = 1$. The on-path IC for agent j is

$$u_j(h_j = 0 | h_{-j}, R) \equiv R_j(v(h), h) \geq R_j^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right) \tag{OA.42}$$

Again, the problem can be relaxed if we can make $R_j^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right)$ smaller, if unimpeded by the credibility constraint at $\hat{h}$, which now is

$$\sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}) + R_j^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right) \tag{OA.43}$$

$$+ \sum_{k \notin \xi(h)} R_k^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right) \leq v(\hat{h}) - \delta J(\hat{h}) \tag{OA.44}$$

having used the fact that $\{j\} \coprod \xi(\hat{h}) \coprod \xi(h)^c = \mathcal{N}$. And again, the left-hand side could be minimized by setting $\sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h})$ to zero without affecting other constraints under the 1-Lipschitz condition using Lemma 1. So the condition for the existence of the solution is again $\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h})$. However, setting $\sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h})$ to zero,[6] despite of minimizing the total payoff to $\{j\} \coprod \xi(h)^c$, doesn't necessarily minimize $R_j^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right)$ as the value of it is $R_j^O(v(\hat{h}), \hat{h})$ instead of zero. I could further increase the value of $\sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h})$, the value of the LHS might also increase until the constraint is binding, *without additional constraints.* Using Lemma OA.3, we know that $R_j^O \left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h} \right)$ is minimized at

$$\bar{x}^\delta(J(\hat{h}); \hat{h}) := \max \left\{ x \in [0, v(\hat{h})] : \hat{h} \cdot R^O \left(v(\hat{h}) - x, \hat{h} \right) + x = v(\hat{h}) - \delta J(\hat{h}) \right\}, \tag{OA.45}$$

using the fact $\{j\} \coprod \xi(h)^c = \xi(\hat{h})^c$. Note the solution exists because $0 + \hat{h} \cdot R^O \left(v(\hat{h}), \hat{h} \right) = v(\hat{h}) - J(\hat{h}) \leq v(\hat{v}) - \delta J(\hat{h}) \leq v(\hat{h}) + \hat{h} \cdot R^O \left(0, \hat{h} \right)$ and the LHS is a continuous function of x . The maximum is attainable because the zeros of a Lipschitz function on a closed interval constitute a compact set.

[5]Note R need not be IC at $\hat{h}$ as it is a deviation profile.

[6]Note, I do not require R to be IC at $\hat{h}$ so it can be set to 0. The alternative contract $\tilde{R}$ that can be proposed needs to be IC, but it's captured in the $J(\hat{h})$.

Summary of existence and uniqueness In summary, the condition for a credible to exist is that for any deviation $\hat{h} \in \mathcal{B}(h)$, the highest value $J(\hat{h})$ that can be alternatively obtained using a credible contract at $\hat{h}$ is smaller than the difference between the asset value $v(\hat{h})$ and the collective holdout payout $\hat{h} \cdot R^O(v(\hat{h}), \hat{h})$, i.e.,

$$\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h). \tag{OA.46}$$

Moreover, the analysis above shows that, for any $J \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$, the value $J(h|\mathcal{C}^\delta(h|J))$ depends only on $J(h + e_i)$ for some $i \in \xi(h)$, and recursively on any $h' \geq h$. On the contrary, the value $J(h - e_j)$ for some $j \notin \xi(h)$ does not affect the value of $J(h|\mathcal{C}^\delta(h|J))$ and recursively so does any h' not in the upper contour set of h: $\{h' : h' \geq h\}$.

Construction of the fixed point: The discussion above allows us to calculate the J^* via the following procedure for $\delta > 0$. In particular, we want to emphasize that we are not calculating $J(h|\mathcal{C}^\delta(h|J))$ for a specific J.

1. First we decompose $H = \{0, 1\}^N$ into $N + 1$ disjoint sets $H^k = \{h : \xi(h) = k\}$ for $k = 0, ..., N$ on which exactly k agents tender.

2. We calculate the $J(h|\mathcal{C}^\delta(h|J^0))$ on $H^0 = \{\mathbf{1}\}$ for any fixed $J^0 \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$. At $h = \mathbf{1}$, none of the credibility constraints matter since all hold out, and the ICs are simply the non-negativity constraints $R_i^O(1|\mathbf{1}_{-i}, \mathbf{1}) = R_i^O(v(\mathbf{1}), \mathbf{1}) \geq 0 \quad \forall i \in \mathcal{N}$. So we can calculate the value function
$$J^*(\mathbf{1}) := J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J^0)) = v(\mathbf{1}) - \mathbf{1} \cdot R^O(v(\mathbf{1}, \mathbf{1})) \tag{OA.47}$$
and the maximum credible punishment at $\mathbf{1}$
$$\bar{x}^\delta(\mathbf{1}) := \bar{x}^\delta(J^*(\mathbf{1}), \mathbf{1}) = v(\mathbf{1}) - \delta J^*(\mathbf{1}) = \delta \mathbf{1} \cdot R^O(v(\mathbf{1}), \mathbf{1}) + (1 - \delta) v(\mathbf{1}). \tag{OA.48}$$
Then, we update our J^0 to J^1 as follows
$$J^1(h) = \begin{cases} J^0(h) & \text{if } h \notin H^0 \\ J^*(h) & \text{if } h \in H^0 \end{cases} \tag{OA.49}$$

$k+3$. Now we carry out the calculation by induction: Suppose $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ are defined on all H^κ for $\kappa = 0, 1, \ldots, k$, and J^{k+1} is also defined. We solve for $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ defined on all H^{k+1} by solving for $J(h|\mathcal{C}^\delta(h|J^{k+1}))$ and update J^{k+1} to J^{k+2}. For any $h \in H^{k+1}$, the relevant IC constraints are
$$R_i(v(h), h) \geq R_i^O(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i) \quad \forall i \in \xi(h). \tag{OA.50}$$
The RHS is known as $h + e_i \in H^k$. We re-iterate the principal's problem following the simplification above:
$$\max_R v(h) - \sum_{i \in \xi(h)} R_i(v(h), h) - \sum_{j \notin \xi(h)} R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h\right). \tag{OA.51}$$
Again, under the assumption that $h \cdot R^O(\cdot, h)$ is 1-Lipschitz, the objective is weakly decreasing in each

on-path payoff $R_j(v(h), h)$ for each $j \in \xi(h)$. And we have

$$J^*(h) \equiv J(h|\mathcal{C}^\delta(h|J^{k+1})) = v(h) - \sum_{i\in\xi(h)} R_i^O\left(v(h+e_i) - \bar{x}^\delta(h+e_i), h+e_i\right) - \sum_{j\notin\xi(h)} R_j^O\left(v(h) - \sum_{i\in\xi(h)} R_i^O\left(v(h+e_i) - \bar{x}^\delta(h+e_i), h+e_i\right), h\right). \quad \text{(OA.52)}$$

To calculate the maximum credible punishment at h, we find the largest solution to the equation $h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)$. By Lemma 1, the maximum solution exists and is unique, and we calculate

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)\}. \quad \text{(OA.53)}$$

We also update J^{k+1} to J^{k+2} as follows

$$J^{k+2}(h) = \begin{cases} J^{k+1}(h) & \text{if } h \notin H^{k+1} \\ J^*(h) & \text{if } h \in H^{k+1} \end{cases} \quad \text{(OA.54)}$$

$N+3$. Finally, after calculating J^* for $k = N-1$, we obtain $J^* = J^{N+1}$, and we need to verify that it satisfies $J^*(h) = J(h|\mathcal{C}^\delta(h|J^*))$. This could be easily done by observing that $J^*(h) = J(h|\mathcal{C}^\delta(h|J^{k+1})) = J(h|\mathcal{C}^\delta(h|J^*))$ for any $h \in H^k$ for $k = 0, 1, ..., N-1$.

Finally, the uniqueness should be obvious from the iteration. Notice that the J^* we calculated in the procedure is independent of the initial choice J^0. But the readers may wonder if there's a fixed point not found through the procedures above. To alleviate this concern, suppose there exist two fixed-points J and $\tilde{J}$ such that $J \neq \tilde{J}$ and $J(h) = J(h|\mathcal{C}^\delta(h|J))$ (resp. $\tilde{J}(h) = J(h|\mathcal{C}^\delta(h|\tilde{J}))$) for any h. Since $J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J))$ doesn't depend on J, it must be that $J(\mathbf{1}) = J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|J)) = J(\mathbf{1}|\mathcal{C}^\delta(\mathbf{1}|\tilde{J})) = \tilde{J}(\mathbf{1})$. Then there must be an h such that $J(h) \neq \tilde{J}(h)$. Let $\underline{k} = \min\{k \geq 1 : \exists h \in H^k : J(h) \neq \tilde{J}(h)\}$. Then on all the action profiles $h \in H^{\underline{k}-1}$, $J(h) = \tilde{J}(h)$, then we would have for all $h \in H^{\underline{k}}$, $J(h) = J(h|\mathcal{C}^\delta(h|J)) = J(h|\mathcal{C}^\delta(h|\tilde{J})) = \tilde{J}(h)$, contradicting the definition of $\underline{k}$. Thus, the fixed-point equation (OA.34) has a unique solution.

□

OA.4 Proof of the Solution to the General Credible Contracts

Proof of Proposition 9. To prove this result, we need to show that i) the initial condition is satisfied and that ii) Equation (18) is satisfied when we plug Equation (19) in. The initial condition is very easy to verify: at $\mathbf{1}$, there are no tendering agents, so the RHS is nonexistent.

Before plugging, we want to state several basic facts about the set of permutations. By definition, $\xi(h) = \xi(h+e_i) \cup \{i\}$, and thus $|\xi(h+e_i)| = |\xi(h)| - 1$. Moreover, consider two sets of permutations $\Sigma(\xi(h+e_i))$ and $\Sigma(\xi(h))$. It's easy to see that $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))|$ but conditional on the kth element being i, the subset $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ is isomorphic to $\Sigma(\xi(h+e_i))$. Moreover, the disjoint union of them is isomorphic to $\Sigma(h)$. That is,

$$\coprod_{i\in\xi(h)} \Sigma(\xi(h+e_i)) \cong \coprod_{i\in\xi(h)} \{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\} \cong \Sigma(\xi(h)) \quad \forall\ k = 1, ..., |\xi(h)| \tag{OA.55}$$

Now we plug the solution in Equation (19) into the recursive (18), the right hand side of (18) is $(1-\delta)v(h) + \delta \sum_{i\in\xi(h)} \alpha_i(v(h+e_i) - \bar{x}(h+e_i))$. The second term is

$$\begin{aligned}
&\delta \sum_{i\in\xi(h)} \alpha_i(v(h+e_i) - \bar{x}(h+e_i)) \\
=&\delta \sum_{i\in\xi(h)} \alpha_i \left(\delta v(h+e_i) - \sum_{k=1}^{|\xi(h+e_i)|} \frac{(-\delta)^{k+1}}{(|\xi(h+e_i)|-k)!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h+e_i+\sum_{s=1}^{k} e_{\sigma(s)}\right)\right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h+e_{\sigma(1)}\right) \\
&+ \sum_{i\in\xi(h)} \sum_{k'=2}^{|\xi(h)|} \frac{(-\delta)^{k'+1}}{(|\xi(h)|-k')!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\alpha_i \prod_{s=1}^{k'-1} \alpha_{\sigma(s)}\right) v\left(h+e_i+\sum_{s=1}^{k'-1} e_{\sigma(s)}\right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h+e_{\sigma(1)}\right) \\
&+ \sum_{k=2}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{i\in\xi(h)} \sum_{\sigma\in\Sigma(\xi(h)):\sigma(k)=i} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h+\sum_{s=1}^{k} e_{\sigma(s)}\right) \\
=&\sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{\sigma\in\Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h+\sum_{s=1}^{k} e_{\sigma(s)}\right)
\end{aligned}$$

where the first quality is the result with $\bar{x}(h+e_i)$ directly plugged in; the second equality is the separation of the first term and the rest, with the replacement $k' = k+1$. Note, we have the $\frac{1}{(|\xi(h)|-1)!}$ term because $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))| = |\xi(h)| \cdot (\xi(h)-1)!$ so the term is used to offset the repetitive counting. In the third equality, we switch the indicator to k, and change the order of the summation using the isomorphism between $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ and $\Sigma(\xi(h+e_i))$. The last line combines the two parts, using the isomorphism in equation (OA.55).

At $h = \mathbf{0}$, the maximum punishment is also $\bar{x}^{\delta}(\mathbf{0}) = v(\mathbf{0}) - \delta J(\mathbf{0})$. Substituting it into $J(\mathbf{0}) = \delta^{-1}(v(\mathbf{0}) - \bar{x}^{\delta}(\mathbf{0}))$ and using $\xi(\mathbf{0}) = \mathcal{N}$ and $|\xi(\mathbf{0})| = N$, we arrive at the expression in the proposition. □

OA.5 Credible contracts with debts

Now, suppose the existing securities are debts. Each agent A_i holds a debt contract with face value D_i. For simplicity, I use the vector $D = \{D_i\}_i$ to denote the profile of existing securities. Given a profile h, the total outstanding debt (not including the potentially newly issued) is given by the inner product $D \cdot h$. Applying the general formulation of the recursive relation of the maximum credible punishment, we can write it for the debt case as follows.

Lemma OA.4. *For debt contracts $D = \{D_i\}_i$, the maximum credible punishment on the profile $h \neq \mathbf{1}$ is* *given by the recursive relation*

$$\bar{x}^{\delta}(h) = \begin{cases} v(h) & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ (1-\delta)(v(h) - D \cdot h) + \delta \underline{x}(h) & \text{otherwise} \end{cases} \tag{OA.56}$$

with the initial condition $\bar{x}^{\delta}(\mathbf{1}) = 0$ *where*

$$\underline{x}(h) := \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h + e_i)} \left[v(h+e_i) - \bar{x}^{\delta}(h+e_i) \right], D_i \right\} \tag{OA.57}$$

is the sum of the minimal payments to hold in the tendering agents.

This lemma says that when the asset value is low enough, the principal can credibly punish the holdouts by giving all the asset value to the tendering agents since she will not be paid anyway. But, if the asset value is high, then such punishment hurts the principal, and there's a limit on the punishment, which consists of two parts: i) the value exceeding the full payment to the holdouts $v(h) - D \cdot h$ lost due to discounting; ii) the discounted payment to each tendering agents if he were to hold out. The second case is similar to the situation where outstanding securities are equities, but the complication comes from the fact that there are two cases on each profile h depending on the magnitude of the asset value versus the total outstanding debt of the holdouts with an additional recursive component for payment to tendering agents. The formulation that the holdouts' debt is subtracted from the asset value seems to suggest that all holdouts would be paid in full in the second case, and it is indeed true:

Lemma OA.5. *Each holdout* $i \notin \xi(h)$ *is either paid nothing or in full at any* $h \neq \mathbf{1}$*. More specifically, the value that can be distributed to the holdouts and the principal herself is*

$$v(h) - \bar{x}^{\delta}(h) \begin{cases} = 0 & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ > D \cdot h & \text{otherwise,} \end{cases} \tag{OA.58}$$

where $\underline{x}(h)$ *is defined in Lemma OA.4.*

Thus, either no holdouts are paid anything, or all of them are paid in full. This allows us to describe the recursive relation using an indicator variable:

Lemma OA.6. *Let* $\boldsymbol{\eta} = \{\eta(h)\}_h \in \{0,1\}^{2^N}$ *be a vector of indicator functions such that* $\eta(h) = 1$ *if and only if* $\delta > 0$ *and* $v(h) - \bar{x}^{\delta}(h) \geq D \cdot h$*. Then the recursive relation in Lemma OA.4 can be described as*

$$\eta(h) = \begin{cases} 0 & \text{if } \delta = 0 \\ \mathbb{1}_{\{v(h) \geq D \cdot h\}} & \text{if } \delta \neq 0 \text{ and } h = \mathbf{1} \\ \mathbb{1}_{\left\{v(h) > D \cdot h + \sum_{i \in \xi(h)} D_i \eta(h+e_i)\right\}} & \text{otherwise} \end{cases}. \tag{OA.59}$$

Whenever $\eta(h) = 1$, the asset value is more than enough to pay off every creditor, tendering or not; And the principal gets paid something instead of nothing, so she cannot credibly punish the holdouts by diverting asset value to the tendering creditors since any diversion hurts herself. Otherwise, when $\eta(h) = 0$, the principal doesn't get paid anything, and the punishment is credible.

An immediate implication of this result is that the level of commitment δ is almost irrelevant to the success of the exchange offer

Proposition OA.6 (Almost Irrelevance and Discontinuity of Commitment)**.** *The vector $\boldsymbol{\eta}$ that solves the recursive relation in Lemma OA.6 is independent of δ for any $\delta > 0$ and $\eta(h) = 0$ for all h if $\delta = 0$. Given the solution $\boldsymbol{\eta}$, the value of the principal is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} D_i \eta(e_i) \tag{OA.60}$$

This result, together with the lemma above, reveals a discontinuity at $\delta = 0$: Almost full commitment is very different from full commitment, but the level of the commitment between 0 and 1 affects neither the resolution of the holdout problem nor the principal's value.

This discontinuity in the value function is a result of the discontinuity of the maximum punishment due to the flat region in the payment function illustrated in Figure OA.3. The RHS of the equation for the maximum punishment is $v(h) - \delta J(h)$ is plotted using the dashed line. The LHS, the total payment to all creditors, as a function of the punishment when the holdout has a debt D is $x + \min\{v - x, D\}$ and is displayed using a solid black line. When the discount rate is $\delta > 0$, the dashed line (in red) is always below the flat region of the payment function, so the maximum punishment is the intersection point $v(h) - D - \delta J(h)$. But when $\delta = 0$, the dashed line (in blue) overlaps with the flat region of the payment function, and the maximum punishment jumps from $v(h) - D$ to v.

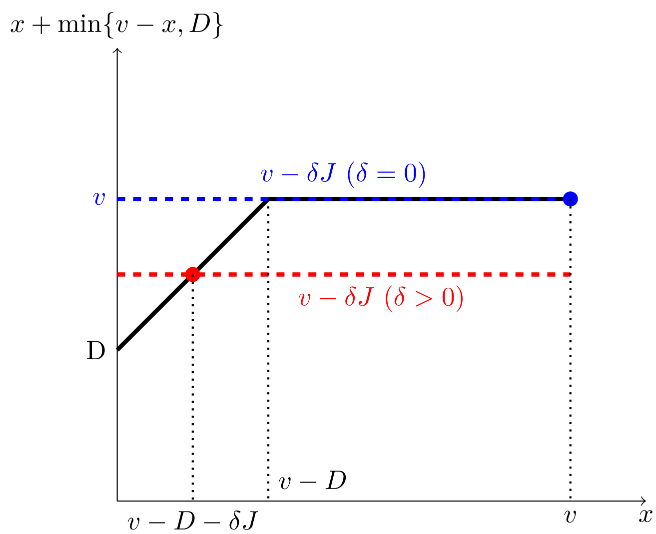

Figure OA.3: Discontinuity of Punishment

Moreover, it is also different from strong credibility. Under strong credibility, the principal needs to pay D_i on path to A_i if $v(e_i) > D_i$ but under $\delta-$credibility, she does so if $v(e_i) > D_i + \sum_{j \neq i} D_j \eta(e_i + e_j)$. I illustrate this point using the three-agent example below.

A deeper characterization of the relationship between strong δ-credibility and δ-credibility is provided in Section OA.8

Numerical Example: Three-Agent Case with Debts The principal has 3 creditors A_i for $i \in \{1, 2, 3\}$ and A_i has outstanding debt $D_i = 10i$. And the value of the asset depends on the number of holdouts $v(h) = 40 + 5h^\top \mathbf{1}$. I.e., the value is 55 (resp. $50, 45, 40$) when 0 (resp. $1, 2, 3$) agents hold out.

Under full commitment, the principal can extract full surplus as per Proposition 2, so the principal's value is $J(\mathbf{0}) = v(\mathbf{0}) = 55$.

Under strong δ-credibility for $\delta > 0$, since $v(e_i) > D_i$ for all i, the principal has to repay everyone in full. His value is $55 - 10 - 20 - 30 = -5$. So, he might not even initiate the restructuring. He will only do so if $v(\mathbf{0}) > 60$.

Under δ-credibility for $\delta > 0$, I calculate the η function using backward induction. $\eta(\mathbf{1}) = 0$ as $v(\mathbf{1}) = 40 < D \cdot \mathbf{1} = 60$. At $h = e_1 + e_3$ (resp. $h = e_2 + e_3$), the asset value 45 is larger than the total outstanding debt 40 (resp. 30), so $\eta(e_1 + e_3) = \eta(e_2 + e_3) = 1$. Now I calculate $\eta(e_3)$:

$$v(e_3) = 50 < D_3 + \sum_{j=1,2} D_j \eta(e_3 + e_j) = D_1 + D_2 + D_3 = 60. \tag{OA.61}$$

So, by definition, $\eta(e_3) = 0$. Similarly one can get $\eta(e_2) = 0$ and $\eta(e_1) = 1$. Thus, the principal's value is $v(\mathbf{0}) - D_1 = 45$.

OA.6 Greater Protection Facilities Restructuring: A Negative Example

I first construct an example showing that a higher property right protection could increase the principal's value, facilitating restructuring. Let there be 3 agents, each with a property value π_i and a claim that resembles a "kinked equity" (or debt if $\beta_i = 0$)

$$R_i^O(v, h) = \alpha_i v + (\beta_i - \alpha_i)(v - \hat{v}_i)\mathbb{1}_{v \geq \hat{v}_i} \quad \forall h : i \notin \xi(h) \tag{OA.62}$$

for some parameters $\{\alpha_i, \beta_i, \pi_i, \hat{v}_i\}_i$. I find a set of parameters such that greater property rights protection facilitates restructuring in the next proposition.

Example OA.1 (Example: Property Rights Facilitates Restructuring)**.** *There exists a set of initial contracts such that a locally small increase in property rights protection facilitates restructuring. In particular, let* $\hat{v}_1 = \hat{v}_3 = 1, \hat{v}_2 = 98/100$*,* $\pi_1 = \pi_2 = 1/100$ *and* $\pi_3 = 99/100$*,* $\alpha_2 = 7/10, \alpha_1 = \alpha_3 = 1/10$*,* $\beta_1 = \beta_2 = 1/10, \beta_3 = 7/10$*. Let* $v(\cdot)$ *be such that* $v(\mathbf{1}) = 0, v(\mathbf{0}) = 3, v(e_i) = 2, v(\mathbf{1} - e_i) = 1$ *for all* i*. The principal's value function* $J(\mathbf{0})$ *is increasing in* π_1 *at the parameters specified above.*

This example shows how property rights protection could facilitate the restructuring by giving the principal less bargaining power in renegotiation and, thus, more commitment to the punishment. The protection still undermines the principal's bargaining power initially, so the compensation off-path must exceed this direct effect. For this to be the case, the structure has to be made asymmetric as it's restricted by the 1-Lipschitz continuity. In my example, R_2^O (resp. R_3^O) has a large dilution sensitivity when the asset value accrues to the holdout is small (resp. large). When A_2 holds out, since π_3 is large, a one-dollar increase in π_1 would reduce A_2's payoff by α_2, i.e., the dilution sensitivity evaluated at point A. This is multiplied by a discount factor $1 - \alpha_3$, reflecting the renegotiation when A_3 also holds out. Similarly, when A_3 holds out, a one-dollar increase in π_1 would reduce A_3's payoff by $\beta_3(1 - \beta_2)$ since it is evaluated at the point B.

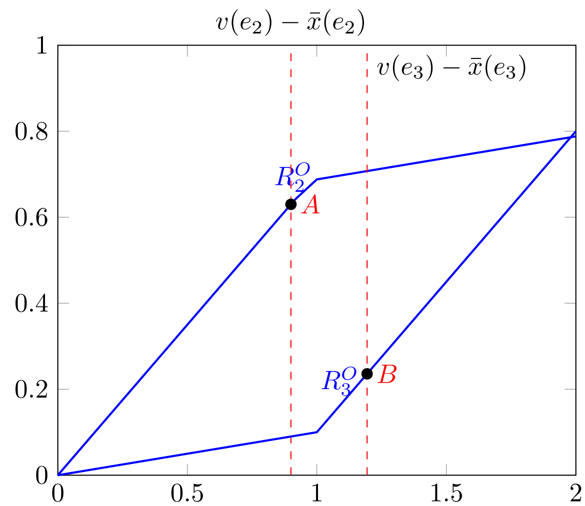

Figure OA.4: Example of Property Rights Facilitating Restructuring: Initial Contracts with Kinks

Despite the quirky example shown above, when the existing securities are the more commonly seen contracts, such as debts or equities, a locally small increase in property rights protection usually exacerbates the holdout problem even in the limited commitment case. Indeed, for equity holdouts, this is even true for large increases in property rights; whereas for debt holdouts, it can be reversed.

OA.7 Mapping to Classics

The framework advanced here incorporates many classic papers in the literature. Specifically, I show how to map these papers onto my framework using the functions $v(\cdot)$, $R_i(\cdot,\cdot)$ and $R_i^O(\cdot,\cdot)$.[7] The results in these papers are shown to be special cases of my model (mostly under full commitment).

OA.7.1 Takeover via Public Tender Offer à la Grossman and Hart (1980).

They model the situation where a raider (the principal) can improve the value of a firm after acquiring a controlling stake in the firm through a public tender offer, i.e., by offering a price to each shareholder to purchase his shares. The firm value is v_0 if the takeover fails, and $v_0 + \Delta v$ if it succeeds, which occurs when more than a fraction of $\bar{h}$ of shares are tendered, i.e., $h^\top \mathbf{1} \geq \bar{h}$ and if the raider pays a private cost c. The raider is plagued with the holdout problem as the shareholder who does not tender benefits from the value improvement once the firm is acquired and thus will demand a price equal to the post-takeover value, leaving little-to-no surplus to the raider.

In my framework, the value creation function takes the form of a step function $v(h) = v_0 + \Delta v \mathbb{1}_{h^\top 1 < \bar{h}}$. Each existing contract has a payoff $R_i^O(v(h), h) = \frac{v(h)-d}{N}$ where d is the dilution factor considered in Grossman and Hart (1980), the value of the asset that the raider can extract after raider obtains control.

The cash offer t_i *unconditional* on getting control would be a flat payoff function which only depends on the action of A_i: $R_i(v(h), h) = t_i \mathbb{1}_{h_i=0}$. For any agent A_i, he decides not to hold out if $R_i(v(\mathbf{0}), \mathbf{0}) \geq R_i^O(v(e_i), e_i)$

[7] Most papers here have a continuum of agents for computational tractability. I show their finite-agent counterpart using my notations as well as their continuous limit, if possible. The continuous limit is not immediately obtained by taking the limit $N \to \infty$ because the asset value v depends on the exact action of each contract holder, and each agent has a claim on it. To circumvent this, I normalize the contract of each agent by N whenever possible, e.g., with debt or equity claims.

which implies for an offer to be incentive compatible, it must exceed the outside option $t_i \geq t_i^* := \frac{v(e_i)-d}{N} = R_i^O(v(e_i), e_i)$ and the principal implements this action with cost c if and only if $v(\mathbf{0}) - \sum_{i=1}^N t_i^* \geq c$. The condition cannot hold when there's no dilution, and the cost is positive, which is the holdout problem they identified.

OA.7.2 Bond Buyback Boondoggle à la Bulow et al. (1988) and Leverage Ratchet Effect à la Admati et al. (2018).

In this example, I illustrate the common friction that underlies the bond buyback boondoggle analyzed in Bulow et al. (1988), and a more recent leverage ratchet effect illustrated in the dynamic model in Admati et al. (2018) with modified notations. In both models, the firm offers cash to buy back existing debts held by external creditors, but creditors who do not sell their debt also benefit from the deleveraging and are not willing to sell unless they are compensated at the post-buyback price. Another manifestation of the holdout problem!

The debtor (principal) has a project that generates a random payoff X following a distribution F, independent of the outstanding debts. There are N creditors (agents): Each owns a debt contract with face value $\frac{D}{N}$. All the debts are of the same seniority. The principal also has a wealth $W(\mathbf{1})$, i.e., internal cash reserve, but only a fraction θ of the project return and wealth is pledgeable to the creditors. And the cost of buying back $N - h^\top \mathbf{1}$ shares of debts is $T(h)$. I let $W(h) = W(\mathbf{1}) - T(h)$ be the remaining internal wealth after implementing action h.

So, using my notation, the project value takes a separable form of the action profile h and the underlying state ω: $v(h)(\omega) = W(h) + X(\omega)$ with the expected value being $\mathbb{E}[v(h)] = W(h) + \mathbb{E}[X] = W(h) + \int_0^\infty x \mathrm{d}F(x)$. Since X is just a random variable here, I drop the explicit dependence on ω whenever no confusion arises. The default threshold $\hat{X}$ is given by $\theta(X + W(h)) \leq \frac{h^\top \mathbf{1}}{N} D \implies X \leq \hat{X} := \frac{h^\top \mathbf{1} D}{\theta N} - W(h)$. So the payoff to each existing contract owned by agent A_i is

$$R_i^O(v(h), h) = \frac{1}{h^\top \mathbf{1}} \min\left\{\theta(X + W(h)), \frac{h^\top \mathbf{1}}{N} D\right\} = \min\left\{\theta \frac{v(h)}{h^\top \mathbf{1}}, \frac{D}{N}\right\} \tag{OA.63}$$

with the expected value being $\mathbb{E}[R_i^O(v(h), h)] = \mathbb{E}\left[\min\left\{\theta \frac{v(h)}{h^\top \mathbf{1}}, \frac{D}{N}\right\}\right]$.

Since creditors only recover a fraction of asset value in default, each of them benefits from less default resulting from deleveraging. Buying back external debt has two effects: it reduces cash reserve and lowers the creditors' payoff; it also lowers the total debt outstanding. Thus, the creditors' recovery since default is less likely. Their main result is a condition under which bond buyback is not beneficial for the principal because of the free-riding effects. I derive this condition in the continuous limit and its finite-agent counterpart in section OA.7.3.

OA.7.3 Derivation of the Bond Buyback Model

Continuous Limit Their main result is a characterization when buying back debt is beneficial to the principal in the limit $N \to +\infty$. Since the existing contracts are fully symmetric, we can use $H = h^\top \mathbf{1}/N$ to denote the fraction of debts that hold out as the state variable in lieu of h. Using the new state variable H, the value of the aggregate debt is

$$\mathbb{E}[\min\{\theta v(H), HD\}] = \int_0^{\hat{X}} \theta(x + W(H))\mathrm{d}F(x) + HD(1 - F(\hat{X})) \tag{OA.64}$$

where $\hat{X} = HD/\theta - W(H)$ is the default threshold.[8] And the marginal value of the debt is[9]

$$\frac{\mathrm{d}}{\mathrm{d}H}\mathbb{E}[\min\{\theta v(H), HD\}] = \theta W'(H)F(\hat{X}) + D(1 - F(\hat{X})) \tag{OA.65}$$

where the second term is the repayment when the firm is not in default, and the first term accounts for the effect on the internal cash reserve through the transaction.

To retire a fraction $\mathrm{d}H$ of the total debt D, the creditors must be compensated at least the average debt value. Equating it to the marginal cost yields

$$\frac{\mathbb{E}[\min\{\theta v(H), HD\}]}{HD} D\mathrm{d}H = W'(H)\mathrm{d}H \implies W'(H) = H^{-1}\mathbb{E}[\min\{\theta v(H), HD\}] \tag{OA.66}$$

The total value accrued to the principal is the difference between the asset value and the debt value

$$\mathbb{E}[v(H)] - \mathbb{E}[\min\{\theta H^{-1} v(H), HD\}]. \tag{OA.67}$$

whose first order derivative w.r.t. H is

$$W'(H) - \frac{\mathrm{d}}{\mathrm{d}H}\mathbb{E}[\min\{\theta v(H), HD\}] = (1 - \theta F(\hat{X}))W'(H) - D(1 - F(\hat{X})) \tag{OA.68}$$

which is positive (i.e., retiring debt hurts the principal) if and only if

$$1 - \theta F(\hat{X}) \geq \frac{HD}{\mathbb{E}[\min\{\theta v(H), HD\}]}(1 - F(\hat{X})) \tag{OA.69}$$

after substituting the expression from equation (OA.66). This is analogous to Equation (6) in Bulow et al. (1988). When this condition holds, the principal benefits from increasing the leverage as the cost of default is also borne by the creditors, and she has no incentive to deleverage, which generates the ratchet effect.

Finite Agent Now, let's try to derive the finite-agent counterpart. Since all agents are symmetric, I let $h^k = (1, \ldots, 1, 0, \ldots, 0)$ be the vector whose first k elements are ones and the rest zero. The number of holdouts is $(h^k)^\top \mathbf{1} = k$. Under action profile h^k, the aggregate debt value is

$$\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \int_0^{\hat{X}^k} \theta(x + W(h^k))\mathrm{d}F(x) + \frac{kD}{N}(1 - F(\hat{X}^k)) \tag{OA.70}$$

[8] Firm defaults whenever $\theta(X + W(H)) < HD$.

[9] Using Leibniz rule, the derivative of the debt value is $\theta W'(H)F(\hat{X}) + \theta(\hat{X} + W(H))\frac{\mathrm{d}\hat{X}}{\mathrm{d}H}f(\hat{X}) + D(1 - F(\hat{X})) - HDf(\hat{X})\frac{\mathrm{d}\hat{X}}{\mathrm{d}H}$ where the second and the fourth term cancels out at $\hat{X}$.

where $\hat{X}^k = \frac{kD}{N\theta} - W(h^k)$ is the default threshold when k creditors hold out. Using integration by parts and substituting the value of $\hat{X}^k$, we can write the debt value as

$$\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \theta F(\hat{X}^k)\hat{X}^k - \theta \int_0^{\hat{X}^k} F(x)\mathrm{d}x + \theta W(h^k)F(\hat{X}^k) + \frac{kD}{N}(1 - F(\hat{X}^k)) \tag{OA.71}$$

$$= \frac{kD}{N} - \theta \int_0^{\hat{X}^k} F(x)\mathrm{d}x \tag{OA.72}$$

And the change in the total debt value when one additional creditor holds out is

$$\mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \frac{D}{N} + \theta \int_{\hat{X}^{k+1}}^{\hat{X}^k} F(x)\mathrm{d}x \tag{OA.73}$$

To retire the debt from an additional agent, the debtor has to pay out the average debt value from the internal cash reserve, and thus, the internal cash reserve changes by

$$W(h^{k+1}) - W(h^k) = \frac{1}{k}\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \frac{D}{N} - \frac{\theta}{k}\int_0^{\hat{X}^k} F(x)\mathrm{d}x \tag{OA.74}$$

The value to the principal at h^k is

$$\mathbb{E}[v(h^k)] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}] \tag{OA.75}$$

The change in the principal's value from h^{k+1} to h^k, if we write completely analogously, is

$$W(h^{k+1}) - W(h^k) - \left\{\mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]\right\} \tag{OA.76}$$

$$= W(h^{k+1}) - W(h^k) - \left[\int_0^{\hat{X}^{k+1}} \theta x \mathrm{d}F(x) + \theta W(h^{k+1})F(\hat{X}^{k+1}) + \frac{(k+1)D}{N}(1 - F(\hat{X}^{k+1}))\right. \tag{OA.77}$$

$$\left. - \left(\int_0^{\hat{X}^k} \theta x \mathrm{d}F(x) + \theta W(h^k)F(\hat{X}^k) + \frac{kD}{N}(1 - F(\hat{X}^k))\right)\right] \tag{OA.78}$$

$$= (1 - \theta F(\hat{X}^{k+1}))W(h^{k+1}) - (1 - \theta F(\hat{X}^k))W(h^k) + \int_{\hat{X}^{k+1}}^{\hat{X}^k} \theta x \mathrm{d}F(x) \tag{OA.79}$$

$$+ \frac{kD}{N}(1 - F(\hat{X}^k)) - \frac{(k+1)D}{N}(1 - F(\hat{X}^{k+1})) \tag{OA.80}$$

$$= (1 - \theta F(\hat{X}^k))[W(h^{k+1}) - W(h^k)] + \theta(F(\hat{X}^k) - F(\hat{X}^{k+1}))W(h^{k+1}) \tag{OA.81}$$

$$+ \int_{\hat{X}^{k+1}}^{\hat{X}^k} \theta x \mathrm{d}F(x) - \frac{D}{N}(1 - F(\hat{X}^k)) - \frac{(k+1)D}{N}(F(\hat{X}^k) - F(\hat{X}^{k+1})) \tag{OA.82}$$

$$= (1 - \theta F(\hat{X}^k))\frac{1}{k}\mathbb{E}[\min\{\theta v(h^k), kD/N\}] - \frac{D}{N}(1 - F(\hat{X}^k)) \tag{OA.83}$$

$$+ \int_{\hat{X}^{k+1}}^{\hat{X}^k} \theta \left(x + W(h^{k+1}) - \frac{(k+1)D}{N}\right) \mathrm{d}F(x) \tag{OA.84}$$

which is positive if

$$1 - \theta F(\hat{X}^k) \geq \frac{kD(1 - F(\hat{X}^k))}{N\mathbb{E}[\min\{\theta v(h^k), kD/N\}]} + \frac{k\int_{\hat{X}^k}^{\hat{X}^{k+1}} \theta \left(x + W(h^{k+1}) - \frac{(k+1)D}{N}\right) \mathrm{d}F(x)}{\mathbb{E}[\min\{\theta v(h^k), kD/N\}]}. \tag{OA.85}$$

This is still the same as in Bulow et al. (1988), but we have one additional term which vanishes in the continuous limit. Since X^k is increasing in k,[10] this term is negative, as $x + W(h^{k+1} - \frac{(k+1D)}{N})$ is zero when evaluated at $x = \hat{X}^{k+1}$. So, the condition is easier to satisfy, as a non-atomic agent partially takes into consideration his own externality.

Alternatively, we could provide a simpler characterization

$$W(h^{k+1}) - W(h^k) - \left\{\mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]\right\} \tag{OA.86}$$

$$= \left(\frac{D}{N} - \frac{\theta}{k}\int_0^{\hat{X}^k} F(x)\mathrm{d}x\right) - \left(\frac{D}{N} - \theta\int_{\hat{X}^k}^{\hat{X}^{k+1}} F(x)\mathrm{d}(x)\right) \tag{OA.87}$$

$$=\theta\int_0^{\hat{X}^{k+1}} F(x)\mathrm{d}x - \frac{k+1}{k}\theta\int_0^{\hat{X}^k} F(x)\mathrm{d}x \tag{OA.88}$$

which is positive if

$$\frac{k}{k+1} > \frac{kD/N - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]}{(k+1)D/N - \mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}]} \tag{OA.89}$$

having used that the integration of the CDF multiplied by θ is the difference between the nominal value of the debt and the market value of the debt

$$\theta\int_0^{\hat{X}^k} F(x)\mathrm{d}x = \frac{kD}{N} - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]. \tag{OA.90}$$

The characterization is only available in the finite-agent case as both sides of the inequality approach one in the continuous limit.

OA.7.4 Distressed Debt Restructuring à la Gertner and Scharfstein (1991).

A firm with dispersed creditors in distress often offers a debt exchange to its creditors. It's the same as the bond buyback model, except that new debts are offered instead of cash. The firm is impeded by the same holdout problem: Creditors who do not accept the exchange offer also benefit from deleveraging. But the problem can sometimes be solved when debt is offered. Gertner and Scharfstein (1991) considers many different cases of existing debt structure and offer types, but I will focus on the comparison between offering pari passu debt vs. senior debt. It's also used to demonstrate that the two-period model here can incorporate a more dynamic structure.

The firm has existing debt D, a fraction q of which is due at date 1, and date-1 interim cash Y. The principal needs investment I to continue the project, and a random cash flow $X \sim F$ will be realized if the project is continued. For simplicity, I will omit the bank debt in Gertner and Scharfstein (1991) and only focus on the public bonds. I also focus on the case when there is no interim shortage of cash as in their propositions 1-3, i.e., $Y > I + qD$.

Each agent has short-term $\frac{qD}{N}$ debt due at the interim date and $\frac{(1-q)D}{N}$ due at date 2. In the "no-cash-shortage" case, the project is always implemented, so the value creation function is $v(h)(\omega) = X(\omega) + Y - I$,

[10] To see this, notice $\hat{X}^{k+1} - \hat{X}^k = \frac{D}{N\theta} - (W(h^{k+1} - W(h^k))$ but the second term is smaller than $\frac{D}{N}$ by equation (OA.74) while the firm term is larger then $\frac{D}{N}$ as $\theta < 1$. This simply says that the default threshold is higher when there are more debts outstanding, even when internal cash is used to repurchase debt.

and the payoff of each original contract is

$$R_i^O(v,h) = \frac{qD}{N} + \frac{1}{h^\top \mathbf{1}} \min\left\{v - \frac{h^\top \mathbf{1}}{N} qD, (1-q)\frac{h^\top \mathbf{1}}{N} D\right\} \tag{OA.91}$$

$$= \min\left\{\frac{1}{h^\top \mathbf{1}} v, \frac{1}{N} D\right\}, \forall i = 1,2,...,N, \forall v > qD \tag{OA.92}$$

The payoff of the new contracts depends on what's being offered. In the section OA.7.5, we derive that for the pari-passu debt and senior debt. When pari-passu long-term debt is offered, it has effectively lower priority than the holdouts, as the short-term debt held by the holdouts is repaid firm, so the firm has to offer more long-term debt than 1-to-1; in contrast, when long-term senior debt is offered, it's paid after the short-term debt, but ahead of the long-term part of the debt held by the holdouts. So, the principal can offer to implement the exchange at a ratio smaller than 1:1.

OA.7.5 Derivation of the Debt Exchange Model

Hypothetically, we assume a fraction β of the short-term debt holders accept the exchange offer and, for simplicity, assume βN is an integer. Let $h^{(1-\beta)N} = (1,...,1,,0,...,0)$ be the action profile where the first $(1-\beta)N$ agents hold out.

Pari-passu Debt Exchange If the principal offer long-term debt pD/N in exchange for the existing short-term debt qD and long-term debt $(1-q)D/N$, and at any profile h, the debt due at the interim date is $h^\top \mathbf{1} \cdot qD/N$ and will be paid off first. The total amount of debt outstanding at date 2 is $(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N$.

The value of the new contract is thus

$$R_i(v,h) = \frac{pD/N}{(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N} \tag{OA.93}$$

$$\min\{v - h^\top \mathbf{1} \cdot qD/N, (1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N\} \tag{OA.94}$$

$$= \min\left\{\frac{pD/N\,(v - h^\top \mathbf{1} \cdot qD/N)}{(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N}, pD/N\right\} \;\forall i : h_i = 0, \tag{OA.95}$$

In particular, under the action profile $h^{(1-\beta)N}$, using $(h^{(1-\beta)N})^\top \mathbf{1} = (1-\beta)N$

$$R_i(v, h^{(1-\beta)N}) = \frac{1}{N} \min\left\{\frac{p\,(v - (1-\beta)qD)}{(1-q)(1-\beta) + p\beta}, pD\right\}, \forall i > (1-\beta)N \tag{OA.96}$$

and the total payment to the tendered creditors is

$$x = \sum_{i=1}^{N} (1-h_i) R_i(v, h^{(1-\beta)N}) = \min\left\{\frac{\beta p\,(v - (1-\beta)qD)}{(1-q)(1-\beta) + p\beta}, \beta pD\right\} \tag{OA.97}$$

The payoff of the holdouts under the action profile $h^{(1-\beta)N}$ is thus[11]

[11] Technically, we should evaluate the outside option at the action profile $h^{(1-\beta)N+1}$, but the difference is small when N is large, and it complicates the analysis as we see in the bond buyback example. So I omit that difference to reproduce the result in Gertner and Scharfstein (1991) and then comment on the case when the difference exists.

$$R_i^O(v-x,h^{(1-\beta)N}) = \min\left\{\frac{1}{(1-\beta)N}\left[v - \min\left\{\frac{\beta p\,(v-(1-\beta)qD)}{(1-q)(1-\beta)+p\beta}, \beta pD\right\}\right], \frac{D}{N}\right\} \tag{OA.98}$$

$$= \frac{1}{N}\min\left\{\frac{1}{1-\beta}\max\left\{\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+p\beta}v + \frac{\beta p\,(1-\beta)qD}{(1-q)(1-\beta)+p\beta}, v-\beta pD\right\}, D\right\} \tag{OA.99}$$

Note, it is equivalent to separate the payoff into the short-term part and the long-term part, i.e.,

$$\min\left\{\frac{1}{(1-\beta)N}\left[(v-(1-\beta)qD) - \min\left\{\frac{\beta p\,(v-(1-\beta)qD)}{(1-q)(1-\beta)+p\beta}, \beta pD\right\}\right], \frac{(1-q)D}{N}\right\} + \frac{qD}{N}. \tag{OA.100}$$

At $p = 1$, the payoff to the new and old contracts can be simplified to

$$R_i(v,h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{v-(1-\beta)qD}{(1-q)(1-\beta)+\beta}, D\right\} \tag{OA.101}$$

$$R_i^O(v-x,h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1}{1-\beta}\max\left\{\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+\beta}v + \frac{\beta(1-\beta)qD}{(1-q)(1-\beta)+\beta}, v-\beta D\right\}, D\right\} \tag{OA.102}$$

Notice whenever $\frac{1}{(1-q)(1-\beta)+\beta}(v-(1-\beta)qD) < D$, we have $v < D$ and therefore[12]

$$R_i^O(v-x,h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1-q}{(1-q)(1-\beta)+\beta}v + \frac{\beta}{(1-q)(1-\beta)+\beta}qD, D\right\}, \forall v < D \tag{OA.103}$$

and taking the difference between the two terms in the min function in $R_i^O(v-x,h^{(1-\beta)N})$ and $R_i(v,h^{(1-\beta)N})$

$$\left[\frac{1-q}{(1-q)(1-\beta)+\beta}v + \frac{\beta}{(1-q)(1-\beta)+\beta}qD\right] - \frac{(v-(1-\beta)qD)}{(1-q)(1-\beta)+\beta} = \frac{q(D-v)}{1-q(1-\beta)} > 0 \tag{OA.104}$$

So, the payoff to the holdouts is always higher than the tendering agents

$$R_i^O(v-x,h^{(1-\beta)N}) \geq R_i(v,h^{(1-\beta)N}) \tag{OA.105}$$

with the inequality being strict when $v < D$. This is equivalent to Proposition 1 in Gertner and Scharfstein (1991) when N approaches infinity.[13]

It will turn out that holding out is not always optimal when N is finite. For the comparison when N is finite, I need to compare the payoff of accepting at $h^{(1-\beta)N}$ with that of holding out at $h^{(1-\beta)N+1}$. When $v > D$,

$$R_i(v,h^{(1-\beta)N}) = \frac{D}{N} = R_i^O(v-x,h^{(1-\beta)N+1}) \tag{OA.106}$$

[12] The difference of the two terms inside the max function in $R_i^O(v-x,h^{(1-\beta)N})$ is $\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+\beta}v + \frac{\beta(1-\beta)qD}{(1-q)(1-\beta)+\beta} - (v-\beta D) = \frac{\beta(D-v)}{1-q(1-\beta)} > 0$.

[13] Ideally, we want to compare the payoff of the tendering with β to the holdout with $\beta - \frac{1}{N}$, but the difference diminishes as N approaches infinity.

but when $v < D$, comparing the payoffs between tendering and holdout yields

$$
\begin{aligned}
& N(R_i^O(v-x, h^{(1-\beta)N}+1) - R_i(v, h^{(1-\beta)N})) && \text{(OA.107)} \\
=& \left[\frac{1-q}{(1-q)(1-\beta+1/N)+\beta-1/N}v + \frac{\beta-1/N}{(1-q)(1-\beta+1/N)+\beta-1/N}qD\right] && \text{(OA.108)} \\
& - \left[\frac{1}{(1-q)(1-\beta)+\beta}(v-(1-\beta)qD)\right] && \text{(OA.109)} \\
=& \frac{q(D-v)}{1-q(1-\beta)} \times \frac{N-1-(1-\beta)Nq}{N-q-(1-\beta)Nq} && \text{(OA.110)}
\end{aligned}
$$

which is positive whenever $N > \frac{1}{1-q(1-\beta)}$ or $N < \frac{q}{1-q(1-\beta)}$. When N goes to infinity, the condition is satisfied, so we obtain the same result. But when the number of agents is finite, in particular, $\frac{q}{1-q(1-\beta)} \le N \le \frac{1}{1-q(1-\beta)}$, holding out may not be optimal as the agent bears his own externality. But the second half of the quality puts a lower bound on the number of agents holding out: at least a fraction $1-\beta > \frac{1}{q}\frac{N-1}{N}$ of the agents hold out.

Senior Debt Exchange In contrast, if the principal offers long-term senior debt pD/N in exchange for the short-term debt qD and long-term debt $(1-q)D/N)$, the holdouts' short-term debts totaling $h^\top\mathbf{1}\cdot qD/N$ are paid-off, and the total amount of senior debt outstanding is $pD(N-h^\top\mathbf{1})$. The payoff to the new contract, i.e., each senior debt contract, is thus

$$
\begin{aligned}
R_i(v,h) =& \frac{pD}{pD(N-h^\top\mathbf{1})}\min\{v-h^\top\mathbf{1}\cdot qD/N, pD(N-h^\top\mathbf{1})/N\} && \text{(OA.111)} \\
=& \min\left\{\frac{1}{N-h^\top\mathbf{1}}(v-h^\top\mathbf{1}\cdot qD/N), \frac{pD}{N}\right\} && \text{(OA.112)}
\end{aligned}
$$

Using $(h^{(1-\beta)N})^\top\mathbf{1} = (1-\beta)N$

$$
R_i(v, h^{(1-\beta)N}) = \min\left\{\frac{1}{\beta N}(v-(1-\beta)qD), \frac{pD}{N}\right\} \tag{OA.113}
$$

and the total payment to the senior debts is

$$
x = \min\{v-(1-\beta)qD, \beta pD\} \tag{OA.114}
$$

while that to each holdout is[14]

$$
\begin{aligned}
R_i^O(v-x, h^{(1-\beta)N}) =& \min\left\{\frac{1}{(1-\beta)N}\left[v-\min\{v-(1-\beta)qD, \beta pD\}\right], \frac{1}{N}D\right\} && \text{(OA.115)} \\
=& \min\left\{\max\left\{\frac{qD}{N}, \frac{v-\beta pD}{(1-\beta)N}\right\}, \frac{D}{N}\right\} && \text{(OA.116)}
\end{aligned}
$$

At $p=1$, the payoffs to the new and old contracts are

$$
R_i(v, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1}{\beta}(v-(1-\beta)qD), D\right\} \tag{OA.117}
$$

[14]Again, we should more pedantically single out the short-term payment and the expression would be the same.

$$R_i^O(v-x, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\max\left\{qD, \frac{v-\beta D}{1-\beta}\right\}, D\right\} \tag{OA.118}$$

Whenever $\frac{1}{\beta}(v-(1-\beta)qD) < D$, we have $v < (q+\beta-\beta q)D$, and therefore $qD > \frac{v-\beta D}{1-\beta}$. Hence

$$R_i^O(v-x, h^{(1-\beta)N}) = \frac{1}{N}\min\{qD, D\} = \frac{qD}{N} < \frac{v-(1-\beta)qD}{\beta N}, \forall v < (q+\beta-\beta q)D \tag{OA.119}$$

So we have

$$R_i(v, h^{(1-\beta)N}) \geq R_i^O(v-x, h^{(1-\beta)N}), \forall v \tag{OA.120}$$

which the inequality being strict when $v < (q+\beta-q\beta)D$. So it's feasible to implement an exchange offer with senior debt, and we confirm Proposition 2 in Gertner and Scharfstein (1991) as N approaches infinity.

Moreover, as

$$R_i^O(v-x, h^{(1-\beta)N+1}) = \frac{qD}{N+1} < \frac{qD}{N} < \frac{v-(1-\beta)qD}{\beta N} \tag{OA.121}$$

the incentive to hold out is even weaker when N is finite.

OA.8 Unifiying Notions of Credibility

So far I have introduced the concepts of strongly δ-credible contracts $\mathcal{S}^\delta(h)$ and δ-credible contracts $\mathcal{C}^\delta(h)$ and it is straightforward that $\mathcal{C}^\delta(h) \subsetneqq \mathcal{S}^\delta(h)$. Clearly, when $\delta = 0$, they coincide in the degenerate case – full commitment. Yet, it is not very clear what the relationship between the two concepts is as the set $\mathcal{C}^\delta(h)$ is much smaller than $\mathcal{S}^\delta(h)$. In this section, I introduce an intermediate notion, k-step δ-credible contracts, to unify the two notions, which capture the case when the principal is committed after k rounds of (re)negotiations. I will show that our recursive definition of the δ-credibility is the limiting case of this intermediate credibility notion when k is sufficiently large.

Definition OA.3 (k-step δ-Credible Contracts)**.** *A contract R is a k-step δ-credible contract for some $\delta \in [0,1]$ at an action profile h if and only if i) it is incentive compatible for the agents at the action profile h and ii) at any unilateral deviation profile $\hat{h}$, it weakly δ-dominates all $(k-1)$-step δ-credible contracts at $\hat{h}$. The 0-step δ-credible contract is simply the set of incentive compatible contracts at h.*

Formally, $\mathcal{C}_k^\delta(h)$, the set of k-step δ-credible contracts at h, is given by

$$\mathcal{C}_k^\delta(h) = \left\{R \in \mathcal{I}(h): \quad R \succeq_\delta \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in \mathcal{C}_{k-1}^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\} \quad \forall\, k = 1, 2, \cdots \tag{OA.122}$$

with the initial condition $\mathcal{C}_0^\delta(h) = \mathcal{I}(h)$.

It is directly from the definition that the strongly δ-credible contracts are simply the 1-step δ-credible contracts, i.e., $\mathcal{S}^\delta(h) = \mathcal{C}_0^\delta(h)$. To link it to the recursively defined δ-credible contracts $\mathcal{C}^\delta(h)$, we want to look at the case when k approaches infinity. Unfortunately, the sequence $\mathcal{C}^\delta(h)$ is not always a monotone sequence, which makes the characterization a little bit harder. Nevertheless, it has the following oscillating structuring.

Lemma OA.7. *The even subsequence of $\{\mathcal{C}_k^\delta(h)\}_k$ is weakly decreasing and the odd subsequence is weakly increasing. That is,*

$$\mathcal{C}_{2k}^\delta(h) \subset \mathcal{C}_{2k-2}^\delta(h) \text{ and } \mathcal{C}_{2k-1}^\delta(h) \subset \mathcal{C}_{2k+1}^\delta(h) \quad \forall h\, \forall k = 1, 2, 3, \cdots \tag{OA.123}$$

Proof. For simplicity, let $J(\hat{h};R) := v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i},R)$.

I prove this lemma by induction. First, I prove that it is true for $k=1$. By definition $\mathcal{C}_2^\delta(h) \subset \mathcal{C}_0^\delta(h) = \mathcal{I}(h)$. For any $R \in \mathcal{C}_1^\delta(h)$, by definition, for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_0^\delta(\hat{h}) \tag{OA.124}$$

and since $\mathcal{C}_2^\delta(\hat{h}) \subset \mathcal{C}_0^\delta(\hat{h})$, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_2^\delta(\hat{h}). \tag{OA.125}$$

Thus, $R \in \mathcal{C}_3^\delta(h)$. I proved the first step of the induction.

Now, I proceed to the second step. Suppose that this is true for $k \in \{1,2,\ldots,\kappa\}$, I want to show that this is true for $k = \kappa+1$.

- I show $\mathcal{C}_{2\kappa}^\delta(h) \subset \mathcal{C}_{2\kappa-2}^\delta(h)$. By definition, for any $R \in \mathcal{C}_{2\kappa}^\delta(\hat{h})$, for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{2\kappa-1}^\delta(\hat{h}) \tag{OA.126}$$

  and since $\mathcal{C}_{2\kappa-3}^\delta(\hat{h}) \subset \mathcal{C}_{2\kappa-1}^\delta(\hat{h})$ by the induction hypothesis, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{2\kappa-3}^\delta(\hat{h}). \tag{OA.127}$$

  Thus, $R \in \mathcal{C}_{2\kappa-2}^\delta(h)$ given $R \in \mathcal{I}(h)$.

- Now I show $\mathcal{C}_{2\kappa-1}^\delta(h) \subset \mathcal{C}_{2\kappa+1}^\delta(h)$. By definition, for any $R \in \mathcal{C}_{2\kappa-2}^\delta(\hat{h})$, for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{2\kappa-2}^\delta(\hat{h}) \tag{OA.128}$$

  and since $\mathcal{C}_{2\kappa}^\delta(\hat{h}) \subset \mathcal{C}_{2\kappa-2}^\delta(\hat{h})$ by the induction hypothesis, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{2\kappa}^\delta(\hat{h}). \tag{OA.129}$$

  Thus, $R \in \mathcal{C}_{2\kappa+1}^\delta(h)$ given $R \in \mathcal{I}(h)$.

Therefore, we conclude that the statement is correct. □

This allows us to obtain the limits of the two subsequences

$$\lim_{k\to\infty} \mathcal{C}_{2k+1}^\delta(h) = \bigcup_{k\geq 1} \mathcal{C}_{2k+1}^\delta \quad \text{and} \quad \lim_{k\to\infty} \mathcal{C}_{2k}^\delta(h) = \bigcap_{k\geq 1} \mathcal{C}_{2k}^\delta. \tag{OA.130}$$

Moreover, the two subsequences are "separated."

Lemma OA.8. *The odd subsequence never exceeds the even subsequence. That is,*

$$\mathcal{C}_{2k+1}^\delta(h) \subset \mathcal{C}_{2k}^\delta(h) \quad \forall h\, \forall\, k = 1,2,3,\cdots \tag{OA.131}$$

And as a corollary, $\lim_{k\to\infty} \mathcal{C}_{2k+1}^\delta(h) \subset \lim_{k\to\infty} \mathcal{C}_{2k}^\delta(h)$.

Proof. For simplicity let $J(\hat{h}; R) := v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R)$.

Fix an h, let

$$\kappa = \inf\{k \geq 1 : \mathcal{C}^{\delta}_{2k+1}(h) \subsetneq \mathcal{C}^{\delta}_{2k}(h)\} \tag{OA.132}$$

which implies both $\mathcal{C}^{\delta}_{2\kappa+1}(h) \subsetneq \mathcal{C}^{\delta}_{2\kappa}(h)$ and, by the minimality of κ, $\mathcal{C}^{\delta}_{2\kappa-1}(h) \subset \mathcal{C}^{\delta}_{2\kappa-2}(h)$.

Therefore, two possibilities between $\mathcal{C}^{\delta}_{2\kappa-1}(h)$ and $\mathcal{C}^{\delta}_{2\kappa}(h)$ and we prove by contradiction that both are not possible.

- $\mathcal{C}^{\delta}_{2\kappa-1}(h) \subset \mathcal{C}^{\delta}_{2\kappa}(h)$. In this case, from $\mathcal{C}^{\delta}_{2\kappa+1}(h) \subsetneq \mathcal{C}^{\delta}_{2\kappa}(h)$ we know $\exists R \in \mathcal{C}^{\delta}_{2\kappa+1}(h)$ but $R \notin \mathcal{C}^{\delta}_{2\kappa}(h)$, which implies for any $\hat{h} \in \mathcal{B}(h)$,

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}^{\delta}_{2\kappa}(h) \tag{OA.133}$$

  while $\exists \tilde{R} \in \mathcal{C}^{\delta}_{2\kappa-1}(\hat{h})$

$$J(\hat{h}; R) < \delta J(\hat{h}; \tilde{R}). \tag{OA.134}$$

  This implies $\mathcal{C}^{\delta}_{2\kappa-1}(h)$ is not a subset of $\mathcal{C}^{\delta}_{2\kappa}(h)$, contradicting the case $\mathcal{C}^{\delta}_{2\kappa-1}(h) \subset \mathcal{C}^{\delta}_{2\kappa}(h)$.

- $\mathcal{C}^{\delta}_{2\kappa-1}(h) \subsetneq \mathcal{C}^{\delta}_{2\kappa}(h)$. This means $\exists R \in \mathcal{C}^{\delta}_{2\kappa-1}(h)$ but $R \notin \mathcal{C}^{\delta}_{2\kappa}(h)$, which implies for any $\hat{h} \in \mathcal{B}(h)$,

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}^{\delta}_{2\kappa-2}(h) \tag{OA.135}$$

  while $\exists \tilde{R} \in \mathcal{C}^{\delta}_{2\kappa-1}(\hat{h})$

$$J(\hat{h}; R) < \delta J(\hat{h}; \tilde{R}). \tag{OA.136}$$

  This suggests $\mathcal{C}^{\delta}_{2\kappa-1}(h)$ is not a subset of $\mathcal{C}^{\delta}_{2\kappa-2}(h)$, contradicting the minimality of κ.

Thu,s we must have $\mathcal{C}^{\delta}_{2k+1}(h) \subset \mathcal{C}^{\delta}_{2k}(h) \quad \forall h \, \forall k = 1, 2, \ldots$

To prove $\lim_{k\to\infty} \mathcal{C}^{\delta}_{2k+1}(h) \subset \lim_{k\to\infty} \mathcal{C}^{\delta}_{2k}(h)$, it is enough to show for any k and any $k' > k$, $\mathcal{C}^{\delta}_{2k+1}(h) \subset \mathcal{C}^{\delta}_{2k'}(h)$. This is true, given

$$\mathcal{C}^{\delta}_{2k+1}(h) \subset \mathcal{C}^{\delta}_{2k'-1}(h) \subset \mathcal{C}^{\delta}_{2k'}(h). \tag{OA.137}$$

The first inclusion holds because the odd subsequence is non-decreasing, and the second holds by the first half of this lemma. □

Now, we can characterize the limit of the two subsequences. By definition[15] $\liminf_{k\to\infty} \mathcal{C}^{\delta}_{k} \subset \limsup_{k\to\infty} \mathcal{C}^{\delta}_{k}$. Using de Morgan's Law and the two lemmata above, I can write them as

$$\limsup_{k\to\infty} \mathcal{C}^{\delta}_{k}(h) = \left(\lim_{k\to\infty} \mathcal{C}^{\delta}_{2k}(h)\right) \bigcup \left(\lim_{k\to\infty} \mathcal{C}^{\delta}_{2k+1}(h)\right) = \lim_{k\to\infty} \mathcal{C}^{\delta}_{2k}(h) \tag{OA.139}$$

$$\liminf_{k\to\infty} \mathcal{C}^{\delta}_{k}(h) = \left(\lim_{k\to\infty} \mathcal{C}^{\delta}_{2k}(h)\right) \bigcap \left(\lim_{k\to\infty} \mathcal{C}^{\delta}_{2k+1}(h)\right) = \lim_{k\to\infty} \mathcal{C}^{\delta}_{2k+1}(h). \tag{OA.140}$$

This allows us to show that $\mathcal{C}^{\delta}(h)$ is the limiting case of the k-step δ-credible contracts.

[15] The standard definition of limsup and liminf is

$$\limsup_{k\to\infty} \mathcal{C}^{\delta}_{k}(h) := \bigcap_{k\geq 1} \bigcup_{j\geq k} \mathcal{C}^{\delta}_{j}(h) \quad \text{and} \quad \liminf_{k\to\infty} \mathcal{C}^{\delta}_{k}(h) := \bigcup_{k\geq 1} \bigcap_{j\geq k} \mathcal{C}^{\delta}_{j}(h) \tag{OA.138}$$

Proposition OA.7. *The recursively defined $\mathcal{C}^\delta(h)$ Definition 6 satisfies*

$$\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \mathcal{C}^\delta(h) \subset \limsup_{k\to\infty} \mathcal{C}_k^\delta(h) \quad \forall h \tag{OA.141}$$

This result suggests that the recursively defined credibility is the limiting case when the number of rounds of negotiations in which the principal cannot commit goes to infinity. In particular, when the $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) = \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$, the limit is well-defined and we have $\lim_{k\to\infty} \mathcal{C}_k^\delta(h) = \mathcal{C}^\delta(h)$. But in general, the liminf and limsup are not identical.

Proposition OA.8. *There exists a set of initial contracts R^O such that $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subsetneq \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$ for some h.*

This result shows that the limit of $\mathcal{C}_k^\delta(h)$ does not always exist as k approaches infinity, and the bounds above cannot be made tighter.

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A Proofs for Section OA.1 (Full Model and Simplification)

Proposition OA.1 (Separation)**.** *Under Weak Consistency (Definition 1), we can rewrite the original design problem as*

$$\max_{h,R} R_0^O\left(v(h) - x(h), h\right) \tag{OA.5}$$

where $x(h) = \sum_{j=1}^{N} (1-h_j)\, R_j(v(h), h)$ *under the agents' IC condition*

$$h_i \in \arg\max_{h_i' \in H_i} (1-h_i')R_i(v(h_{-i}, h_i'), (h_{-i}, h_i')) \tag{OA.6}$$

$$+h_i' R_i^O\left(v(h_{-i}, h_i') - x(h_{-i}, h_i'), (h_{-i}, h_i')\right) \;\; \forall i \tag{OA.7}$$

where $x(h_{-i}, h_i') = \sum_{j=1}^{N} (1-h_j)\, R_j(v(h_{-i}, h_i'), (h_{-i}, h_i'))$

Proof. To prove this statement, we only need to show that for any R and $R^{O|R}$ satisfying weak consistency (Definition 1), it can be written in a separate form as in the statement. First, under the weak consistency, the payoff to the existing contract when A_i chooses h_i' is

$$R_i^O(v(h_{-i}, h_i'), (h_{-i}, h_i')|R) = R_i^O\left(v((h_{-i}, h_i')) - x(h_{-i}, h_i'), (h_{-i}, h_i')\right). \tag{OA.142}$$

Substituting it to Equation (OA.4), we obtain Equation (OA.6), so the two ICs coincide. We also need to show that the objective function is identical: again, substituting it to Equation (OP) and expanding it, we have

$$v(h) - \sum_{i=1}^{N}(1-h_i)\cdot R_i(v(h), h) - \sum_{i=1}^{N} h_i \cdot R_i^O\left(v(h) - \sum_{j=1}^{N}(1-h_j)\cdot R_j(v(h), h), h\right) \tag{OA.143}$$

which by definition is the same as equation (OA.5).

Thus, the two problems are equivalent.

□

Proposition OA.2 (Equivalence)**.** *For any consistent exchange offer* (H, h^*, R) *such that* $h^* \neq \mathbf{0}$ *is implementable for the principal, there exists an alternative consistent exchange offer, with the same action space* H*, in which* $h = \mathbf{0}$ *is implementable, and the principal obtains the same payoff as under the original exchange offer.*

Proof. For a given exchange offer (H, h^*, R) that is incentive compatible, we construct a new exchange offer $(H, \mathbf{0}, \hat{R})$ such that is also incentive compatible. Since the relevant payoff is only "around" the equilibrium payoff, i.e., $\mathbf{0}_{-i} \times H_i$, we only need to specify the payoff on these action profiles.

For the payoff on path, let

$$\hat{R}_i(v(\mathbf{0}), \mathbf{0}) = (1-h_i)R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^*|h_{-i}^*, R) + \frac{v(\mathbf{0}) - v(\boldsymbol{h^*})}{N}. \tag{OA.144}$$

Let's check that the principal obtains the same payoff. Under the new exchange offer, the principal's payoff is

$$v(\mathbf{0}) - \sum_{i=1}^{N} \hat{R}_i(v(\mathbf{0}), \mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \left[(1-h_i)R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^*|h_{-i}^*, R) + \frac{v(\mathbf{0}) - v(h^*)}{N}\right] \tag{OA.145}$$

$$= v(h^*) - \sum_{i=1}^{N} \left[(1-h_i)R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^*|h_{-i}^*, R)\right] \tag{OA.146}$$

which is the principal's payoff under (H, h^*, R). So this suggests it is feasible on path and that the principal obtains exactly the same payoff.

Now we proceed to specify the off-path payoffs and show it's feasible and incentive compatible.

For agent A_i let

$$\hat{R}_i(v(\mathbf{0}_{-i}, h_i), (\mathbf{0}_{-i}, h_i)) = \begin{cases} (1-h_i)R_i(v(h^*), h^*) + \frac{v(\mathbf{0}) - v(\boldsymbol{h}^*)}{N} & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{OA.147}$$

and

$$\hat{R}_j(v(\mathbf{0}_{-i}, h_i), (\mathbf{0}_{-i}, h_i)) = \begin{cases} (1-h_j^*)R_j(v(h^*), h^*) + \frac{v(\mathbf{0}) - v(h^*)}{N} & \text{if } h_i = 0 \\ \frac{v(\mathbf{0}_{-i}^*, h_i)}{N-1} & \text{otherwise} \end{cases} \tag{OA.148}$$

where

$$x(h^*|R) = \sum_{i=1}^{N} h_i^* R_i(v(h^*), h^*) \tag{OA.149}$$

It is easy to see that the new contract is feasible: when $h_i = 0$, the payoff coincides with the on-path payoff specified; when $h_i \neq 0$, the total payoff is the total asset value available $v(\mathbf{0}_{-i}^*, h_i)$. Also, since deviation leads to zero payoff, every agent has an incentive to play 0 whenever others do. Thus, this new exchange offer is incentive compatible and delivers exactly the same payoff to the principal.

□

B Proofs for Section OA.2 (Non-contingent Exchange Offers with Financing)

Proposition OA.3. *The necessary and sufficient condition for the existence of a cash exchange offer that implements $h = \mathbf{0}$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{OA.14}$$

Moreover, the principal is willing to implement the exchange offer if and only if

$$v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) \geq c. \tag{OA.15}$$

Proof. First, I will show the condition (OA.14) is necessary. Suppose an exchange offer $\{t_i\}_i$ exists. And we denote the sum $T = \sum_{i=1}^{N} t_i(0)$. Simplifying the conditions (OA.13), we obtain

$$T \leq R_0^O\,(v(0), 0) + W \leq v(0) + W \tag{OA.150}$$

which is independent of F. It says that the borrowing is unconstrained as long as the principal is solvent. Plug in the definition of T and the individual IC of the agents (OA.11), and I obtain the condition (OA.14) in the proposition.

To see why it is sufficient, let's construct an exchange offer as follows

$$t_i(h_i) = \begin{cases} R_i^O(v(e_i), e_i) & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{OA.151}$$

and the principal borrows

$$F = \max\left\{0, \sum_{i=1}^N R_i^O(v(e_i), e_i) - W\right\}. \tag{OA.152}$$

It is easy to verify that all the constraints are satisfied when the inequality (OA.14) holds.

With the cost c, the principal can guarantee his own wealth W without implementing the exchange offer. And the payoff to the principal, if the offer is implemented, is

$$W + v(0) - \sum_{i=1}^N R_i^O(v(e_i), e_i) - c. \tag{OA.153}$$

Comparing the two scenarios, we obtain the condition in the proposition. □

C Proofs for Section OA.5 (Credible contracts with debts)

Lemma OA.4. *For debt contracts $D = \{D_i\}_i$, the maximum credible punishment on the profile $h \neq \mathbf{1}$ is given by the recursive relation*

$$\bar{x}^\delta(h) = \begin{cases} v(h) & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ (1-\delta)(v(h) - D \cdot h) + \delta \underline{x}(h) & \text{otherwise} \end{cases} \tag{OA.56}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$ where

$$\underline{x}(h) := \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h+e_i)}\left[v(h+e_i) - \bar{x}^\delta(h+e_i)\right], D_i\right\} \tag{OA.57}$$

is the sum of the minimal payments to hold in the tendering agents.

Proof. When $h = \mathbf{1}$, there is no tendering agents so by definition $\bar{x}^\delta(\mathbf{1}) = 0$.

Consider any $h \neq \mathbf{1}$, the principal's value function at h is given by

$$J(h) = v(h) - \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h+e_i)}\left[v(h+e_i) - \bar{x}^\delta(h+e_i)\right], D_i\right\} \tag{OA.154}$$

$$- \min\left\{v(h) - \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h+e_i)}\left[v(h+e_i) - \bar{x}^\delta(h+e_i)\right], D_i\right\}, D \cdot h\right\} \tag{OA.155}$$

$$= v(h) - \min\left\{v(h), D \cdot h + \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h+e_i)}\left[v(h+e_i) - \bar{x}^\delta(h+e_i)\right], D_i\right\}\right\} \tag{OA.156}$$

$$= \max \left\{ 0, v(h) - D \cdot h - \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^\delta (h + e_i) \right], D_i \right\} \right\} \quad \text{(OA.157)}$$

The corresponding equation for the maximum punishment is

$$x + \min \{ v(h) - x, D \cdot h \} \leq v(h) - \delta J(h) = (1 - \delta) v(h) \quad \text{(OA.158)}$$

$$+ \delta \min \left\{ v(h), D \cdot h + \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^\delta (h + e_i) \right], D_i \right\} \right\} \quad \text{(OA.159)}$$

When $D \cdot h + \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h+e_i)} \left[v(h + e_i) - \bar{x}^\delta (h + e_i) \right], D_i \right\} \geq v(h)$ or $\delta = 0$, the inequality always holds because

- The LHS is at most $v(h)$: $\min \{ v(h) - x, D \cdot h \} + x = \min \{ v(h), D \cdot h + x \} \leq v(h)$
- The RHS is simply $v(h)$

so the largest punishment is $\bar{x}^\delta = v(h)$;

Otherwise, there's an interior solution as the LHS varies from $D \cdot h$ to $v(h)$ while the RHS is a constant in-between:

- It is strictly smaller than $v(h)$ because $v(h) - \delta J(h) < v(h)$ by the positivity of $J(h)$ and δ.
- It is larger than $D \cdot h$ because both $v(h)$ and

  $$\min \left\{ v(h), D \cdot h + \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^\delta (h + e_i) \right], D_i \right\} \right\}$$

  are larger than $D \cdot h$.

The interior solution is given by solving $x + D \cdot h = RHS$ when the RHS is strictly smaller than $v(h)$, which yields

$$\bar{x}^\delta (h) = (1 - \delta) \left(v(h) - D \cdot h \right) + \delta \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^\delta (h + e_i) \right], D_i \right\} \quad \text{(OA.160)}$$

Thus we complete the proof. □

Lemma OA.5. *Each holdout $i \notin \xi(h)$ is either paid nothing or in full at any $h \neq \mathbf{1}$. More specifically, the value that can be distributed to the holdouts and the principal herself is*

$$v(h) - \bar{x}^\delta (h) \begin{cases} = 0 & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ > D \cdot h & \text{otherwise,} \end{cases} \quad \text{(OA.58)}$$

where $\underline{x}(h)$ is defined in Lemma OA.4.

Proof. The proof is obtained by simply calculating $v(h)-\bar{x}^{\delta}(h)$ using the recursive equation in Lemma OA.4

$$v(h)-\bar{x}^{\delta}(h)=\begin{cases}0 \quad \text{if } \sum_{i\in\xi(h)}\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right],D_i\right\}\geq v(h)-D\cdot h \text{ or } \delta=0\\ \delta\left(v(h)-D\cdot h-\sum_{i\in\xi(h)}\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right],D_i\right\}\right)+D\cdot h\end{cases} \tag{OA.161}$$

and the non-negativity of the first term in the second case. Since $v(h)-D\cdot h-\sum_{i\in\xi(h)}\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right],D_i\right.$ is strictly positive, the second case is always positive. □

Lemma OA.6. *Let $\boldsymbol{\eta}=\{\eta(h)\}_h\in\{0,1\}^{2^N}$ be a vector of indicator functions such that $\eta(h)=1$ if and only if $\delta>0$ and $v(h)-\bar{x}^{\delta}(h)\geq D\cdot h$. Then the recursive relation in Lemma OA.4 can be described as*

$$\eta(h)=\begin{cases}0 & \text{if } \delta=0\\ \mathbb{1}_{\{v(h)\geq D\cdot h\}} & \text{if } \delta\neq 0 \text{ and } h=\mathbf{1}\\ \mathbb{1}_{\left\{v(h)>D\cdot h+\sum_{i\in\xi(h)}D_i\eta(h+e_i)\right\}} & \text{otherwise}\end{cases}. \tag{OA.59}$$

Proof. The case when $\delta=0$ is trivial since $\eta(h)=0$ for all h by definition.

At $h=\mathbf{1}$, since no punishment can be imposed $\bar{x}^{\delta}(\mathbf{1})=0$, $\eta(\mathbf{1})=1$ if and only if $v(h)\geq D\cdot h$ by definition.

At any $h\neq\mathbf{1}$, by Lemma OA.5 the condition $v(h)-\bar{x}^{\delta}(h)\geq D\cdot h$ is satisfied if and only if $\sum_{i\in\xi(h)}\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\right.\right.$ $v(h)-D\cdot h$. Also, whenever $v(h+e_i)-\bar{x}^{\delta}(h+e_i)>0$, we have $v(h+e_i)-\bar{x}^{\delta}\geq D\cdot(h+e_i)$ by Lemma OA.5 and hence $\eta(h+e_i)=1$. Therefore, whenever this is the case, $\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right],D_i\right\}=D_i$ as $\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right]\geq\frac{D_i}{D\cdot(h+e_i)}D\cdot(h+e_i)=D_i$. So the condition $\sum_{i\in\xi(h)}\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^{\delta}(h+e_i)\right],\right.$ $v(h)-D\cdot h$ can be rewritten as $\sum_{i\in\xi(h)}D_i\eta(h+e_i)>v(h)-D\cdot h$. And we obtain the recursive relation in the lemma. □

Proposition OA.6 (Almost Irrelevance and Discontinuity of Commitment)**.** *The vector $\boldsymbol{\eta}$ that solves the recursive relation in Lemma OA.6 is independent of δ for any $\delta>0$ and $\eta(h)=0$ for all h if $\delta=0$. Given the solution $\boldsymbol{\eta}$, the value of the principal is*

$$J(\mathbf{0})=v(\mathbf{0})-\sum_{i=1}^{N}D_i\eta(e_i) \tag{OA.60}$$

Proof. Since the recursion in Lemma OA.6 doesn't depend on δ for any $\delta>0$, neither would the solution. When $\delta=0$, by definition $\eta(h)=0$.

Given $\boldsymbol{\eta}$, the holdout A_i would be paid in full if $\eta(e_i)=1$ and nothing otherwise. So the principal has to pay A_i exactly what he would get if he deviates, i.e., D_i. And this gives the principal a value of $v(\mathbf{0})-\sum_{i=1}^{N}D_i\eta(e_i)$ □

D Proofs for Section OA.6 (Greater Protection Facilities Restructuring: A Negative Example)

Proof of Example OA.1. Since the asset value $v(\mathbf{1})=0$, when all three agents hold out, they get nothing more than their property value, so to convince one of them, say A_i, to tender, the principal only needs to pay him π_i, and the principal obtains a value

$$J(\mathbf{1}-e_i) = v(\mathbf{1}-e_i) - \pi_i - \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - \pi_i, \mathbf{1}-e_i) \tag{OA.162}$$

Solving for the maximum x such that $x + \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - x, \mathbf{1}-e_i) \leq J(\mathbf{1}-e_i)$ yields $\bar{x}(\mathbf{1}-e_i) = \pi_i$ given the parametric assumption on the slopes of R_j^O.

Now consider the holdout profile e_i. The principal obtains a value $J(e_i) = v(e_i) - \underline{x}(e_i) - R_i^O(v(e_i) - \underline{x}(e_i), e_i)$ where $\underline{x}(e_i) = \sum_{j\neq i}\left[R_j^O(v(e_i+e_j) - \pi_k, e_i+e_j) + \pi_j\right]$ for $k \neq i, j$.

Again, solving for the maximum x such $x + R_i^O(v(e_i) - x, e_i) \leq v(e_i) - J(e_i)$ yields $\bar{x}(e_i) = \underline{x}(e_i)$. Taking derivatives with respect to π_j gives $\frac{\mathrm{d}\bar{x}(e_i)}{\mathrm{d}\pi_j} = 1 - \frac{\partial}{\partial v} R_k^O(v(e_i+e_k) - \pi_j, e_i+e_k)$

The principal's value at $h = \mathbf{0}$ is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^3\left[R_i^O(v(e_i) - X(e_i), e_i) + \pi_i\right]$. Taking the derivative with respect to π_i gives

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} = -1 + \sum_{j\neq i} \frac{\partial}{\partial v} R_j^O(v(e_j) - \bar{x}(e_j), e_j)\left[1 - \frac{\partial}{\partial v} R_k^O(v(e_j+e_k) - \pi_i, e_j+e_k)\right] \tag{OA.163}$$

Given the parameters in the proposition, we have $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_1} = -1 + \alpha_2(1-\alpha_3) + \beta_3(1-\beta_2) = \frac{13}{50} > 0$ as $\bar{x}(e_2) = 1.1$ and $\bar{x}(e_3) = 0.806$. □

E Proofs for Section OA.8 (Unifiying Notions of Credibility)

Proposition OA.7. *The recursively defined $\mathcal{C}^\delta(h)$ Definition 6 satisfies*

$$\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \mathcal{C}^\delta(h) \subset \limsup_{k\to\infty} \mathcal{C}_k^\delta(h) \quad \forall h \tag{OA.141}$$

Proof. For simplicity let $J(\hat{h}; R) := v(\hat{h}) - \sum_{i=1}^N u_i(\hat{h}_i|\hat{h}_{-i}, R)$.

I first show that $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \mathcal{C}^\delta(h)$. For any $R \in \liminf_{k\to\infty} \mathcal{C}_k^\delta(h)$, there exists a k such that for all $j \geq k$, $R \in \mathcal{C}_j^\delta(h)$, i.e., $R \in \bigcap_{j\geq k} \mathcal{C}_j^\delta(h)$ which implies for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{j-1}^\delta(\hat{h}) \quad \forall j \geq k \tag{OA.164}$$

This can be equivalently written as

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}) \quad \exists k \tag{OA.165}$$

which, since $\bigcap_{k\geq 1}\bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}) \subset \bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h})\, \forall k$, implies

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcap_{k\geq 1}\bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}) \tag{OA.166}$$

Since $\bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h})$ is a decreasing sequence, $\bigcap_{k\geq 1}\bigcup_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}) = \bigcap_{k\geq 0}\bigcup_{j\geq k} \mathcal{C}_j^\delta(\hat{h}) = \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$

and therefore

$$\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \{R \in \mathcal{I}(h) : J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \limsup_{k\to\infty} \mathcal{C}_k^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\} \tag{OA.167}$$

$$\subset \{R \in \mathcal{I}(h) : J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} \mathcal{C}_k^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\} \tag{OA.168}$$

where the second inclusion holds because $\liminf_{k\to\infty} \mathcal{C}_k^\delta(\hat{h}) \subset \limsup_{k\to\infty} \mathcal{C}_k^\delta(\hat{h})$. This shows that for any $R \in \liminf_{k\to\infty} \mathcal{C}_k^\delta(h)$, we have $J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} \mathcal{C}_k^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)$, which satisfies the definition of $\mathcal{C}^\delta(h)$, and therefore $R \in \mathcal{C}^\delta(h)$. Thus, we proved $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \mathcal{C}^\delta(h)$.

Now I proceed to show that $\mathcal{C}^\delta(h) \subset \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$. For any $R \in$, by definition, we have

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}^\delta(\hat{h}). \tag{OA.169}$$

And since $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subset \mathcal{C}^\delta(\hat{h})$, we have

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} \mathcal{C}_k^\delta(h). \tag{OA.170}$$

Since $\bigcap_{j\geq k} \mathcal{C}_j^\delta(h)$ is an increasing sequence in k, we have

$$\liminf_{k\to\infty} \mathcal{C}_k^\delta = \bigcup_{k\geq 0} \bigcap_{j\geq k} \mathcal{C}_j^\delta(h) = \bigcup_{k\geq 1} \bigcap_{j\geq k} \mathcal{C}_{j-1}^\delta(h). \tag{OA.171}$$

In order to show $R \in \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$, by definition, we need to show $R \in \bigcup_{j\geq k} \mathcal{C}_j^\delta(h) \, \forall k \geq 1$, which means for any $k \geq 1$, there is a $j(k) \geq k$ such that $R \in \mathcal{C}_{j(k)}^\delta(h)$, which means for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \mathcal{C}_{j(k)-1}^\delta(\hat{h}) \, \forall k \geq 1. \tag{OA.172}$$

This could be equivalently written as

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1} \mathcal{C}_{j(k)-1}^\delta(\hat{h}). \tag{OA.173}$$

Now, it remains to show that if

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1} \bigcap_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}), \tag{OA.174}$$

then there exists a $j(k) \geq k$ such that

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1} \mathcal{C}_{j(k)-1}^\delta(\hat{h}), \tag{OA.175}$$

which amounts to showing that

$$\exists j(k) \geq k : \bigcup_{k\geq 1} \mathcal{C}_{j(k)-1}^\delta(\hat{h}) \subset \bigcup_{k\geq 1} \bigcap_{j\geq k} \mathcal{C}_{j-1}^\delta(\hat{h}) = \lim_{k\to\infty} \mathcal{C}_{2k+1}^\delta(\hat{h}). \tag{OA.176}$$

For any k, there's a $j(k) \geq k$ and is an even number, then by the fact that the odd subsequence is increasing (Lemma OA.7), we have $\mathcal{C}_{j(k)-1}^\delta(\hat{h}) \subset \lim_{k\to\infty} \mathcal{C}_{2k+1}^\delta(\hat{h})$. Thus, we proved $\mathcal{C}^\delta(h) \subset \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$.

□

Proposition OA.8. *There exists a set of initial contracts* R^O *such that* $\liminf_{k\to\infty} \mathcal{C}_k^\delta(h) \subsetneq \limsup_{k\to\infty} \mathcal{C}_k^\delta(h)$ *for some* h*.*

Proof. To prove this, we show that when the existing securities are equities and $\delta > 0$, $\mathcal{C}_2^\delta(\mathbf{0}) = \mathcal{I}(\mathbf{0}) \subsetneq \mathcal{C}_1^\delta(\mathbf{0})$ and thus by induction, $\mathcal{C}_{2\kappa}^\delta(\mathbf{0}) = \mathcal{I}(\mathbf{0}) \subsetneq \mathcal{C}_{2\kappa+1}^\delta(\mathbf{0}) = \mathcal{C}_1^\delta(\mathbf{0})$.

We have shown that

$$\sup_{R\in\mathcal{I}(\mathbf{0})} J(\mathbf{0}; R) = v(\mathbf{0}) > \sup_{R\in\mathcal{C}_1^\delta(\mathbf{0})} J(\mathbf{0}; R) = v(\mathbf{0}) - \delta \sum_{i=1}^{N} \alpha_i v(e_i) \tag{OA.177}$$

Thus, it must be the case that $\mathcal{I}(\mathbf{0}) \subsetneq \mathcal{C}_1^\delta(\mathbf{0})$.

To see why $\mathcal{C}_2^\delta(\mathbf{0}) = \mathcal{I}(\mathbf{0})$, notice we have proven that when the existing securities are equities, no contracts can do better than simply using cash (Proposition 3), and the same is true at any e_i. Formally

$$J(e_i; R) \geq \sup_{R\in\mathcal{C}_1^\delta(e_i)} J(e_i; R) \quad \forall R \in \mathcal{I}(e_i) \tag{OA.178}$$

Therefore, in the definiton of $\mathcal{C}_2^\delta(\mathbf{0})$, the condition

$$R \succeq_\delta \tilde{R} \quad \forall \tilde{R} \in \mathcal{C}_1^\delta(e_i) \quad \forall e_i \tag{OA.179}$$

always holds and thus $\mathcal{C}_2^\delta(\mathbf{0}) = \mathcal{I}(\mathbf{0})$. □