A General Theory of Holdouts*

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Abstract

This paper presents a unified framework for analyzing the holdout problem, a pervasive economic phenomenon in which value creation is hindered by the incentive to free-ride on other agents' participation. My framework nests many classic applications, such as takeovers and debt restructuring, and highlights the role of the commitment power: The holdout problem can be resolved using contingent contracts with commitment, e.g., by a unanimity rule if the principal can commit to calling off the deal when anyone holds out. In contrast, a lack of commitment substantially alters the optimal offers depending on the payoff sensitivities of the existing contracts, which explains the absence of the unanimity rule despite its efficacy, and cross-sectional heterogeneity in offers. (E.g., senior debt used in debt restructuring but not in takeovers.) Furthermore, I show that stronger partial commitment can backfire via renegotiation, exacerbating the holdout problem. This non-monotonicity reconciles contradictory empirical findings on the use of CACs in the sovereign debt market and sheds light on various policies. Lastly, the paper shows stronger investor protection could facilitate instead of hinder restructuring under limited commitment. *JEL codes:* G34, G38, C78, D86.

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[The] effectiveness [of punishment] is seen as resulting from its inevitability.

Michel Foucault, *Discipline and Punish*

1 Introduction

Holdout problems are pervasive. They occur whenever a socially beneficial transaction fails because one of the parties in the transaction free-rides on the participation of the other parties, *holding out*, hoping to obtain a larger individual payoff later on.[1] Sovereign debt renegotiations, corporate debt restructuring, and corporate takeovers are some of the situations in which one party proposes new contracts in exchange for old ones held by dispersed agents, and holdout problems arise. The social costs of these problems can be quite sizeable. For instance, in the recent Argentinian sovereign debt restructuring, Elliot Management and five other funds held out on the Argentinian government's proposal to restructure its debt after the country defaulted on its $132 billion debt in 2001, preventing it from accessing world financial markets for fifteen years.[2,3] It has cost Argentina an estimated 30% loss in the equity value of all the Argentine firms listed in the US (Hébert and Schreger, 2017).

Theoretically, holdout problems are somewhat surprising. The reason is that a proposal requiring unanimous consent by all parties in the transaction is enough to address the holdout problem. It eliminates the incentive of any party to free ride by rendering the decision of each pivotal.[4] This easy fix to the holdout problem, unanimity, has almost never been observed except in land assembly.[5] Instead, we see different

[1]Inconsequential to my purpose, Miceli (2011) distinguishes holdout problems from free-riding problems as demand vs. supply side. I use them interchangeably: A "free-riding problem" is an ontological description of the underlying incentive, whereas a "holdout problem" is a phenomenological account of the symptoms.

[2]The six funds were Aurelius, Bracebridge Capital, Davidson Kempner, EM Ltd. (A hedge fund held by Kenneth Dart, who was dubbed "el enemigo número uno de Argentina"), Montreux Partners, and NML Capital, an off-shore unit of Elliott.

[3]Argentina's exclusion from the international capital market is largely attributed to the lawsuits with the holdout creditors and the legal risk associated (Schumacher et al., 2021). On the other hand, it is argued that the exclusion lasted for 15 years because Argentina "had the economic and political resources to fight distressed debt fund" and had "no urgency to access the international credit market." (Guzman, 2020, p.733)

[4]Indeed, Grossman and Hart (1980, fn 3, p.43) argues unanimity is impractical because the holdout would anticipate a secret payment from the raider to bribe him into the offer and there might be sleeping investors. Now, the best-price rule (Exchange Act Rule 10d-10, see 17 CFR § 240.14d-10) forbids such bribes and holdouts are usually financial experts in these high stake transactions so a similar idea of making them pivotal by deploying contingency should address the issue were it the only concern.

[5]In certain jurisdictions such as Pennsylvania, Maine, and some European regions, the raider, whenever

solutions to the same holdout problem being used. For instance, the Argentinian government settled on a cash payment of $4.65 billion with the holdouts. AMC Entertainment, the world's largest movie theater chain, restructured its dispersedly-held bonds and solved the holdout problem by using high-priority debt, reducing its outstanding debt by over $500 million.[6] In contrast, Elon Musk took over Twitter with an all-cash offer. None of these solutions requires unanimity!

Why do they forgo unanimity? Why do we see different solutions to problems with the same economic structures? Why was cash used in the Twitter takeover and senior debt in the AMC debt restructuring?

Overview In this paper, I offer a general theory of holdouts that explains the observed variation in the solutions to the holdout problem and the absence of unanimity rules. This general framework nests classic models such as takeovers (Grossman and Hart, 1980), corporate debt restructuring (Gertner and Scharfstein, 1991), bond buybacks (Bulow et al., 1988) and the leverage ratchet effect (Admati et al., 2018). The model features a principal and multiple agents with contractual claims on an underlying asset.[7] There are gains from the trade (i.e., asset value appreciation) if the principal can exchange outstanding claims for new ones, but agents may want to free-ride on others' participation and hold out. The principal can exert punishment to discourage holdouts but cannot credibly commit to the punishment she proposed. The novel insight on how variation in the set of initial contracts affects the principal's credibility to commit gives rise to the observed heterogeneous solutions to the holdout problem.

In the model, the contracts held by agents have payoffs that are jointly determined i) by the value of the underlying asset and ii) by the contractual holding structure, that is, who holds what contracts. The principal has a residual claim on the asset, and it affects her incentive to propose the new contracts. Each agent's payoff from the new contract can depend on his decision to accept or reject the principal's proposal, as well as the asset value and remaining initial contracts. I make two assumptions. First,

reaching a controlling stake, is required by the *mandatory bid rule* to proceed with 100% of the shareholders before she can allocate assets away from or losses to the acquired firm, as a protection for the minority shareholders. See Burkart and Panunzi (2003) and Betton et al. (2008).

[6]In AMC's case, the creditors received secured second-lien notes in exchange for their unsecured senior subordinated notes, and the holdouts, which previously had seniority in-between, were promoted to first-lien.

[7]In the Argentinian case, the agents are creditors, the asset is Argentina's tax revenue, and the contracts are the general-obligation government bonds issued under New York law. The principal is the Argentinian government, who would like to commit to never making a second offer so as to discourage holdouts.

there is a collective action problem: Each agent accepts the offer or holds out without coordinating with the others. Second, the payoffs of the initial set of contracts held by the holdouts are affected by the new set of contracts, for example, when some agents are granted seniority at the expense of others. The principal aims to design a new set of contracts that all agents accept.[8] The holdout problem impedes it as one agent can increase his payoff by refusing the principal's offer when the rest of the agents accept theirs. This free-rider problem restricts the set of contracts the principal can offer.

This participation constraint alone is not hard to satisfy, but on top of it, there is another constraint: The principal cannot commit to implementing the proposal she has offered when any agent holds out. For instance, the principal may have embedded a punishment mechanism in her proposal to deter holdouts. The problem of commitment arises if the principal does not find it optimal to implement the punishment once the deviation has occurred. The principal would like to commit to not renegotiating with any agent, deviating or not, because renegotiation undermines the credibility of the punishment. She cannot, and that further restricts the set of feasible contracts.

Full-Commitment Benchmark The full-commitment case is a useful benchmark for contrasting solutions. If the principal can commit, then the holdout problem can be solved. The reason is that she can always offer each agent a contract that awards him slightly more than what the initial contract would yield absent the asset value enhancement, but only if the new contracts are unanimously agreed upon by all agents. The reason, as mentioned, is that unanimity renders each agent pivotal. In this case, the principal can always extract the full surplus associated with the value enhancement of the underlying asset.[9] As long as the principal can commit to punishing holdouts, she will require unanimity no matter the setting, be it a take-over or debt restructuring. It obviously does not explain the observed cross-sectional heterogeneity in contracts.

[8] Note this is not the unanimity rule, which requires the threat of calling off the entire transaction when anyone holds out. Here, the principal may nevertheless continue the deal when someone holds out.

[9] In fact, she can extract not just the surplus but the full value of the asset. She does this by using a contingent contract that resembles "consent payment" in practice, giving the tendering agents a penny and nothing to the holdouts. A consent payment "effectively bribes bondholders to vote in favor of a restructuring, thereby trapping them in a prisoner's dilemma." (Donaldson et al., 2022, p.2) It survived judiciary scrutiny in the US and is also ruled legal by the English High Court in *Azevedo v. Imcopa (2012)*, provided that it is i) openly disclosed, ii) offered to all creditors, and iii) on an equal basis.

Result 1 on Initial Contracts Heterogeneity in outcomes arises once I relax the principal's commitment to punishing holdouts. To see this, let us look at two canonical examples. Consider first a corporate debt restructuring in which the agents' initial contracts are debt contracts. The principal (the firm in distress) can dilute the payoff of the holdouts by granting priority to the tendering agents (creditors). This is a credible threat as the dilution only hurts the holdouts, not the principal herself: She would only get paid after the holdouts are paid in full. Indeed, this is the solution suggested by the literature (e.g., Gertner and Scharfstein, 1991) and used in practice. Consider next a takeover by a raider in which all agents have equity claims. Now, by granting priority, the principal (the raider) would hurt herself with dilution because she has the same priority as the holdouts, and therefore, she would have an incentive to renegotiate any punishment away. The optimal solution turns out to be offering cash, which involves no punishment, albeit at a premium, because the agents need to be compensated for the rent they would obtain if they were to hold out when the rest of the agents tender.

Intuitively, the principal needs to offer a new contract with a credible punishment to deter holdouts, but the punishment is credible insofar as it does not hurt the principal herself, which depends on the payoff sensitivity of the holdout's initial contract. Punishing the holdouts requires diluting the payoff of their initial contract. The punishment is credible if the dilution is fully borne by the holdout (e.g., debt in default). The credibility problem arises when the dilution is also partially borne by the principal. Specifically, I show this occurs whenever the payoff of the holdout's initial contract moves less than one-to-one with the underlying value (i.e., a payoff sensitivity smaller than one).

This result explains the heterogeneity of solutions across applications and the absence of more sophisticated contractual solutions in takeovers. Unlike corporate debt restructuring, where over 66% of exchange offers involve offering seniority (Bratton and Levitin, 2018), in takeovers, the dominant forms of offers are cash or the acquirers' stocks.[10] Malmendier et al. (2016) find that more than 92% of the successful takeovers use cash or stock offers with an equal split and pay an average premium of about 50% (Also see Betton et al., 2008). My model rationalizes these findings: Dilution is credible in corporate debt restructuring but not in takeovers, as it also hurts the raider. The

[10]Stock offers are often used in the presence of financial constraints or relative overvaluation (See Rhodes-Kropf and Viswanathan, 2004). In my framework, there is no distinction between acquirer equity and cash offers, as both are non-contingent on the target asset value and capital structure.

optimal tool in takeovers is simply cash.

Result 2 on Commitment Unsurprisingly, a lack of commitment makes holdout problems harder to solve. There are various policy proposals to strengthen the principal's commitment to punishing holdouts, for example, the introduction of Collective Action Clauses (CACs). A CAC allows the sovereign principal to implement a restructuring using a (super-)majority vote[11] and limits the dissenting creditors' ability to initiate litigations. Full commitment is always optimal: If not, the principal could simply commit to whatever she would do with limited commitment. A naïve generalization would be that higher commitment helps. However, I show it is not always the case: Higher partial commitment can backfire, hindering restructuring.

The reason is that, whereas higher commitment allows the principal to impose more stringent punishment on the holdouts (a direct effect), it also allows the principal to obtain a higher value from renegotiation following a rejection, making the principal more likely to renegotiate, and lowering the punishment that can be credibly imposed on the holdouts (an indirect effect). This indirect effect can outweigh the direct effect, leading to a lower value to the principal, especially when the principal starts with a low level of commitment: Renegotiation is more likely when the commitment is low. This force gives rise to a non-monotone effect of commitment and alerts policymakers that gradual increases in commitment could exacerbate holdout problems.

This result resonates with evidence that policies increasing commitment can either alleviate or exacerbate holdout problems. Indeed, there are seemingly contradictory findings about CACs. Almeida (2020) suggests that the introduction of CACs would give the sovereign too much commitment[12] to punishing the holdouts ex post, leading to a higher borrow cost ex ante. However, Chung and Papaioannou (2021) finds it actually lowers the borrowing cost. The difference is that the latter looks at a partial inclusion of CACs, a small increase in commitment, while the former looks at a full inclusion. The contradictory findings are reconciled in my model: A small increase in commitment can make restructuring harder. Also consistent with this result, Carletti et al. (2021) finds the mandatory replacement of unanimity with supermajority voting lowers the

[11] Generally speaking, there are two main types of CACs: Single-limp and multi-limp, most commonly, two-limb. A single-limp CAC requires an aggregated vote across all series of bonds, and the restructuring plan has to reach supermajority approval, while a two-limb CAC would require the plan to get a majority approval within each class of bonds. For more details, see Gelpern and Heller (2016) and Fang et al. (2021).

[12] Their original phrase is that it *weakens* the sovereign's commitment to fulfilling the debt service.

yields of the sovereign bonds, whereas Donaldson et al. (2022) finds making one class of bonds easier to restructure increases the yields. Similarly, in takeovers, Chen et al. (2022) finds that the inclusion of a bidder termination clause, which slightly strengthens the raider's commitment to calling off the deal,[13] increases the offer premium, making takeovers more costly.

Extension on Property Rights The solutions to the holdout problems, by and large, are achieved by deploying *dilution*: the principal designs new contracts to exert a *contractual externality* on the holdouts off path, reducing the value of the existing ones and thus the incentive to hold out. There are cases where agents' interests or claims are protected by property rights, which cannot be diluted by contractual externalities,[14] e.g., houses in land assembly and debt secured by collateral.

Usually, property rights protections are perceived to exacerbate the holdout problems.[15] This is true under full commitment: Each agent needs to be compensated more in order for him to tender since the value protected by property rights cannot be diluted by new contracts. However, when the commitment is limited, the relationship can be overturned: Stronger property rights protection also makes renegotiation harder for the principal. Indeed, the incentive to renegotiate is reduced when the principal's benefit from renegotiation is reduced, which is the case when agents' rights are well protected in renegotiation. This allows the principal to commit to imposing stronger punishment initially, which, on the contrary, facilitates restructuring.

Contribution The general framework nests classic works on the holdout problems, such as Grossman and Hart (1980), Bulow et al. (1988) and Gertner and Scharfstein (1991), by including arbitrary existing contracts, and goes beyond in two dimensions: A more general contracting space and a flexible commitment assumption. Without the ad-hoc restriction on the contracting space, the holdout problem no longer exists since the contracts can be contingent upon everyone's action and make them pivotal.

[13]The bidders would nevertheless have the fiduciary or regulatory rights to termination even without the provisions.

[14]I adopt the notions of contractual rights and property rights as defined in Ayotte and Bolton (2011) that contractual right is a right against the contracting party whereas property right against everyone.

[15]For example, Demiroglu and James (2015) finds that loans held more by collateralized loan obligations (CLOs) exhibit greater holdout problems and are more difficult to restructure. Holland (2022) shows using survey data in Colombia that greater property rights protection exacerbates holdout problems in real estate development.

Limited commitment is often allowed in the sovereign debt literature, usually to debt repayment and new borrowing, but they typically do not consider optimal contracting. For example, Pitchford and Wright (2012) looks at the delay caused by the negotiation with a cash settlement. The main insight of the paper on how optimal exchange offers depend on the interaction of commitment and existing contracts' payoff sensitivity can only be obtained when all three elements are considered. Notably, Segal (1999) also provides a general framework for contracting with externalities, but he mainly considers optimal allocation *given* the externalities, while designing externalities is part of the principal's problem in this paper. Most analysis in his paper only concerns non-contingent transfers, except in the general commitment mechanism section, in which the optimality of unanimity is identified. He also alludes to the inefficiency of limited commitment and shows how it compares with the commitment case with non-contingent transfers[16] but leaves the contractual design in the face of the limited commitment to future research, and that's my focus.

Readers should be alerted that the abovementioned solutions are private solutions that the principal devises to overcome agents' incentive to hold out given the institutional constraints. The optimal institution design needs to have more elements to be in the objective: For example, it has to balance the ex-ante financing and the ex-post restructuring, which could either conflict with (Bolton and Jeanne, 2007, 2009) or complement (Donaldson et al., 2020) each other. The paper nevertheless provides a broader picture for the ex-post consideration.

In what follows, in Section 2, I lay out the model setting, provide two simplifying results, and show how this framework nests classic applications. In Section 3, I show the existence of the holdout problem with a non-contingent contract and an extreme gauging result that solves all holdout problems when the principal has full commitment. Section 4 relaxes the commitment assumption and introduces the notion of credible contracts. There, I show how commitment and the initial contracts interact: Holdout problems can be solved for some initial contracts but not for others; and the commitment has a non-monotone effect. Section 5 extends the analysis to the case when initial contracts are not fully dilutable, and I show that counterintuitively, higher protections

[16]The notion of limited commitment assumed is that the principal cannot commit in public offers but can commit in subsequent private renegotiation. This corresponds to the strong credibility I develop later in Section 4.1. I also relax it and consider the case when the principal cannot commit even in subsequent renegotiation.

of the agents can alleviate the holdout problem. Section 6 provides the theoretical underpinnings of the notion of credibility and unifies the concepts used throughout. Section 7 surveys the literature, and Section 8 discusses various model assumptions, renegotiation protocols, and empirical relevance.

2 Model Setup

2.1 Baseline Setup

Agents, Asset, and Actions. There are N agents (A_i), indexed by $i \in \mathcal{N} := \{1, 2, \dots, N\}$ and one principal (P). Each agent is endowed with a security, a claim on an asset whose value is endogenous. P can enhance the asset value by restructuring the claims, and she[17] does so via an exchange offer: The principal proposes new securities in exchange for the existing ones, and each agent independently chooses to accept his offer or *hold out*.

Let $v(h)$ be the value of the asset as a function of the *holdout profile* $h = (h_1, h_2, \dots, h_N)^\top$ where $h_i \in H_i$ is the holdout decision chosen by A_i in the set $H_i \subset [0, 1]$ specified by P. A_i accepts a fraction $1 - h_i$ of the new offer and sticks to a fraction h_i of his original contract.[18] I require $h_i = 1$ to be in the choice set, i.e., $\{1\} \subset H_i$, for all i, so that all agents are able to hold out entirely. In addition, we say the exchange offer admits *no rationing* if $\{0\} \subset H_i$ for all i. That is, all agents are able to exchange the entirety of their claims. Without loss of generality, I assume that the exchange offer admits no rationing since the firm could offer any agent the same contract as his old one. I use $e_i = (0, 0, \dots, 1, \dots, 0)^\top \in \mathbb{R}^N$ to denote the unit vector of length N whose ith element is 1 and all other elements are 0.

I assume $v(h)$ is a *weakly decreasing* function of h: $v(h^a) \leq v(h^b)$ if and only if $h^a \geq h^b$,[19,20] or equivalently $h^a_i \geq h^b_i$ for all i, with equality if and only if $h^a = h^b$. This

[17]Throughout, the principal is referred to as "she" and the agents as "he".
[18]It may not be feasible for h_i to be non-integers. For example, a house might not be divisible in the land acquisition case.
[19]For example, projects that naturally require unanimous consent can be encoded as a step function $v(h) = v_0 + \Delta v \mathbb{1}_{\{h=\mathbf{0}\}}$, and similarly for other thresholds. Note this is different from using a unanimity or majority rule by the principal. I discuss the microfoundations of this assumption in Section 8.1.
[20]Also notice that since $v(\cdot)$ is only weakly decreasing, it may not necessarily be optimal for everyone to tender. Indeed, having everyone tender is optimal in the full-commitment case as shown in Proposition 18, and also in the limited-commitment case with additional regularity conditions. I will on the implementation of $h = \mathbf{0}$ for expositonal purpose.

assumption is intuitive: Houlding out destroys the asset value. The value of the asset decreases as more agents hold out. In the baseline model, I assume the asset value $v(h)$ is a deterministic function of the holdout profile h, but the analysis could be extended to the case of random functions and write $v(h)(\omega)$ for the explicit dependence on the state ω. For example, a firm may still have uncertain cash flow after a restructuring and end up in bankruptcy as in Donaldson et al. (2020).

Payoffs Let $R^O(w,h) : \mathbb{R}_+ \times [0,1]^N \to \mathbb{R}_+^N$, be a function that maps what can be distributed to initial security holders w and the agents' holdout profile h to payoffs, given the Original securities held by the agents. Notice that potentially, $w \neq v$. For instance, it may be the case that the amount to be distributed to initial claimants is only $w = v - x$, with $x > 0$ being the value of the asset that accrues to new claims created in the restructuring. The function $R^O(\cdot,\cdot)$ encodes both the original set of claims as well as the underlying system of conflict resolution among securities, such as a bankruptcy code. We write $R_i^O(w,h)$ as the ith entry in that vector and assume that payoffs are, trivially, feasible, that is, $h \cdot R^O(w,h) := \sum_{i=1}^N h_i R_i^O(w,h) \leq w$ for all w and non-negative, $R_i^O(w,h) \geq 0$, for all w and i. Finally, the payoff of the principal,[21] whose index is 0, is written as

$$R_0^O(w,h) := w - \sum_{i=1}^N h_i R_i^O(w,h). \tag{1}$$

The function $R^O(\cdot,\cdot)$ does not capture the effect on payoffs resulting from the new securities offered by the principal, only the payoffs associated with the original securities. I denote by $R(v,h) : \mathbb{R}_+ \times [0,1]^N \to \mathbb{R}_+^N$ the payoffs of the new securities. Since the payoffs to the old securities are affected by the new ones, which are not encoded in R^O, we use $\tilde{R}^O(v,h)$ to represent the payoffs to the initial securities when the asset value is v. Clearly, $R^O(\cdot,\cdot)$, $R(\cdot,\cdot)$, and $\tilde{R}^O(\cdot,\cdot)$ are not independent of each other, and their relation will be explained further below in Section 2.2.

[21]The principal need not have an explicit claim on the asset as his identity as the residual claimant is determined by the contractual relationship with the agents. I will, possibly interchangeably, use the more vague term "contracts" to capture this idea.

Renegotiation The principal cannot commit to not renegotiating his initial exchange offer (more details in Section 4). The focus on renegotiation proof exchange offers allows me to use the same notation h for the renegotiated outcome. Finally, the principal can always call off the deal; in this case, the payoffs are simply evaluated at $h = 1$.

Cost The principal faces a random cost c, whose value is realized before announcing the exchange offer but incurred only if the plan is carried out, that is, $h \neq 1$. It could be interpreted as the outside option of the principal or the cost of carrying out the plan (e.g., investment, attorney fees, etc). Thus, the principal is willing to carry out the plan if and only if her benefit from the plan, (1), exceeds the cost, c. Throughout, I assume the cost is small, $c < v(0) - v(1)$, so it is always socially efficient to carry out the deal. The randomness of this cost is not essential for the analysis but captures unobserved heterogeneity that can potentially be important to explain the variation in outcomes in otherwise similar situations.

Exchange Offers To summarize, in the spirit of the revelation principle, I formalize the notion of exchange offers as follows:

Definition 1 (Direct Exchange Offer)**.** *A direct exchange offer is a tuple $(H, h, R, \tilde{R}^O)$ where*

- $H = \prod_{i=1}^{N} H_i$ *is the product space of A_i's action space H_i such that $\{0, 1\} \subset H_i \subset [0, 1]$;*
- $h = (h_1, h_2, \ldots, h_N) \in H$ *is the (recommended) holdout profile of the agents;*
- R *is a mapping from $\mathbb{R}_+ \times H$ to $\mathbb{R}^N$ where the ith element $R_i(v, h)$ determines the unit payoff of A_i's new contract given the asset value is v and the holdout profile h;*
- $\tilde{R}^O$ *is a mapping from $\mathbb{R}_+ \times H$ to $\mathbb{R}_+^{N+1}$ where the ith element $\tilde{R}_i^O(v, h)$ determines the unit payoff of A_i's old contract (or principal's if $i = 0$) given the asset value is v and the holdout profile h*

such that

- *the allocation is feasible:*

$$\sum_{i=0}^{N} h_i \, \tilde{R}_i^O(v, h) + \sum_{i=1}^{N} (1 - h_i) \, R_i(v, h) = v \tag{2}$$

- *the action h_i is incentive compatible:*

$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i' | h_{-i}, R, \tilde{R}^O) \tag{3}$$

where

$$u_i(h_i | h_{-i}, R, \tilde{R}^O) := (1 - h_i)\, R_i(v, h) + h_i\, \tilde{R}_i^O(v, h) \tag{4}$$

is A_i's payoff given the action profile $h = (h_{-i}, h_i)$ and the corresponding project value v.

The use of the word "recommended" might surprise the reader: Why would the principal recommend a holdout profile as part of the exchange offer? As others before in the mechanism design literature (e.g., Myerson, 1983), I allow the principal to provide a public coordination device by recommending an action profile to overcome the concern of multiple equilibria outside those proposed by the principal.

Principal's original problem (OP) The principal aims to design an exchange offer $(H, h, R, \tilde{R}^O)$ in exchange for the old contract. I consider first the case of full commitment. In this case, the constrained optimization problem of the principal is

$$\max_{H,h,R,\tilde{R}^O} v(h) - \sum_{i=1}^{N} (1 - h_i) \cdot R_i(v(h), h) - \sum_{i=1}^{N} h_i \cdot \tilde{R}_i^O(v(h), h) \tag{OP}$$

such that the action is incentive compatible

$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i | h_{-i}, R, \tilde{R}^O) \quad \forall i \in \mathcal{N}. \tag{5}$$

To understand the principal's payoff, it is helpful to consider the situation where some agents tender fully, whereas others hold out. The principal's payoff is the value of the asset given the profile h, minus the payoff that accrues to the tendering agents, $R_i\,(v(h), h)$, minus what accrues to holdouts $\tilde{R}_i^O(v(h), h)$, which is a function of the old contracts.

In the absence of commitment, the principal is subject to an additional credibility constraint. This case is investigated in Section 4.

2.2 Simplified Problem (SP) of the Principal

The generality of the proposed framework makes it difficult to characterize the problem, but it can be simplified as follows. First, I impose a condition that the principal cannot alter the existing contractual relationship among securities using the new securities proposed in the exchange offer. In other words, the relative payment to two holdouts has to stay the same. For instance, the principal may want to write a contract with a tendering agent by which the priority structure between two non-tendering agents is flipped. The assumption, which I refer to as *weak consistency*, excludes this type of exchange offers. Second, without loss of generality, it is enough for the principal to focus on exchange offers in which all agents tender. Formal statememt can be found in Section A.1 and A.2 in the appendix.

Start with weak consistency. It is defined as follows.

Definition 2 (Weak Consistency)**.** *An exchange offer is weakly consistent if the payoff to non-tendering agents, $\tilde{R}^O$, equals the payoff of the original securities evaluated at the asset value minus the part that accrues to tendering agents. That is, if $x := \sum_{i=1}^{N}(1-h_i)\, R_i(v,h)$, is the part of the value of the asset that accrues to tendering agents, then*

$$\tilde{R}_i^O(v,h) = R_i^O(v-x,h)\; \forall i = 0,1,..,N. \tag{6}$$

Weak consistency[22] captures the intuition that the principal can create externalities on the holdouts by diluting them through new contracts, but the dilution cannot be selective. In particular, she cannot make an exchange offer that dilutes holdouts without, as the residual claimant, diluting herself.[23]

[22]This is a weaker version of the consistency axiom widely used in the study of bankruptcy problems in the cooperative game theory literature, e.g., in Aumann and Maschler (1985) and Moulin (2000). It has also been used in the study of multilateral bargaining games as in, for example, Lensberg (1988) and Krishna and Serrano (1996). The difference between the consistency axiom and weak consistency is that consistency requires this condition to hold for *any* subset of the securities, while I only require it to hold between the new and old contracts. Informally, given an allocation rule $R^O(\cdot,\cdot,\cdot)$ is a map from the set of N agents $\mathcal{N}$, the total value available $v > 0$, and a vector of claims $\boldsymbol{d} \in \mathbb{R}_+^N$, to an allocation vector $R^O(\mathcal{N}, v, \boldsymbol{d}) \in \mathbb{R}_+^N$ where agent A_i receives $R_i^O(\mathcal{N}, v, \boldsymbol{d})$, the rule is consistent if, for any subset $\mathcal{N}_0 \subset \mathcal{N}$, the allocation among the agents in the subset is identical to the original allocation as long as the total resource available is the total resource allocated to $\mathcal{N}_0$ under the original allocation and the agents in the subset $\mathcal{N}_0$ have the exactly same claim $\boldsymbol{d}|_{\mathcal{N}_0}$. Or in formula, $R^O(\mathcal{N}_0, v - \sum_{j\notin\mathcal{N}_0} R_j^O(\mathcal{N}, v, \boldsymbol{d}), \boldsymbol{d}|_{\mathcal{N}_0}) = R^O(\mathcal{N}, v, \boldsymbol{d})|_{\mathcal{N}_0}$. Thomson (1990) and Maschler (1990) have a comprehensive survey on this topic.

[23]An implication of weak consistency is that the principal cannot divert values to herself, for instance, by issuing new super-senior debt to herself. In practice, there are situations where the principal can effectively divert value to herself with the help of a third party. In Müller and Panunzi (2004), they described a procedure

As for the second simplification that it is enough for the principal to focus on exchange offers in which all agents tender, it builds on a simple idea: If it is optimal for an agent to retain a fraction or the entirety of the initial security for a given exchange offer, the principal could equivalently offer the claim the agent has in his hand post-restructuring. This way, the agent would at least find it equally optimal to accept the entire exchange offer. There might be two technical issues: i) With the new offers, there might be actions that are not initially available; ii) The asset value is higher when the agent accepts, so the outside option is more valuable. I address them in Section A.2 in the appendix.

Lastly, with some abuse of notation, I write the payoff associated with the new exchange offer and the original contract as

$$R_i(h_i|h_{-i}) := R_i(v(h_{-i}, h_i), (h_{-i}, h_i)) \tag{7}$$

$$R_i^O(h_i|h_{-i}, R) := R_i^O\left(v(h_{-i}, h_i) - \sum_{j=1}^{N}(1 - h_j) \cdot R_j(v(h_{-i}, h_i), (h_{-i}, h_i)), (h_{-i}, h_i)\right) \tag{8}$$

to highlight the incentives and actions of a particular agent. We write the total payoff of agent A_i as

$$u_i(h_i|h_{-i}, R) := h_i R_i^O(h_i|h_{-i}, R) + (1 - h_i)R_i(h_i|h_{-i}) \tag{9}$$

and the principal's value at h from an exchange offer R as

$$J(h|R) := v(h) - \sum_{i=1}^{N} u_i(h_i|h_{i-}, R). \tag{10}$$

Thus, using Proposition 18 in the Appendix, we can simplify the principal's problem to

$$\max_R J(\mathbf{0}|R) \quad \text{s.t.} \quad R_i(0|\mathbf{0}_{-i}) \geq R_i^O(1|\mathbf{0}_{-i}, R) \quad \forall i \in \mathcal{N}. \tag{SP}$$

Below, I consider the case in which the principal lacks commitment, and thus, an additional credibility constraint enters into the optimization problem.

called bootstrap acquisition where the acquirer could use the target as the collateral to raise senior debt from a third-party lender and pocket in the proceeds from borrowing. As they analyzed, doing so could appropriate value from the existing shareholders and facilitate the takeover. The legality of this practice is challenged but not overturned.

2.3 Mapping to Classic Papers

The framework advanced here incorporates many classic papers in the literature. Specifically, I show how to map these papers onto my framework using the functions $v(\cdot)$, $R_i(\cdot,\cdot)$ and $R_i^O(\cdot,\cdot)$.[24] The results in these papers are shown to be special cases of my model (mostly under full commitment).

Takeover via Public Tender Offer à la Grossman and Hart (1980). They model the situation where a raider (the principal) can improve the value of a firm after acquiring a controlling stake in the firm through a public tender offer, i.e., by offering a price to each shareholder to purchase his shares. The firm value is v_0 if the takeover fails, and $v_0 + \Delta v$ if it succeeds, which occurs when more than a fraction of $\bar{h}$ of shares are tendered, i.e., $h^\top \mathbf{1} \geq \bar{h}$ and if the raider pays a private cost c. The raider is plagued with the holdout problem as the shareholder who does not tender benefits from the value improvement once the firm is acquired and thus will demand a price equal to the post-takeover value, leaving little-to-no surplus to the raider.

In my framework, the value creation function takes the form of a step function $v(h) = v_0 + \Delta v \mathbb{1}_{h^\top \mathbf{1} < \bar{h}}$. Each existing contract has a payoff $R_i^O(v(h), h) = \frac{v(h)-d}{N}$ where d is the dilution factor considered in Grossman and Hart (1980), the value of the asset that the raider can extract after raider obtains control.

The cash offer t_i *unconditional* on getting control would be a flat payoff function which only depends on the action of A_i: $R_i(v(h), h) = t_i \mathbb{1}_{h_i=0}$. For any agent A_i, he decides not to hold out if $R_i(v(\mathbf{0}), \mathbf{0}) \geq R_i^O(v(e_i), e_i)$ which implies for an offer to be incentive compatible, it must exceed the outside option $t_i \geq t_i^* := \frac{v(e_i)-d}{N} = R_i^O(v(e_i), e_i)$ and the principal implements this action with cost c if and only if $v(\mathbf{0}) - \sum_{i=1}^N t_i^* \geq c$. The condition cannot hold when there's no dilution, and the cost is positive, which is the holdout problem they identified.

Bond Buyback Boondoggle à la Bulow et al. (1988) and Leverage Ratchet Effect à la Admati et al. (2018). In this example, I illustrate the common friction that underlies

[24] Most papers here have a continuum of agents for computational tractability. I show their finite-agent counterpart using my notations as well as their continuous limit, if possible. The continuous limit is not immediately obtained by taking the limit $N \to \infty$ because the asset value v depends on the exact action of each contract holder, and each agent has a claim on it. To circumvent this, I normalize the contract of each agent by N whenever possible, e.g., with debt or equity claims.

the bond buyback boondoggle analyzed in Bulow et al. (1988), and a more recent leverage ratchet effect illustrated in the dynamic model in Admati et al. (2018) with modified notations. In both models, the firm offers cash to buy back existing debts held by external creditors, but creditors who do not sell their debt also benefit from the deleveraging and are not willing to sell unless they are compensated at the post-buyback price. Another manifestation of the holdout problem!

The debtor (principal) has a project that generates a random payoff X following a distribution F, independent of the outstanding debts. There are N creditors (agents): Each owns a debt contract with face value $\frac{D}{N}$. All the debts are of the same seniority. The principal also has a wealth $W(\mathbf{1})$, i.e., internal cash reserve, but only a fraction θ of the project return and wealth is pledgeable to the creditors. And the cost of buying back $N - h^{\top}\mathbf{1}$ shares of debts is $T(h)$. I let $W(h) = W(\mathbf{1}) - T(h)$ be the remaining internal wealth after implementing action h.

So, using my notation, the project value takes a separable form of the action profile h and the underlying state ω: $v(h)(\omega) = W(h) + X(\omega)$ with the expected value being $\mathbb{E}[v(h)] = W(h) + \mathbb{E}[X] = W(h) + \int_0^\infty x \mathrm{d}F(x)$.

Since X is just a random variable here, I drop the explicit dependence on ω whenever no confusion arises. The default threshold $\hat{X}$ is given by $\theta(X + W(h)) \leq \frac{h^{\top}\mathbf{1}}{N} D \implies X \leq \hat{X} := \frac{h^{\top}\mathbf{1}D}{\theta N} - W(h)$. So the payoff to each existing contract owned by agent A_i is

$$R_i^O(v(h)(\omega), h) = \frac{1}{h^{\top}\mathbf{1}} \min\left\{\theta(X + W(h)), \frac{h^{\top}\mathbf{1}}{N} D\right\} = \min\left\{\theta \frac{v(h)(\omega)}{h^{\top}\mathbf{1}}, \frac{D}{N}\right\} \tag{11}$$

with the expected value being $\mathbb{E}[R_i^O(v(h), h)] = \mathbb{E}\left[\min\left\{\theta \frac{v(h)}{h^{\top}\mathbf{1}}, \frac{D}{N}\right\}\right]$.

Since creditors only recover a fraction of asset value in default, each of them benefits from less default resulting from deleveraging. Buying back external debt has two effects: it reduces cash reserve and lowers the creditors' payoff; it also lowers the total debt outstanding. Thus, the creditors' recovery since default is less likely. Their main result is a condition under which bond buyback is not beneficial for the principal because of the free-riding effects. I derive this condition in the continuous limit and its finite-agent counterpart in section A.3.

Distressed Debt Restructuring à la Gertner and Scharfstein (1991). A firm with dispersed creditors in distress often offers debt exchange to its creditors. It's the same as the bond buyback model, except that new debts are offered instead of cash. The firm is impeded by the same holdout problem: Creditors who do not accept the exchange offer also benefit from deleveraging. But the problem can sometimes be solved when debt is offered. Gertner and Scharfstein (1991) considers many different cases of existing debt structure and offer types, but I will focus on the comparison between offering pari passu debt vs. senior debt. It's also used to demonstrate that the two-period model here can incorporate a more dynamic structure.

The firm has existing debt D, a fraction q of which is due at date 1, and date-1 interim cash Y. The principal needs investment I to continue the project, and a random cash flow $X \sim F$ will be realized if the project is continued. For simplicity, I will omit the bank debt in Gertner and Scharfstein (1991) and only focus on the public bonds. I also focus on the case when there is no interim shortage of cash as in their propositions 1-3, i.e., $Y > I + qD$.

Each agent has short-term $\frac{qD}{N}$ debt due at the interim date and $\frac{(1-q)D}{N}$ due at date 2. In the "no-cash-shortage" case, the project is always implemented, so the value creation function is $v(h)(\omega) = X(\omega) + Y - I$, and the payoff of each original contract is

$$R_i^O(v,h) = \frac{qD}{N} + \frac{1}{h^\top \mathbf{1}} \min\left\{v - \frac{h^\top \mathbf{1}}{N} qD, (1-q)\frac{h^\top \mathbf{1}}{N} D\right\} \tag{12}$$

$$= \min\left\{\frac{1}{h^\top \mathbf{1}} v, \frac{1}{N} D\right\}, \forall i = 1, 2, \ldots, N, \forall v > qD \tag{13}$$

The payoff of the new contracts depends on what's being offered. In the section A.4, we derive that for the pari-passu debt and senior debt. When pari-passu long-term debt is offered, it has effectively lower priority than the holdouts, as the short-term debt held by the holdouts is repaid firm, so the firm has to offer more long-term debt than 1-to-1; in contrast, when long-term senior debt is offered, it's paid after the short-term debt, but ahead of the long-term part of the debt held by the holdouts. So, the principal can offer to implement the exchange at a ratio smaller than 1:1.

3 Optimal Exchange Offer with Full Commitment

In this section, I provide two benchmark results. First, I show that holdout problems occur whenever the principal is only allowed to offer non-contingent contracts (i.e., cash), and the cost of implementing the exchange offer c is not too small.[25] Second, if, instead, the contracts are fully contingent, the principal can uniquely implement an equilibrium that extracts the full value of the assets.

3.1 Optimal Non-Contingent Exchange Offers

Suppose first that the principal can only offer cash. A cash offer is a one-shot payment $t_i(h_i)$ to agent i, which is only a function of the agent's decision to tender $h_i = 0$, independent of v and of h_{-i}.[26] These cash transfers can only come from the principal's equilibrium allocation plus her initial wealth W, if any. Notice that this implicitly assumes perfect capital markets. For instance, if the exchange offer includes some cash transfers, then the principal is able to borrow F from an outside lender[27] and commit to repaying. Alternative assumptions are discussed in Section C.

The following assumption restricts the analysis to the interesting cases.

Assumption A1 (Moderate Cost)**.** *The cost is neither too small nor too large*

$$v(\mathbf{0}) > c > v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{A1}$$

The first inequality is there to guarantee that it is socially efficient to implement $h = \mathbf{0}$. The second inequality says if the principal has to give each agent what they obtain

[25] Of course, since whether the holdout problem occurs depends on the type of new contracts offered, it may not be limited to cash offers. For example, Gertner and Scharfstein (1991) and Donaldson et al. (2020) illustrate that the holdout problem also arises with pari-passu debt offering. But in most studies, a cash-like payoff is considered, e.g., in takeover (Grossman and Hart, 1980; Bagnoli and Lipman, 1989; Holmström and Nalebuff, 1992) and bond buyback (Bulow et al., 1988; Admati et al., 2018).

[26] A subtlety here is that with cash offers, the equivalence result in Proposition 18 in the Appendix does not necessarily carry over, especially with linear pricing.

[27] In the bulk of our analysis, we will not tap into the deal's financing issue, as most contracts are written as a claim to the asset value. The exception is using cash to pay the existing contract holder *ex ante*. We introduce financing only in an artificial manner, using safe debt, to enable the principal to pay agents ex ante. There is no financing friction, so the failure of the exchange offer is not due to financial constraints. A more serious discussion of how financing friction impedes takeovers can be found in Burkart et al. (2014). Of course, alternatively, I could simplify the issue by assuming that the cash is paid ex post or that the principal has a deep pocket. In either case, no financing arises. The modeling choice has no material effect on the equilibrium, and I extend the model setup to incorporate all the scenarios.

under the old contract if they hold out, she would not want to initiate the exchange offer;[28] Otherwise, the holdout problem does not occur.

Then, if $h = \mathbf{0}$ is to be implemented via a cash transfer, it must be the case that

1. Each A_i is paid at least as much as what he would otherwise get by holding out

$$t_i(0) \geq R_i^O\left(v(e_i), e_i\right), \forall i \in \mathcal{N} \tag{14}$$

2. Total payment can be financed via the internal cash W and borrowing F from an external financier

$$\sum_{j=1}^{N} t_j(0) \leq F + W, \tag{15}$$

   where

$$F \leq R_0^O\left(v(\mathbf{0}), \mathbf{0}\right) + \left(F + W - \sum_{j=1}^{N} t_j(0)\right). \tag{16}$$

   That is, F is safe debt. Notice then that the principal's payments are only restricted by his initial wealth, W, and the value of the asset under the exchange offer and not by any financial friction.[29]

Armed with these, the next proposition shows the condition under which the holdout problem arises.

Proposition 1. *The necessary and sufficient condition for the existence of a cash exchange offer that implements $h = \mathbf{0}$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{17}$$

Moreover, the principal is willing to implement the exchange offer if and only if

$$v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) \geq c. \tag{18}$$

[28] Notice that the RHS of inequality (A1) could be negative, for instance, when outstanding claims are debt (See Example 3.1). In this case, the holdout problems occur even if there's no cost. When the outstanding claims are equity, it is always non-negative and it converges to zero when the number of agents goes to infinity, and a single holdout doesn't affect the asset value.

[29] This can be easily seen by using equation (1) and rewriting equation (16) as $\sum_{j=1}^{N} t_j(0) \leq v(\mathbf{0}) + W$. Finally, throughout, and without any loss of generality, I have assumed that the interest rate is zero.

The next Corollary is now immediate.

Corollary 1 (Holdouts with cash offers)**.** *Under Assumption A1, the first best $h = \mathbf{0}$ cannot be implemented via an exchange offer with only non-contingent contracts, i.e., paying cash.*

The corollary simply states that under Assumption A1 Moderate Cost the classic holdout problem occurs: A simple cash transfer is not enough to compensate each agent for his reservation value under the deviation, $R_i^O(v(e_i), e_i)$. The key force in a typical holdout problem is that the incentive compatibility constraint of any single agent becomes more difficult to satisfy as more of the rest of the agents tender.[30] Effectively, the principal has to pay agent A_i $R_i^O(v(e_i), e_i)$ rather than $R_i^O(v(\mathbf{1}), \mathbf{1})$, which is the value of agent i's original claim. The reason is that the asset value v increases as more people tender, and more of the value of the asset accrues to holdouts.

This makes addressing the holdout problem using cash prohibitively costly, and an efficient value enhancement cannot be obtained.

Example 3.1. *Suppose a situation with 3 creditors, each with an outstanding debt claim with a face value of $D_i = 6$. Assume that the asset value is $v(h) = 9 + \sum_i (1 - h_i)$. Each creditor would be paid $9/3 = 3$ without asset value improvement and up to $(9 + 3)/3 = 4$ when all of them tender. If the principal can renegotiate with all creditors collectively, then she could offer any price between 3 and 4 to each claimant and the first best obtains. Next, consider the situation in which this collective negotiation is not feasible. In this case, if all but one agent tender, the holdout could get paid in full, i.e., 6 out of the asset value 11, and this leaves the principal a residual value of 5, which allows her to pay each tendering agent at most 2.5, which is worse than their initial value. Of course, each agent thinks of himself as the marginal holdout and demands 6; thus, the holdout problem cannot be solved with a simple cash offering.*[31]

The intuition for this is straightforward. As the number of agents who tender increases, it becomes increasingly more difficult to get other agents to tender. There are two forces at work that induce a form of strategic substitutability amongst agents. First, the asset value is higher when more agents tender; and second, there are few competing claims on the asset. To see this, if three agents hold out, each holdout will get 3 out of the asset value 9, but when two agents hold out, each gets 5 out of the asset value 10. The value of the outside options grows even faster than

[30] The argument alludes to the slightly more restrictive assumption A2 that the original securities are increasing in the underlying value. But for the Proposition 1 per se, it holds even without this assumption.

[31] Note the RHS of the Equation (A1) is $12 - 6 \times 3 = -6$, so a positive cost is not needed to generate the holdout problem.

the asset value growth as more agents tender. □

3.2 Optimal Contingent Exchange Offers

Consider now a richer contracting space: The principal can offer contingent contracts, i.e., contracts whose payoffs depend on both the asset value and the decision of each agent to tender or not, which indirectly depends on the type of contracts other agents end up with, whether the original contract or a new one under the exchange offer. In this case, the principal will not only solve the holdout problem, but she will also be able to extract the full value of the asset and implement it as a unique equilibrium.

To see this, start by recalling the definition of unique implementation of Segal (2003) and Halac et al. (2020).

Definition 3 (Unique Implementation)**.** *The principal can uniquely implement an action profile h and guarantee herself a value w i) if there exists a consistent exchange offer (H, h, R) such that h is an equilibrium in the subgame played by the agents and ii) for any $\varepsilon > 0$, there exists a consistent exchange offer $(H^{\varepsilon}, h, R^{\varepsilon})$ such that h is the unique equilibrium in the subgame, in which the principal obtains a payoff of at least $w - \varepsilon$.*

Introducing this perturbation ε is purely technical as the set of exchange offers that admits a unique equilibrium is not necessarily closed.[32] With this definition at hand, I derive

Proposition 2 (Extreme Gauging)**.** *With fully contingent contracts, the principal can* uniquely *implement the action profile $h = \mathbf{0}$ and guarantees herself a value of $v(\mathbf{0})$.*

To get the gist of the proof, notice the IC facing an agent is that the on-path payoff from tendering must be greater than the off-path payoff from holding out

$$R_i(v(\mathbf{0}), \mathbf{0}) \geq R_i^O\left(v(e_i) - \sum_{j \neq i} R_j(v(e_i), e_i), e_i\right). \tag{19}$$

In the off-path payoff, which is the right-hand side of (19) (the constraint in problem (SP)), the total payment to all other agents $\sum_{j \neq i} R_j(v(e_i), e_i)$ "dilutes" the value that A_i is able to claim. Notice that, in principle, the principal can commit to paying the

[32] For a more detailed discussion, see Section 4 in Segal (2003) (and footnote 9 in particular).

tendering agents more, up to the full value of the asset $v(e_i)$, as a punishment for the holdout. The equilibrium will thus feature the principal offering an arbitrarily small fraction of the asset to each agent. If any one agent deviates and holds out, she will then distribute the entirety of the asset to the tendering agents. This occurs off-equilibrium path. It is here the ability of the principal to commit matters. The reason is that when the principal assigns the entirety of the asset to tendering agents, she also dilutes her claim. Instead, in the absence of commitment, the principal will have an incentive to renegotiate, rendering this exchange offer non-credible. It is in this case that I turn to next.

4 Optimal Exchange Offer with Limited Commitment

What happens them when the pincipal cannot commit to punish holdouts off-equilibrium path? In this case, the principal may be tempted to renegotiate with holdouts. Specifying the exact sequence of renegotiation might be convoluted as it might involve infinite rounds of bargaining and an agreement may never be achieved as shown in Anderlini and Felli (2001). Clearly, absent private information, if the principal finds ex post optimal to do something else, she could have already anticipated it and written it in the original contract. Thus, instead of looking for what happens in renegotiation, I look for contracts that are renegotiation-proof: the principal prefers just executing the original contracts even if an agent deviates. This strictly shrinks the space of contracts the principal can propose initially and rules out some non-credible threats that the principal might want to renegotiate away.

Before introducing a formal definition of credibility it is helpful to add some additional notation as well as making some standard assumptions that will simplify the presentation of the results below. First, let the set of incentive compatible contracts at h be given by

$$\mathcal{I}(h) := \left\{ R : [\underline{v}, \bar{v}] \times H \to [0, \bar{v}^N] \,\middle|\, h_i \in \arg\max_{h_i' \in H_i} u_i(h_i' | h_{-i}, R) \; \forall i \in \mathcal{N} \right\}. \tag{20}$$

Second, I impose two regularity conditions on the existing contracts:

Assumption A2 (Increasing and 1-Lipschtiz)**.** *The collective payoff to the agents who do not* *tender at h*

$$h \cdot R^O(\cdot, h) = \sum_{i=1}^{N} h_i R_i^O(\cdot, h) \tag{21}$$

is increasing and 1-Lipschitz for all h.

This assumption is commonly used in the security design literature.[33] This condition says that the original contracts have an increasing and continuous payoff under any holdout profile h. Moreover, the payoff function has a slope weakly less than 1. That is, whenever the underlying increases by one dollar, the incremental payoff to the existing contracts cannot exceed one dollar. Most commonly seen contracts, such as equity, debt, and call options, satisfy this condition.

Credibility issues arise arises only when agents deviate. Throughout, I consider only *unilateral deviations*. A profile $\hat{h}$ is a unilateral deviation of h if and only if $\hat{h} = h + e_i$ or $\hat{h} = h - e_i$ for some i, which is equivalent to $||\hat{h} - h|| = 1$. I use $\mathcal{B}(h) = \{\hat{h} \in \{0, 1\}^N : ||\hat{h} - h|| = 1\}$ to denote the unit "ball" around h.

Lastly, I introduce the language of δ-domination, which characterizes the principal's incentive to deviate, that is, whether to carry out the exchange offer R or to propose a different exchange offer $\tilde{R}$ at a certain holdout profile h.

Definition 4 (δ-domination)**.** *A contract R weakly δ-dominates another contract $\tilde{R}$ ($R \succeq_\delta \tilde{R}$) at h, for a number $\delta \in [0, 1]$, if $J(h|R) \geq \delta J(h|\tilde{R})$.*

There are two possible interpretations of the parameter δ. First, δ can be thought of as a delay cost equivalent to a discount rate as in Rubinstein and Wolinsky (1992) and DeMarzo and Fishman (2007). Second, it can also be interpreted as the exogenous probability that the contract is voided and the principal is allowed to re-propose a new offer as in Crawford (1982) and Dovis and Kirpalani (2021).[34] Either way, δ parametrizes the principal's lack of commitment: The higher it is the lower her ability to commit.

[33]For example, in the definition of feasible contracts in DeMarzo et al. (2005), they require the payoff to each party to be increasing, which implies 1-Lipschitz continuity.

[34]To see this explicitly, let $\hat{\delta}$ be the discount rate instead, and the principal is allowed to delay the payoff and re-propose a new contract $\tilde{R}$ with some exogenous probability p, then the current proposed contract is preferred if $J(h|R) \geq (1-p)\hat{\delta}J(h|R) + \hat{\delta}pJ(h|\tilde{R})$.

Rearranging the terms, the current proposed contract R δ-dominates contract $\tilde{R}$ at h for $\delta = \frac{\hat{\delta}p}{1-(1-p)\hat{\delta}}$, which is a strictly increasing in p for all $\hat{\delta} \in (0, 1)$ since $\frac{\partial}{\partial p}\frac{\hat{\delta}p}{1-(1-p)\hat{\delta}} = \frac{\hat{\delta}(1-\hat{\delta})}{(1-(1-p)\hat{\delta})^2} > 0 \, \forall \hat{\delta} \in (0, 1)$. Thus, for a fixed $\hat{\delta}$, a higher probability of renegotiation corresponds to a higher δ.

Notice that when $\delta = 0$ we are back to the full commitment case, whereas when $\delta = 1$, the principal can essentially renegotiate at no additional cost. For most of the analysis, I will focus on the lowest-commitment case $\delta = 1$ and omit δ if it equals to one whenever no confusion arises. I also drop "weakly" or "at h" whenever there's no confusion.

4.1 Strongly Credible Contracts

4.1.1 Strongly Credible Contracts and the Principal's Problem

I introduce two definitions of credibility. I introduce first what I refer to as Strongly Credible Contracts and later, in section 4.2, a weaker definition I refer to as just Credible Contracts. Strong credibility illustrates in a simple manner how the lack of commitment interacted with the set of initial securities, producing a variety of solutions to the holdout problem. Instead, Credible Contracts illustrate why more commitment is not always good for the principal. It is important to emphasize that strong credibility is not needed to show the two main results: Credibility is enough, and strong credibility is introduced just for clarity, tractability and intuition. Finally, all strongly credible contracts are also credible contracts.

Definition 5 (Strong δ-credibility)**.** *A contract $R : [\underline{v}, \bar{v}] \times H \to [0, \bar{v}^N]$ is* strongly δ-credible at *h if*

(a) It is incentive compatible at h, that is, it belongs to $\mathcal{I}(h)$ (see expression (20) above).

(b) Upon any unilateral deviation $\hat{h}$, it weakly δ-dominates any incentive compatible contracts $\tilde{R}$ at $\hat{h}$ for the principal.

Condition (a) means intuitively that Strongly Credible Contracts must be incentive compatible. Condition (b) means that even when one agent deviates, the principal will find in her interest "to stick with" the initial offer R rather than any other incentive compatible contract $\tilde{R}$. I denote the set of strongly δ-credible contracts by

$$\mathcal{S}^{\delta}(h) = \left\{ R \in \mathcal{I}(h) : \ R \geq_{\delta} \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in \mathcal{I}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\}. \tag{22}$$

Again, I drop δ and simply call it a strongly credible contract when δ equals 1.

The principal's value function on the set $\mathcal{S}^\delta(h)$ is defined by[35]

$$J(h|\mathcal{S}^\delta(h)) := \sup_{R\in\mathcal{S}^\delta(h)} J(h|R). \tag{23}$$

Notice that $\mathcal{S}^\delta(h) \subset \mathcal{I}(h)$, so the problem is more restrictive than the full-commitment case, on account of the principal's credibility constraint (see (SP) in section 2.2).

4.1.2 Commitment and diversity of exchange offers: Characterization

To characterize the solution to the principal's problem in (23),[36] I first show that it can be equivalently expressed using a single-dimensional optimization problem: The principal wants to minimize the total payoff to all agents upon the deviation of a single holdout while maximizing the possible punishment to the holdout. Then, as a second step, I show that the extent to which the punishment can be credibly increased depends on the shape of the holdout's payoff (this is Lemma 1 below). This, coupled with Lemma 2 on the disagreement point in renegotiation, gives rise to the diversity of exchange offers in Proposition 3. It is in Lemma 2 that Strong Credibility is used.

Consider then the deviation of agent A_i. The principal wants to find the least costly way to punish him. She does so by imposing a penalty of x that accrues in turn to the tendering agents. The principal wants to minimize the total payment to all agents, tendering or not. Thus, she solves for

$$\inf_{x\geq 0} x + R_i^O\left(v(e_i) - x, e_i\right), \tag{24}$$

where $x = \sum_{j\neq i} \tilde{R}_j(v(e_i), e_i)$ is the punishment to A_i under $\tilde{R}$.

The optimization problem in (24) illustrates the principal's trade-off: A larger punishment x would lower the payment to the holdout A_i, but it would also directly increase the payoff to the tendering agents. This can potentially lower the principal's payoff (recall that this is $v(e_i)$ minus the expression (24) at the optimum; see (10)). With

[35] The notation $J(h|R)$ in (10) can be seen as a special case of this definition, where the set is a singleton.

[36] Note that while it is convenient to look at the implementation of $h = 0$, the Proposition 18 in the Appendix does not guarantee renegotiation-proofness. It is also optimal under some additional mild conditions on R^O and v, so for expositional purposes, I will focus on the implementation of $h = 0$.

some abuse of notation define

$$f(\cdot) := R_i^O(\cdot, e_i).$$

Then the following lemma illustrates the relation between the shape of the payoff of agents A$_i$'s initial contract and the range of punishments that do not hurt the principal by lowering her payoff.

Lemma 1. *Suppose $f(\cdot)$ is a weakly increasing 1-Lipschitz function*[37] *and a is a positive number. The solution to the following problem*

$$\min_{x\in[0,a]} g(x) := x + f(a-x) \tag{25}$$

is obtained at $x = 0$ and the minimum value is $f(a)$. Moreover, if $f(\cdot)$ has a left derivative $f'(a) < 1$, the solution is unique. Otherwise, any $x \in [0, \bar{x}]$, where $\bar{x} = \inf\{x : f'(a-x) < 1\}$, solves the problem and any $x > \bar{x}$ does not.

Lemma 1 says that there is no punishment that does not lower the principal's payoff whenever the (left) slope of the holdout's payoff is strictly smaller than 1. In this case, lowering the punishment always increases the principal's payoff. The relation between the shape of the payoff of the agents' initial contract and the principal's problems of commitment is now apparent: The principal's commitment to punishing will not be credible whenever the slope of the holdout's initial contract has a slope strictly less than one.

The next Lemma characterizes the maximum payoff the principal can obtain *under the deviation*. Given that the principal can only renegotiate with the tendering agents

[37] To see how this Lipschitz condition affects the optimization problem, let's heuristically discuss what happens without it. Since $f(\cdot)$ is a weakly increasing function, it has, at most, a zero-measure set of discontinuous points and is differentiable almost everywhere. It only admits jump discontinuities by Lebesgue's Theorem, which also stipulates the non-differentiable points are either discontinuous, vertical tangent points or kinky points. The optimal solution cannot be just to the right of a jump point. Otherwise, the principal can reduce the total payment by increasing x by a small ϵ and reduce the objective by a lot. The same argument implies it cannot be at a vertical tangent point. So, any interior solution must either satisfy the first-order condition or be at a kinky point. When the first condition is satisfied, it means $1 = f'(a-x)$, i.e., any small increase or decrease in x would just be offset by the response in $f(a-x)$. Put another way, in the context of the model, the claims of the holdouts resemble debt locally at the optimal punishment. Finally, let's discuss the kinky point. One could increase x without violating any constraints at the optimum. This implies the function $x + f(a-x)$ must have a non-negative right derivative at the optimum $\bar{x}$, i.e., $f(\cdot)$ has a left derivative weakly smaller than one at $a - \bar{x}$. To focus on the interesting case and avoid tedious technical discussions on the unrealistic cases, we assume 1-Lipschitz.

and not with the holdout, the best she can do is to offer nothing to the tendering agents (recall that, under strong credibility, she can commit in subsequent renegotiations) and obtain what is left of the asset after the holdout has been paid given his initial contract.

Lemma 2. *Under Assumption A2, the highest payoff the principal can obtain at the deviating profile e_i with an IC contract $\tilde{R} \in \mathcal{I}(e_i)$ is*

$$v(e_i) - R_i^O(v(e_i), e_i). \tag{26}$$

Does Lemma 2 say that there is no credible dilution under the deviation? No. As shown in Lemma 1, this will depend on the shape of the payoff of the holdout's original contract. The next proposition brings together both lemmata to show the conditions under which punishments are credible and how they affect the principal's exchange offer.

Proposition 3. *When $N \geq 2$, under Assumption A2, the principal cannot obtain a strictly higher value at $h = \mathbf{0}$ with a strongly credible contingent contract than offering cash if and only if for all $i \in \mathcal{N}$*

$$\left.\frac{\partial}{\partial w} R_i^O(w, e_i)\right|_{w \uparrow v(e_i)} < 1. \tag{27}$$

where $\uparrow$ indicates the limit from the left.[38] *Consequently, if this condition is satisfied, holdout problems cannot be solved with any strongly credible contingent offers under Assumption A1.*

Recall that in the full commitment case and under assumption A1, Corollary 1 shows that cash can never implement the first best. However the principal is not restricted to cash. If she can propose any exchange offer, she can extract the full value of the asset from the agents. Proposition 3 says instead that if she cannot commit, she may be unable to do better than cash, even when she can use any arbitrary exchange offer. This occurs whenever condition (27) is met. The next section illustrates two practical examples of when the principal can and cannot do better than cash.

I refer throughout to the derivative in (27) as agent A_i payoff sensitivity of the original contract at $v(e_i)$ or *payoff sensitivity* for short. Similarly, we speak of the principal's payoff sensitivity, which is

$$1 - \left.\frac{\partial}{\partial w} R_i^O(w, e_i)\right|_{w \uparrow v(e_i)} \tag{28}$$

[38] Since $R_i^O(\cdot, e_i)$ is only meaningful on $[0, v(e_i)]$ and punishment usually reduces the value, we only look at the left derivatives. It always exists given continuity at $v(e_i)$.

given that the principal is the residual claimant.

4.1.3 Commitment and the diversity of exchange offers: Examples

Consider two canonical examples. In a restructuring case, the agents' contract is debt. In a takeover case, the agents' initial contract is the equity of the target. Consider debt contracts first. The payoff sensitivity of the debt contract is one if the company is in default, in which case the value of the debt of the holdout moves one to one with the value of the asset (recall that we are only considering bilateral deviations). It is zero if it's not getting paid at all or if it's already getting paid in full. As for equity, the payoff sensitivity is one only if the holdout owns the entire equity stake after all debt and other senior claims have been paid in full. It's zero when the company cannot repay its maturing debt, and it is strictly between zero and one if there are other equity holders. It is because these contracts have different payoff sensitivities that they induce, in turn, different problems of commitment for the principal, which in turn result in different solutions to the holdout problem in debt restructuring or takeovers. The following corollary now follows immediately from Proposition 3.

Corollary 2. *When each agent's initial contract is debt, the principal can obtain a higher value than offering cash using a contingent contract; when his initial contract is equity, no contingent contracts give a higher value to the principal than simply offering cash.*

Since it's simply an application of the Proposition 3 and is of empirical interest in itself, we lay out the proofs directly in the following examples.

Example 1: Debt. Let's consider the case when the holdout A_i has debt $D_i \geq 0$. His payoff is

$$R_i^O(w, e_i) = \begin{cases} w & \text{if } w < D_i \\ D_i & \text{otherwise} \end{cases} \tag{29}$$

The maximum credible threat is

$$x_i = \inf\left\{x \geq 0 : \frac{\partial}{\partial w} R_i^O(v(e_i) - x, e_i) < 1\right\} = \begin{cases} 0 & \text{if } v(e_i) > D_i \\ v(e_i) & \text{otherwise} \end{cases} \tag{30}$$

The next proposition shows how the different size of the agents' claims changes the nature of the holdout problem when the principal cannot commit not to renegotiate with the holdouts.

Proposition 4. *When existing securities are debt contracts $D = \{D_i\}_i$, the principal's value function is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} D_i \mathbb{1}_{D_i < v(e_i)} \tag{31}$$

under the strong δ-credibility constraint.

The comparison with the case of full commitment illustrates the mechanism at work. Under full commitment, the principal will extract the full value of the asset: The principal can always punish the holdout by transferring the full value of the asset to the tendering agents. Instead, with limited commitment, the principal cannot credibly commit to punishing the holdout when doing so results in a lower payoff for the principal herself. This occurs whenever the holdout has a "small" debt claim on the asset, $D_i < v(e_i)$. In this case, given that the holdout gets paid in full,[39] any punishment can only be at the expense of the principal, and thus the commitment problem arises. Indeed, as shown in (28), the payoff sensitivity of the principal is one: Any punishment results in a one-to-one drop in the value of her payoff. As a result of this commitment problem, the principal's payoff is reduced precisely by the quantity $\sum_{i=1}^{N} D_i \mathbb{1}_{D_i < v(e_i)}$. If, instead, the holdout is a "large" debt holder, $D_i > v(e_i)$, he will not be paid off in full (his payoff sensitivity is one). Now, the principal can credibly commit to punishing him precisely because her payoff is not affected by the punishment (the payoff sensitivity of the principal in (28) is 0).

The comparison illustrates the different treatments of bank debts versus public bonds in a typical restructuring evidenced in, say, James (1995). Small creditors (bondholders) often have stronger incentives to hold out and are more difficult to punish, so they typically receive preferential treatment, whereas large creditors (banks) internalize their pivotality and can be more credibly punished, so they often make a compromise.

[39]To see why, remember that the principal wants to punish the holdout, but the only way to punish the holdout is to give more values to the tendering agents. However, doing so is even more costly given the 1-Lipschitz condition of the holdout's payoff. In addition, the principal is committed in renegotiation, so she can use the extreme gauging technique as in 2 and pay the tendering agents nothing. This is relaxed in the next section when we use a weaker requirement for credibility. Indeed, the principal may not need to pay a small creditor in full even if she can under the weaker credibility constraint.

Earlier work explains the difference by focusing on the pivotality of large vs small creditors but this paper shows the ability to punish is also a key determinant.

Example 2: Equity. Suppose now that the holdout A_i has an equity claim of share $\alpha_i < 1$. His payoff function is

$$R_i^O(w, e_i) = \alpha_i w \tag{32}$$

and thus, the maximum possible punishment is

$$x_i = \inf\left\{x \geq 0 : \frac{\partial}{\partial w} R_i^O(v(e_i) - x, e_i) < 1\right\} = 0 \tag{33}$$

No punishment is strongly credible! Indeed, in this case, any punishment for the holdout would result in a loss for the principal. The reason is that punishing the holdout reduces his payoff only by $\alpha_i < 1$ whereas the payoff of the principal is instead reduced by $1 - \alpha_i > 0$ (see (28)). Therefore, the principal always wants to renegotiate in the presence of holdouts. Thus, a contingent offer cannot be better than using only cash. This rationalizes the absence of senior debt offering in takeovers despite the persistent high premium attached to many of them:[40] Contingent contracts cannot do better than cash.

The next result illustrates how the principal's payoff varies with commitment under strong credibility when agents are endowed with equity.

Proposition 5. *When existing securities are equities $\alpha = \{\alpha_i\}_i$, the principal's value function on the set of strongly δ-credible contracts is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \delta \sum_{i=1}^{N} \alpha_i v(e_i) \tag{34}$$

which is higher when the commitment is higher (δ is smaller).

Start with the full commitment case, $\delta = 0$. In this case, the principal can extract the full value of the asset (see Proposition 2). Consider now the case of no commitment at all, $\delta = 1$. Then the principal has to give the holdout the share of the asset that he owns under the deviation, $\alpha_i v(e_i)$. Anywhere in between, the principal is able to capture $1 - \delta$

[40] Malmendier et al. (2016) finds that more 92% successful takeovers offer non-contingent contracts such as cash or the stock of the acquirer firm, with an average premium of 46.24%.

of the value of the agent's share of the asset. The reason discounting matters is because in effect, the more the principal cares about the future, which is when renegotiation occurs, the less she is commited to the present exchange offer. As a result the exchange offer today needs to leave more to the agents the more the principal cares about the next round of renegotiation.

In fact the result that the payoff of the principal is decreasing in δ is more general than Proposition 5 may suggest. Under strong credibility we can show the following

Proposition 6. *The principal's value function $J(\mathbf{0})$ on the set of strongly δ-credible contracts is weakly decreasing in δ for any existing contracts R^O.*

It is only "weakly decreasing" since in some cases, as when agents are endowed with debt, the value function is a constant function of δ as in Proposition 4.

A feature of the notion of strong credibility is that it assumes that the principal has little commitment in the initial proposal but is able to commit to the alternative proposal in the renegotiation stage. Empirically, it may be plausible to assume that the laws governing the on-path negotiation and off-path renegotiation are different or that the principal may only be able to propose exchange offers during an exclusive window, as under the US bankruptcy code. Or renegotiation is in private as in Segal (1999).[41] Still, in cases such as sovereign debt restructuring, the ability of the principal to commit is the same irrespective of the renegotiation stage. In the next section, I consider a definition of credibility that considers this weaker form of credibility.

4.2 Credibility: A recursive definition

4.2.1 Credible Contracts: Existence and Uniqueness

In this section, I refine the notion of a *credible* contract to be such that the principal can propose some alternative contracts to replace the initially proposed one, but only if they are also *credible*. Its rationale and connection to the literature are discussed in Section 8.2. I begin by modifying the previously defined notion of strongly credible contracts as follows.

[41] Slightly differently, Segal (1999) assumes the principal cannot commit in the public offer, but when she deviates to privately renegotiate with a single agent, she can commit. Secrecy is not the main concern, as private renegotiation can be anticipated absent private information. The key difference is that the principal only wants to renegotiate the offer after some agents hold out in my model. I cannot preclude the incentive for her to deviate to a bilateral negotiation with a single agent: She always wants to do so given her ability to create super seniority at the expense of others. But this is usually forbidden by law.

Definition 6 (δ-Credible Contracts)**.** *A contract R is a δ-credible contract for some $\delta \in [0, 1]$ at an action profile h if and only if*

(a) it is incentive compatible for the agents at the action profile h, and

(b) at any unilateral deviation profile $\hat{h}$, it weakly δ-dominates all δ-credible contracts at $\hat{h}$

Similarly, $C^\delta(h)$, the set of δ-credible contracts at h, can be denoted by

$$C^\delta(h) = \left\{ R \in \mathcal{I}(h) : R \geq_\delta \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in C^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\}. \tag{35}$$

To understand $\delta-$credibility, the comparison between $C^\delta(h)$ and $\mathcal{S}^\delta(h)$, the set of strongly credible contracts (see expression (22)), is helpful. In the set of strongly credible contracts, we considered renegotiation offers that are only incentive compatible, that is, $\tilde{R} \in \mathcal{I}(h)$. Now, instead, the renegotiation offers have to be incentive compatible and, roughly, "credible going forward." Notice then that the set of strongly credible contracts is contained in the set of $\delta-$credible contracts, that is, $S^\delta(h) \subset C^\delta(h)$.

The set of $\delta-$credible contracts is defined recursively, and thus, issues of existence and uniqueness need to be addressed before continuing with the characterization of the problem. The next proposition establishes the existence and uniqueness of $C^\delta(h)$.

Proposition 7 (Existence and Uniqueness)**.** *The set of $\delta-$credible contracts $\{C(h)\}_h$ exists, it is non-empty and unique.*

A general characterization of $\delta-$credible contracts is postponed until section 4.2.3. An important result is that $\delta-$credibility introduces an interesting non-monotonicity in the payoff the principal as a function of the degree of commitment, δ. The intuition for this important result can be readily grasped in an example.

4.2.2 Credible Contracts: A Numerical Example

Proposition 6 showed that under strong credibility, the payoff of the principal is monotone in δ: "More" commitment (lower δ) always increases her payoff. Instead, under $\delta-$credibility, this result does not hold. The intuition is as follows. When the ability of the principal to commit improves, there are two effects operating in different directions. First, fixing renegotiation incentives "tomorrow," stronger commitment improves the payoff of the principal today. But, of course, tomorrow is not fixed:

Stronger commitment also improves the principal's position in renegotiation tomorrow, which increases her payoff then, making her more likely to renegotiate tomorrow. This, in turn, makes her less committed to punishing the holdout today. Depending on which of these two effects dominates, more commitment can increase or decrease the principal's payoff.

To illustrate this non-monotonicity, consider a three-agent case, all endowed with equity claims. Let the equity share of A_i be $\alpha_i = 1/3$ for $i \in \{1, 2, 3\}$. The asset value is 8 if all of the agents tender, 7 if one agent holds out, and 6 if two agents hold out. The asset value is normalized to 0 when all of them hold out.

We calculate the principal's value when she has a discount factor of δ using backward induction. Consider first the situation in which two agents, say A_1 and A_2, already held out. I assert that the principal can credibly give $6(1-\delta)$ to the tendering agent A_3: The principal doesn't need to give anything to A_3 to tender because A_3 obtains nothing when he holds out, but the principal can still give him some value x_3 as a punishment on A_1 and A_2, say, through senior debt, without hurting herself. Why? The principal obtains $\frac{1}{3} \times (6-0) \times \delta$ if she renegotiates and offers 0 to A_3. (Recall that no punishment is always optimal given the 1-Lipschitz condition.) Without renegotiation, she would obtain $\frac{1}{3}(6 - x_3)$. Comparing the payoff in the two scenarios, the principal is not willing to negotiate if $x_3 \le \bar{x}_3 := 6(1-\delta)$. By symmetry, this is the maximum punishment the principal can impose on any two holdouts, and each holdout obtains $\frac{1}{3} \times (6 - \bar{x}_3) = 2\delta$.

Now consider the case when only one agent, say A_1, holds out. The principal has to give A_2 and A_3 at least 2δ each. Suppose the principal initially promised to give A_2 and A_3 a total value of $x > 4\delta$. By renegotiating, she obtains a value $\frac{2}{3} \times \delta \times (7 - 2\delta \times 2)$. Without renegotiation, she obtains a value $\frac{2}{3}(7 - x)$. Comparing the two scenarios, the principal would not renegotiate if $x \le 7 - 7\delta + 4\delta^2 \equiv (1-\delta) \times 7 + \delta \times 2 \times 2\delta$. And the holdout would obtain a value of $\frac{1}{3}(7 - (7 - 7\delta + 4\delta^2)) = \frac{1}{3}(7\delta - 4\delta^2)$. Therefore, the principal can initially promise only to pay each agent $\frac{1}{3}(7\delta - 4\delta^2)$ since this is the maximum payoff they each would obtain were they to hold out. The principal's value is thus

$$8 - 3 \times \frac{1}{3}(7\delta - 4\delta^2) = 8 - 7\delta + 4\delta^2. \tag{36}$$

I plot this function in Figure 2. Notice that it is non-monotone: The principal's value is decreasing in commitment (increasing in δ) when $\delta > 7/8$.

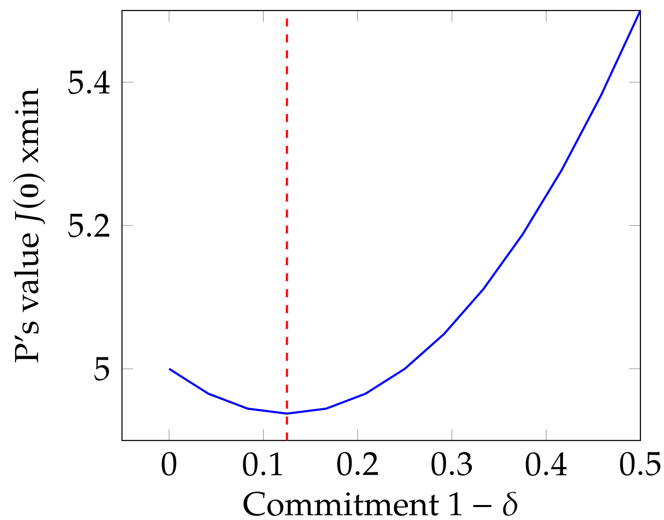

Figure 1: Principal's value function $J(\mathbf{0}) = v(\mathbf{0}) - \delta v_1 + \frac{2}{3}\delta^2 v_2$ when $v(\mathbf{0}) = 8, v_1 = 7, v_2 = 6$

4.2.3 Credible Contracts: General Characterization

I derive, along with the proof of the existence and uniqueness, a recursive characterization of the solutions using the principal's value function and the maximum possible punishment. The challenge with directly solving the problem is that the dimensionality of the contracting space is too large. This problem can be overcome by reducing the problem into a single-dimensional optimization problem, the maximum possible punishment on each holdout profile h.

Proposition 8. *The pair of vectors $\{J^*(h), \bar{x}^\delta(h)\}_{h\in\{0,1\}^N}$ is the pair of the principal's value function J^* and the maximum punishment $\bar{x}^\delta$ at each node h if and only if they satisfy the following recursive relation*

$$J^*(h) = v(h) - \underline{x}(h) - \sum_{j\notin\xi(h)} R_j^O\left(v(h) - \underline{x}(h), h\right) \tag{37}$$

where

$$\underline{x}(h) := \sum_{i\in\xi(h)} R_i^O\left(v(h+e_i) - \bar{x}^\delta(h+e_i), h+e_i\right) \tag{38}$$

is the minimum punishment to implement h, and

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h\cdot R^O(v(h)-x, h) + x = v(h) - \delta J^*(h)\} \tag{39}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$.

The general characterization allows me to explicitly solve the case of the takeover for any number of agents. I first present a recursive characterization of the amount of credible punishment the principal can impose on each action profile. Then, I will provide a closed-form solution to this recursive equation, which provides an explicit formula for the amount of punishment that is credible using a contingent contract.

Lemma 3. *When* $\{R_i^O\}_i$ *are equity contracts, i.e.,* $R_i^O(v, h) = \alpha_i v$ *for all* h*, the maximum possible punishment on the action profile* h *satisfies the recursive relation*

$$\bar{x}^\delta(h) = (1-\delta)v(h) + \delta \sum_{i \in \xi(h)} \alpha_i (v(h+e_i) - \bar{x}^\delta(h+e_i)) \quad \forall h \neq \mathbf{1} \tag{40}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$ *if either* $\sum_{i=1}^N \alpha_i = 1$ *or* $v(\mathbf{1}) = 0$.

The maximum possible punishment the principal can credibly impose at h, i.e., $\bar{x}^\delta(h)$, is a convex combination of the payoff she can credibly give to the tendering agents at h and the total asset value, weighted by the discount rate.

(a) The first term $(1-\delta)v(h)$ is the deadweight loss due to renegotiation: the size of the pie shrinks by $(1-\delta)v(h)$ whenever she wants to renegotiate, so she could impose at least that much to the holdout by paying the tendering agents.

(b) The second term is the sum of the discounted payoff to each tendering agent, which is as much as his holdout payoff. Since the principal has to pay at least what each tendering agent would receive if he holds out, she is not willing to renegotiate with them if the promised value is less than the discounted value of what the principal would otherwise have to pay each.

The initial condition says no punishment is feasible when everyone holds out if i) all agents hold all equity or if ii) there are some agents outside the game, but the asset value is zero. Otherwise, if there's some third-party agent who holds a fraction of the firm and the asset is not worthless, then the principal is able to create some punishment when all agents hold out by diverting some asset value to this third-party agent.

When $\delta = 1$, i.e., the principal has the least commitment and can renegotiate at no additional cost, for each of the tendering agents, the maximum payoff that can be credibly promised to him is his contractual payoff from the asset value available to him when he deviates: $v(h+e_i) - \bar{x}^\delta(h+e_i)$,

The lemma gives a hint on the alternating structure of the punishment: A severer punishment upon further deviation would reduce the maximum credible punishment on path because each tendering agent A_i, if otherwise holding out, would receive a lower payoff due to a higher threat. This makes promising a higher payoff to A_i at h less credible as the principal has a higher incentive to renegotiate. On the contrary, a higher asset value $v(h+e_i)$ on deviation profile $h+e_i$ would increase the maximum punishment at h as the tendering agents would get more if they hold out and hence must be compensated more at h.

Proposition 9. *For equity contracts, the maximum possible punishment on action profile h takes the following alternating multi-linear form*

$$\bar{x}(h) = (1-\delta)v(h) + \sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{\sigma \in \Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)} \right) \tag{41}$$

where $\xi(h) = \{i : h_i = 0\}$ is the set of tendering agents and $\Sigma(\xi(h))$ is the set of all the permutations on $\xi(h)$. The highest payoff the principal can credibly obtain at $\mathbf{0}$ is

$$J(\mathbf{0}) = v(\mathbf{0}) + \sum_{k=1}^{N} \frac{(-\delta)^k}{(N-k)!} \sum_{\sigma \in \Sigma(\mathcal{N})} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v\left(\sum_{s=1}^{k} e_{\sigma(s)} \right). \tag{42}$$

This result shows how contractual structure and the asset value at each k-step deviation profile $h + \sum_{s=1}^{k} e_{\sigma(s)}$ affects the maximum possible credible punishment at h. The first component $(-\delta)^{k+1}$ captures the alternating structure. Since we only to count the k-step deviation path from h once, the sum over all the permutations on $\xi(h)$ over-count the number of paths since it also includes all the paths further deviating from the k-step deviation profile, and the term $\frac{1}{(|\xi(h)|-k)!}$ is used to offset the repeated counting.[42]

I also derive the more complicated δ-credible contracts when debts are outstanding in Section D.1, which exhibits discontinuity and non-responsiveness. The special

[42] This is similar to the factorial in the Shapley value where all possible paths of length N are summed over. Differently, here we sum over all possible paths of length $N-k$ starting at a particular node with k tendering agents.

case illustrates even the non-monotonicity also depends crucially on the set of initial contracts.

5 Property Rights

5.1 Modeling Property Rights

The previous analysis assumes the dilutability of all existing contracts. In reality, property rights protection[43] insulates them from being diluted: Secured debts are protected by the property rights of the collateral from subordination.[44] Holdout in the land acquisition can nevertheless stick to the value of his house if he does not accept the offer.[45] Contractual rights provide protection against the contracting party (the principal), whereas property rights also provide protection against everyone else (Ayotte and Bolton, 2011). This section aims to answer how the ability to solve holdout problems is affected by property rights protection. It turns out that with limited commitment, higher investor protection could lead to an easier resolution of the holdout problem.

To capture property rights protection, in each agent A_i's payoff, there is now an additional term $\pi_i \geq 0$, called "property value", if he holds out. This term is independent of other agents' action and does not come from the value creation of the project.[46] That is, the utility at h, when the value distributed among holdouts is $v - x$,

[43] I do not discuss the optimal allocation of property rights here. Readers can resort to Segal and Whinston (2013) for reference.

[44] Secured interest, though, can be diluted in DIP financing, for example, via *uptier transactions*. It is usually subject to court scrutiny, and obtaining the approval is hard, though not impossible. In the milestone case LCM XXII Ltd. v. Serta Simmons Bedding, LLC, the debtor issued two tranches senior to its existing first-lien debts and the court confirmed its legality.

[45] There are subtle differences between the two types of property rights: In the latter case, the "house" is destroyed once the land owner accepts the offer, and the surplus is generated by allowing the developer to utilize a bigger chunk of the land; In the former, the "collateral" is released once the secured creditor accepts the offer. (Whether the new offer is secured by collateral doesn't matter since the value distribution is immediate.) However, they can be unified in modeling by viewing the unencumbered collateral as the value created from the exchange offer instead of the value of the old collateral. I will treat the properties as if they are "houses" in the general definition and show that it can include "collaterals" by normalizing the asset value.

[46] By assuming this, I exclude another layer of coordination problem when the property is owned collectively among the agents; or more complicated cases where a piece of collateral has multiple liens over it. Moreover, this formulation may not cover other types of investor protections that are state-contingent. For example, creditors insured by credit default swaps would get the additional payment only when the borrower defaults.

is $R_i^O(v-x,h)+\pi_i$.[47] And consequently the problem to implement h can be written as

$$\max_{R(\cdot,\cdot)} v(h) - \sum_{i=1}^{N}(1-h_i)\,R_i(v(h),h) - \sum_{i=1}^{N} h_i\,R_i^O\left(v(h) - \sum_{i=1}^{N}(1-h_i)\,R_i(v(h),h),h\right) \quad (43)$$

subject to the agents' IC constraints

$$h_i \in \arg\max_{h_i' \in H_i}(1-h_i')R(v(h_{-i},h_i'),(h_{-i},h_i')) \quad (44)$$

$$+h_i'\left[R_i^O\left(v(h_{-i},h_i') - \sum_{i=1}^{N}(1-h_i')\,R_i(v(h_{-i},h_i'),(h_{-i},h_i')),(h_{-i},h_i')\right)+\pi_i\right]\ \forall i \quad (45)$$

and the credibility constraints $R \in \mathcal{C}^\delta(h)$.[48]

We assume accepting the offer is always efficient even taking the properties that are destroyed into consideration:[49]

Assumption A3 (Monotonicity with property rights)**.** *$v(h_{-i},0) > v(h_{-i},1) + \pi_i$ for all h_{-i} for all $i \in \mathcal{N}$.*

The main result of this section is to show that higher property rights protection always makes restructuring harder under full commitment, but it can make restructuring easier under limited commitment. Nonetheless, for commonly used securities such as debt and equities, a small increase in protection always leads to a more difficult situation.

First, it's worth noticing the simplification result in Proposition 18 no longer holds as the property rights cannot be diluted by contractual externalities, and thus Proposition 2 would not hold. But the principal is still extremely powerful by deploying contingency: She can extract all the value unprotected by the property rights by creating contractual externalities. Thus, higher property rights protection hinders restructuring.

Proposition 10. *With full commitment, greater property rights protection exacerbates the*

[47] Note if the property is a collateral and the value goes back to the firm when the creditor accepts the offer and is available to be paid to other agents, we could define an alternative value $\tilde{v}(h) := v(h) + (1-h_i)\pi_i$ and replace the occurrence of v by $\tilde{v}$ in the formulation of the problem. We model this way because the notation is simpler.

[48] The definition of the credible contracts is the same except the additional term π_i in the agent's payoff of holding out in the set of incentive compatible contracts. The existence and uniqueness of credible contracts with property rights protection can be proved similarly to Proposition 7 *mutatis mutandis*.

[49] Note if we use the other notation as in footnote 47, this is simply monotonicity of $\tilde{v}$: $\tilde{v}(h_{-i},0) > \tilde{v}(h_{-i},1)$.

holdout problem. More specifically, the principal's value at $\mathbf{0}$ *is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \pi_i \tag{46}$$

which is always decreasing in π_i *for all* i.

Intuition is simple: The principal only needs to compensate each claim holder the amount of the property; the remaining claims can be diluted by the contractual externalities. Thus, more protection implies more compensation for the existing contract holder and lower value for the principal.

5.2 Greater Protection Facilities Restructuring: A Negative Example

I first construct an example showing that a higher property right protection could increase the principal's value, facilitating restructuring.

Let there be 3 agents, each with a property value π_i and a claim that resembles a "kinked equity" (or debt if $\beta_i = 0$)

$$R_i^O(v, h) = \alpha_i v + (\beta_i - \alpha_i)(v - \hat{v}_i)\mathbb{1}_{v \geq \hat{v}_i} \quad \forall h : i \notin \xi(h) \tag{47}$$

for some parameters $\{\alpha_i, \beta_i, \pi_i, \hat{v}_i\}_i$. I find a set of parameters such that greater property rights protection facilitates restructuring in the next proposition.

Proposition 11. *There exists a set of initial contracts such that a locally small increase in property rights protection facilitates restructuring. In particular, let* $\hat{v}_1 = \hat{v}_3 = 1, \hat{v}_2 = 98/100$, $\pi_1 = \pi_2 = 1/100$ *and* $\pi_3 = 99/100$, $\alpha_2 = 7/10, \alpha_1 = \alpha_3 = 1/10, \beta_1 = \beta_2 = 1/10, \beta_3 = 7/10$. *Let* $v(\cdot)$ *be such that* $v(\mathbf{1}) = 0, v(\mathbf{0}) = 3, v(e_i) = 2, v(\mathbf{1} - e_i) = 1$ *for all* i. *The principal's value function* $J(\mathbf{0})$ *is increasing in* π_1 *at the parameters specified above.*

This example shows how property rights protection could facilitate the restructuring by giving the principal less bargaining power in renegotiation and, thus, more commitment to the punishment. The protection still undermines the principal's bargaining power initially, so the compensation off-path must exceed this direct effect. For this to be the case, the structure has to be made asymmetric as it's restricted by the 1-Lipschitz continuity. In my example, R_2^O (resp. R_3^O) has a large payoff sensitivity when the

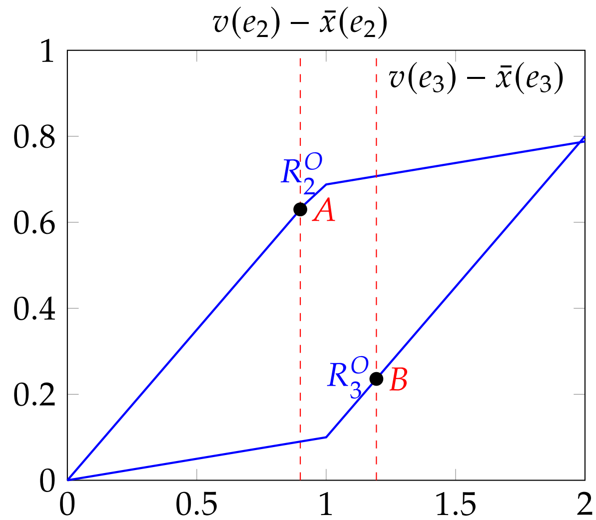

Figure 2: Principal's value function $J(\mathbf{0}) = v(\mathbf{0}) - \delta v_1 + \frac{2}{3}\delta^2 v_2$ when $v(\mathbf{0}) = 8, v_1 = 7, v_2 = 6$

asset value accrues to the holdout is small (resp. large). When A_2 holds out, since π_3 is large, a one-dollar increase in π_1 would reduce A_2's payoff by α_2, i.e., the payoff sensitivity evaluated at point A. This is multiplied by a discount factor $1 - \alpha_3$, reflecting the renegotiation when A_3 also holds out. Similarly, when A_3 holds out, a one-dollar increase in π_1 would reduce A_3's payoff by $\beta_3(1 - \beta_2)$ since it is evaluated at the point B.

Despite the quirky example I show above, when the existing securities are the more commonly seen contracts, such as debts or equities, a locally small increase in property rights protection usually exacerbates the holdout problem even in the limited commitment case. Indeed, for equity holdouts, this is even true for large increases in property rights while for debt, it can be reversed.

5.3 Effect of Property Rights with Equity Holdouts

In contrast, when existing contracts are equities, no matter the structure, a higher property rights protection never leads to an easier resolution of the holdout problem.

Proposition 12 (Property rights hinder equity restructruring)**.** *For any equity contracts $\{\alpha_i\}_i$, the prinicpal's value $J(\mathbf{0})$ under δ-credibility for any $\delta \in (0, 1]$ is decreasing in π_i for all $i \in \mathcal{N}$.*

The result says that in spite of the countervailing forces that greater property rights protection bolsters her commitment, this indirect force will nonetheless not exceed the direct force that makes restructuring harder. The reason is that each indirect effect is weighted by the equity payoff sensitivities $\{\alpha_i\}_i$ which also sum up to less than one off path.

To see the force more clearly, let's look at a specific example. Let the existing contracts be equities $\alpha = \{\alpha_i\}_{i=1}^3$ such that $\langle \alpha, \mathbf{1} \rangle = 1$. And to simplify the exposition, I assume $\delta = 1, v(\mathbf{1}) = 0$. The property values are $\pi_i \geq 0$.

Example 5.1 (Property rights hinder equity restructuring: 3-agent example)**.** *With limited commitment, the value function of the principal at* $\mathbf{0}$ *with equities outstanding is decreasing in each* π_i*,*

$$\frac{\partial}{\partial \pi_i} J(\mathbf{0}) = -\left(1 - \sum_{j \neq i} \alpha_i (1 - \alpha_k)\right) < 0 \textit{ for } k \neq j, i \quad \forall i. \tag{48}$$

The closed-form solution for the sensitivity of the principal's value to property rights protection illustrates the trade-off of the two forces. The direct effect is a one-to-one reduction in P's value and the indirect effect is summarized in the other term. This renegotiation channel is shadowed when equities are in place because higher protection of A_1 also makes punishing A_2 easier, but only at a rate smaller than 1: It is the equity sensitivity to the asset, α_2. Similarly, the effect of punishing A_3 is also dampened by the equity sensitivity α_3. Since the sum of all equity shares adds up to 1, the indirect effect is always smaller than one.

5.4 Effect of Property Rights with Debt Holdouts

The effect of property rights is more nuanced when existing securities are debt contracts. For any locally small increase in property rights protection, it always makes restructuring harder, but when the increase is large, it could backfire. I show the two effects in the next two propositions.

Proposition 13 (Property rights generically hinder debt restructruring)**.** *For any debts contracts* $\{D_i\}_i$*, the prinicpal's value* $J(\mathbf{0})$ *under* δ*-credibility for any* $\delta \in (0, 1]$ *is generically locally decreasing in* π_i *for all* i*. That is,*

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} < 0 \tag{49}$$

at any differentiable points.

When creditors are protected by property rights, the force that makes renegotiation harder for the principal does not get transmitted to the initial bargaining due to the fact that a holdout creditor is either repaid in full or not at all. Thus, the effect of a small change in the protection that increases the punishment does not get transmitted from the off path renegotiation since the maximum credible punishment has a discontinuity and is flat in each region. Notice, however, that this effect only applies to a small increase in π_i away from the boundary.

Now, I show that when the existing contracts are debt, a non-locally-small increase in property rights protection could indeed facilitate debt restructuring. Let there be two agents: agent A_i has a debt value of $D_i = 1$ for all $i \in \{1, 2\}$. The asset value is $v(\mathbf{1}) = 0$, $v(e_i) = 2$ for all i and $v(\mathbf{0}) = 3$. And for simplicity, we assume $\delta = 1$. For the property value, we focus on the region where $\pi_i \in [1/2, 3/2]$ for all i.

Proposition 14. *With limited commitment, the principal's value in the 2-creditor example is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{2} \left[D_i \mathbb{1}_{\{v(e_i) \geq \pi_j + D_i\}} + \pi_i \right] \tag{50}$$

Given the parameters above, the principal's value increases when the property rights of A_j increases from any value $\pi_j \in (1/2, 1)$ to any $\pi_j + \Delta\pi_j \in (1, 3/2)$.

This result says the effect is different when a change in property rights is large enough to "switch the regime". When π_j is small, the principal needs to pay A_i in full if he holds out because she cannot credibly pay more to A_j to punish A_i. But when π_j is slightly larger, above the threshold, she can more credibly pay A_j to punish A_i, which reduces her initial compensation to A_i.

These results echo the finding that higher creditor protection could facilitate or hinder restructuring in Donaldson et al. (2020). Both non-monotonicity stems from the principal's lack of commitment: She cannot commit to a renegotiation policy here and to a bankruptcy filing policy in theirs. Here, higher property rights protection of the creditors has a direct effect of making the restructuring harder but an indirect effect of making the principal more credible when punishing other creditors. In theirs, a more creditor-friendly policy has a direct effect of making priority more attractive but an indirect effect of making a bankruptcy filing less likely, reducing the appeal of priority.

6 Unifiying Notions of Credibility

So far I have introduced the concepts of strongly δ-credible contracts $\mathcal{S}^{\delta}(h)$ and δ-credible contracts $C^{\delta}(h)$ and it's straightforward that $C^{\delta}(h) \subsetneq \mathcal{S}^{\delta}(h)$. Clearly, when $\delta = 0$, they coincide in the degenerate case – full commitment. Yet, it is not very clear what the relationship between the two concepts is as the set $C^{\delta}(h)$ is much smaller than $\mathcal{S}^{\delta}(h)$. In this section, I introduce an intermediate notion, k-step δ-credible contracts, to unify the two notions, which capture the case when the principal is committed after k rounds of (re)negotiations. I will show that our recursive definition of the δ-credibility is the limiting case of this intermediate credibility notion when k is sufficiently large.

Definition 7 (k-step δ-Credible Contracts)**.** *A contract R is a k-step δ-credible contract for some $\delta \in [0, 1]$ at an action profile h if and only if i) it is incentive compatible for the agents at the action profile h and ii) at any unilateral deviation profile $\hat{h}$, it weakly δ-dominates all $(k-1)$-step δ-credible contracts at $\hat{h}$. The 0-step δ-credible contract is simply the set of incentive compatible contracts at h.*

Formally, $C_k^{\delta}(h)$, the set of k-step δ-credible contracts at h, is given by

$$C_k^{\delta}(h) = \left\{ R \in \mathcal{I}(h) : \ R \geq_{\delta} \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in C_{k-1}^{\delta}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\} \qquad \forall\, k = 1, 2, \cdots \quad (51)$$

with the initial condition $C_0^{\delta}(h) = \mathcal{I}(h)$.

It's direct from the definition that the strongly δ-credible contracts are simply the 1-step δ-credible contracts, i.e., $\mathcal{S}^{\delta}(h) = C_0^{\delta}(h)$. To link it to the recursively defined δ-credible contracts $C^{\delta}(h)$, we want to look at the case when k approaches infinity. Unfortunately, the sequence $C^{\delta}(h)$ is not always a monotone sequence, which makes the characterization a little bit harder. But nevertheless it has the following oscillating structuring.

Lemma 4. *The even subsequence of $\{C_k^{\delta}(h)\}_k$ is weakly decreasing and the odd subsequence is weakly increasing. That is,*

$$C_{2k}^{\delta}(h) \subset C_{2k-2}^{\delta}(h) \text{ and } C_{2k-1}^{\delta}(h) \subset C_{2k+1}^{\delta}(h) \quad \forall h\, \forall k = 1, 2, 3, \cdots \qquad (52)$$

This allows us to obtain the limits of the two subsequences

$$\lim_{k\to\infty} C^{\delta}_{2k+1}(h) = \bigcup_{k\geq 1} C^{\delta}_{2k+1} \quad \text{and} \quad \lim_{k\to\infty} C^{\delta}_{2k}(h) = \bigcap_{k\geq 1} C^{\delta}_{2k}. \tag{53}$$

Moreover, the two subsequences are "separated."

Lemma 5. *The odd subsequence never exceeds the even subsequence. That is,*

$$C^{\delta}_{2k+1}(h) \subset C^{\delta}_{2k}(h) \quad \forall h \,\forall\, k = 1, 2, 3, \cdots \tag{54}$$

And as a corollary, $\lim_{k\to\infty} C^{\delta}_{2k+1}(h) \subset \lim_{k\to\infty} C^{\delta}_{2k}(h)$.

Now let's introduce the standard definition of limsup and liminf.

$$\limsup_{k\to\infty} C^{\delta}_{k}(h) := \bigcap_{k\geq 1}\bigcup_{j\geq k} C^{\delta}_{j}(h) \quad \text{and} \quad \liminf_{k\to\infty} C^{\delta}_{k}(h) := \bigcup_{k\geq 1}\bigcap_{j\geq k} C^{\delta}_{j}(h) \tag{55}$$

And by definition $\liminf_{k\to\infty} C^{\delta}_{k} \subset \limsup_{k\to\infty} C^{\delta}_{k}$. Using de Morgan's Law and the two lemmata above, I can write them as

$$\limsup_{k\to\infty} C^{\delta}_{k}(h) = \left(\lim_{k\to\infty} C^{\delta}_{2k}(h)\right) \bigcup \left(\lim_{k\to\infty} C^{\delta}_{2k+1}(h)\right) = \lim_{k\to\infty} C^{\delta}_{2k}(h) \tag{56}$$

$$\liminf_{k\to\infty} C^{\delta}_{k}(h) = \left(\lim_{k\to\infty} C^{\delta}_{2k}(h)\right) \bigcap \left(\lim_{k\to\infty} C^{\delta}_{2k+1}(h)\right) = \lim_{k\to\infty} C^{\delta}_{2k+1}(h). \tag{57}$$

This allows us to show that $C^{\delta}(h)$ is the limiting case of the k-step δ-credible contracts.

Proposition 15. *The recursively defined $C^{\delta}(h)$ in Definition 6 satisfies*

$$\liminf_{k\to\infty} C^{\delta}_{k}(h) \subset C^{\delta}(h) \subset \limsup_{k\to\infty} C^{\delta}_{k}(h) \quad \forall h \tag{58}$$

This result suggests that the recursively defined credibility is the limiting case when the number of rounds of negotiations in which the principal cannot commit goes to infinity. In particular, when the $\liminf_{k\to\infty} C^{\delta}_{k}(h) = \limsup_{k\to\infty} C^{\delta}_{k}(h)$, the limit is well-defined and we have $\lim_{k\to\infty} C^{\delta}_{k}(h) = C^{\delta}(h)$. But in general, the liminf and limsup are not identical.

Proposition 16. *There exists a set of initial contracts R^O such that* $\liminf_{k\to\infty} C_k^\delta(h) \subsetneq \limsup_{k\to\infty} C_k^\delta(h)$ *for some h.*

This result shows that the limit of $C_k^\delta(h)$ does not always exist as k approaches infinity, and the bounds above cannot be made tighter.

7 Literature

The paper speaks to a large body of holdout problems in practice. Specific problems have been extensively studied, but most of them restrict their attention to specific contractual forms of existing and newly offered contracts and usually do not emphasize the commitment issue. Grossman and Hart (1980) is probably the first to study the holdout problem in the takeover case where a raider offers cash to buy equity shares from a continuum of shareholders. The holdout problem exists in this context because the atomic shareholders do not internalize the externality created by its free-riding. This assumption was relaxed by Holmström and Nalebuff (1992) and Bagnoli and Lipman (1988), who paid more attention to non-atomic shareholders in mixed strategy, trying to solve the holdout situation. Other papers also try to solve the problems by relaxing some constraints in the original setting. Shleifer and Vishny (1986) consider the case with a large shareholder and show it significantly alters the outcome because the large shareholder can split the gain from the takeover between its own shares and the raider's. But it only works because commitment is implicitly assumed. Burkart et al. (2014) studies how legal protection affects the bidding strategy in takeovers. Burkart and Lee (2022) compares free-riding à la Jensen–Meckling in activism vs. free-riding à la Grossman–Hart in takeovers. Gertner and Scharfstein (1991), Bernardo and Talley (1996) and Donaldson et al. (2020) study the holdout problem in the corporate debt restructuring. They demonstrate that offering priority in exchange offers via senior debt could offset the incentive to free-ride as priority dilutes existing creditors' payoff. However, the value priority is endogenously dependent on the probability and recovery in bankruptcy. Thus, to facilitate restructuring, the firm might distort the investment policy. Sovereign restructuring differs from corporate as there's no formal seniority structure, and there is s a greater commitment issue.[50] Bulow et al. (1988); Bulow

[50]Here, I omit many macro models on sovereign debt that do not address the holdout problem.

and Rogoff (1989) study the limit of sovereign bond buyback using cash due to the holdout problems. Kletzer (2003) finds that in a dynamic setting, the principal benefits from a collective action clause as it facilitates bargaining, while a unanimity rule leads to a war of attrition and inefficient outcomes. The difference is that each individual lender can propose to the borrower in their model. Pitchford and Wright (2012) also studies the case when a sovereign can renegotiate with each creditor one by one and has no commitment. Bolton and Scharfstein (1996); Bolton and Jeanne (2007, 2009) discuss the ex-ante vs. ex-post trade-off of making some classes of bonds difficult to restructure. Haldane et al. (2005) and Weinschelbaum and Wynne (2005) also study the holdout problems and the use of CACs. Grossman et al. (2019), Sarkar (2017), Kominers and Weyl (2011, 2012), Miceli and Segerson (2012) study the holdout issues in land acquisition and development. The paper differs from them in that we tend to look at a more abstract setting, allowing for heterogeneity in both the investor composition and contractual forms.

The paper falls broadly in the literature of mechanism design with limited commitment, with two notable distinctions. Most papers study the limited commitment of the principal in mechanism design, such as Bester and Strausz (2000, 2001),Bisin and Rampini (2006) and Doval and Skreta (2022), focus on the issue of information leakage: The principal cannot commit not to use the information the agents reported, and hence the revelation principle might no longer hold when the principal lacks commitment. The literature has assured the audience there is a class of canonical mechanisms that are easy to formulate and rich enough to be payoff- or outcome-equivalent to any mechanisms. This paper studies the complete information environment but with endogenous outside options. Another difference is that most mechanism design papers either have private information or moral hazard but usually do not have existing contracts as an outside option with endogenous values, except for the literature on type-dependent outside options.

The existence of contracts as an outside option is a feature in the literature on the dissolution of a partnership, for example, Cramton et al. (1987), McAfee (1992), Fieseler et al. (2003), Moldovanu (2002), Jehiel and Pauzner (2006), de Frutos and Kittsteiner (2008), Figueroa and Skreta (2012), Loertscher and Wasser (2019). Typically, they discuss the reallocation of the ownership with transfers when each agent has a private value of the asset. Differently from mine, they usually only consider equity contracts in place[51]; there's also no notion of value enhancement and hence no holdout problem. They find the initial endowment matters for the dissolution, while my results emphasize the importance of the contractual forms.

The paper solves a mechanism design problem with an endogenous outside option. One strand of the literature focused mostly on the case where the value of the outside option depends on the agents types.[52] In mine, the value of the outside depends on the action of other agents and the principal's offer. A similar case is considered in Halac et al. (2020) where an entrepreneur seeks to raise capital from heterogeneous agents, and each agent's participation has an externality on others. Their main focus is unique implementation, and they restrict their attention to a return schedule instead of more general contracts. Endogenous outside options are also common in the mechanism design problem with ratification and veto constraints,[53] where a status quo game is played when any agent vetoes the mechanism. A veto could create an externality by signaling agents' types and affect the beliefs in the status quo game, while in this paper, the externality is created by the new contracts the principal could propose. Similar consideration also emerges in the contracting with externality literature.[54] Segal (1999) considered many applications, as in this paper, but he models the commitment by assuming the principal has no incentive to deviates to private bilateral offers which she *can* commit. Moreover, the role of existing contracts is under-explored since he takes the preference for the actions as primitive.

The paper contributes to the theory of credible mechanisms and their implementation, particularly using a negotiation-proof contract. However, the notion of credibility is adapted to the holdout setting. The closest notion is Farrell and Maskin (1989) in which they consider a repeated game, and for an equilibrium to be credible, its continuation equilibrium must also be credible. So, it cannot involve punishment with Pareto-dominated equilibrium since otherwise the agents cannot commit not to renegotiate to a better continuation equilibrium. They provide a notion of weakly renegotiation-proof (WRP) equilibrium by requiring any continuation equilibrium not dominated by others

[51]Since they usually do not have uncertainty, equity is not different from debt or other contracts.

[52]For instance, Lewis and Sappington (1989), Jullien (2000), Figueroa and Skreta (2009), Liu (2016), and Sun et al. (2018).

[53]Several related papers include Cramton and Palfrey (1995), Compte and Jehiel (2009), Dequiedt (2006),Laffont and Martimort (2000), Jackson and Wilkie (2005)

[54]To name a few, Jehiel et al. (1996), Segal (1999), Segal (2003), Segal and Whinston (2003), Gomes (2005).

and of a strong renegotiation-proof equilibrium (SRP) requiring none of its continuation equilibrium to be strictly dominated by a WRP. This is similar but not identical to mine in two aspects: in theirs, the stage game is one shot, and players choose actions simultaneously, while in mine, the principal moves first. This leaves essentially one value for the principal as she would choose the equilibrium with the highest value. Also, the equilibrium definition is not recursive as they only require SRP not to be dominated by WRP. Parallel work by Bernheim and Ray (1989) tries to formalize the idea a WRP equilibrium must be undominated by another WRP in continuation, which they call *internal consistency*, and discuss some conceptual difficulty that arises in the infinite horizon: the set of internally consistent equilibrium is interdependent and not necessarily unique. They further add the *external consistency* requirement that players do not choose WRPs that are dominated by another at the beginning, which, unfortunately, may not exist. Ray (1994) modifies the requirement and obtains a truth internally consistent renegotiation-proof equilibrium. Both issues do not exist in my model as, despite the fact that the renegotiation can take infinite rounds, the specific structure in the holdout problem makes it effectively finite. Rubinstein and Wolinsky (1992) also a similar renegotiation-proof contracting problem in the bilateral trading setting with unverifiable information where the key is for both parties to report their true values willingly. Despite the big difference in the setting, they obtain a similar result to mine: The only renegotiation-proof contract is a state non-contingent when assuming a costless renegotiation is feasible whenever the outcome is inefficient. They also show the set of renegotiation-proof contracts is larger when they introduce time and discounting in the renegotiation process, similar to my requirement of δ-dominance. Bergin and MacLeod (1993) considers a recursive definition but uses an axiomatic approach. Strulovici (2017) and Evans and Reiche (2015) study the renegotiation-proof contracts in the incomplete information setting. Strulovici (2022) provides a characterization in continuous time with persistent states. Chakravorty et al. (2006) consider the same problem when the planner may not want to go through the mechanism for some disequilibrium play in the setting of social choice. Different from this paper, they define a notion of credibility by requiring it to be a "best response" for some preference profile in support of the prior beliefs. They obtain some negative results and show they persist even when they adopt a weaker notion of credibility by requiring it to be consistent with the prior about social utility function instead of social choice correspondence. A notion similar to them is studied in the auction setting by Akbarpour and Li (2020) where the auctioneer can safely deviate when the deviation can be perceived as if it's consistent with another agent's type profile. And they require that a credible auction cannot have such a safe deviation. Shavell and Spier (2002) studied the cases when the principal can neither commit to the punishment when the agent complies nor not to punish the agent when he defies. They show that the equilibrium outcome differs greatly in the infinite horizon setting from the finite horizon. In finance, DeMarzo and Sannikov (2006); DeMarzo and Fishman (2007); DeMarzo et al. (2012) discuss the optimal renegotiation-proof contracts in a continuous-time framework.

The extreme gauging result in the full-commitment benchmark echos the classic result in Crémer and McLean (1988) and McAfee and Reny (1992) that a principal could extract full surplus when the agents' type are slightly correlated. Segal (1999) also derived the unanimity result under commitment. Both this paper and theirs achieve incentive compatibility by imposing severe, possibly non-credible, punishment off-path. Heifetz and Neeman (2006) and Chen and Xiong (2013) study the genericity and robustness of full-rent extraction results for general mechanisms.

The non-monotonicity result that higher commitment doesn't always lead to a higher payoff also appears in other contexts. Kovrijnykh (2013) derives a similar non-monotonic effect of commitment in lending contracts. The key intuition is similar: "just as commitment increases the lender's payoff in an optimal equilibrium, it increases his payoff from the most profitable deviation."(Kovrijnykh, 2013, p.2850) However, she mainly focuses on bilateral bargaining and renegotiation, while mine is multilateral. She models repeated interaction, while mine is essentially static but with the possibility of entering a multi-period renegotiation in case of a deviation. In hers, the contract is void with some exogenous probability, while in mine, the renegotiation of the current contract is endogenously determined by the principal's payoff from continuing the proposal upon deviation. In Donaldson et al. (2020a,b), they proxy commitment with pledgeability (Proposition 1) and collateralizability (Proposition 4) and show that both might lead to lower ex-ante payoff: higher pledgeability might hurt borrowers due to excessive power to dilute initial creditor at the interim financing stage, leading to the impossibility of lending ex ante; higher collateralizability could harm borrowers by over-collateralization, which leads to impossible interim financing.

8 Discussions

8.1 Discussions of Assumptions

Asset Value Microfoundation The paper assumes the asset value is decreasing in the holdout profile but is silent about why. I present several canonic ways of microfounding this assumption here, based on agency theory, costly default, and liquidity injection.

Imagine first the case of takeovers where each initial shareholder has a share of α_i. After acquiring the firm, the raider could exert an effort $e \in \mathbb{R}_+$ to improve the asset value from 1 to e, which incurs a quadratic cost e^2. Given the holdout profile h, the raider has a fraction $1 - h^\top \alpha$, and he optimally chooses the effort to maximize his payoff from his equity shares, i.e.,

$$\max_e (1 - h^\top \alpha) e - e^2. \tag{59}$$

The optimal effort and the corresponding asset value is $v(h) = e^* = 1 - h^\top \alpha$, a decreasing function of h as I assumed earlier.

Imagine a different case of debt restructuring where each creditor A_i holds a debt with a face value of D_i. There is an underlying asset whose value e follows a distribution G, independent of the capital structure. There is a chance for the firm to file bankruptcy, which destroys a fraction $1 - \lambda$ of the asset value, but the firm is able to obtain a fraction β of the remaining asset value. So the firm files if and only if

$$\lambda \beta e \geq e - h^\top D \implies e \leq \frac{1}{1 - \lambda\beta} h^\top D \tag{60}$$

The expected value before the underlying asset value realization is thus

$$v(h) = \mathbb{E}[e] - \int_0^{(1-\lambda\beta)h^\top \boldsymbol{D}} \lambda v \mathrm{d}G(v). \tag{61}$$

This function is more complicated and non-linear but is also a decreasing function of h.

In a DIP financing scenario, the firm offers securities to existing creditors in exchange for liquidity injection. Let l_i be the liquidity the A_i injects into the firm and $\boldsymbol{l} = (l_1, \cdots, l_N)$; then the asset value would be

$$v(h) = v(\mathbf{0}) + (\mathbf{1} - h)^\top \boldsymbol{l} \tag{62}$$

which is a linear decreasing function of h.

Security Design and 1-Lipschitz Continuity This paper considers a large set of feasible contracts, which is typical in the security design literature. One restrictive assumption I put is the 1-Lipschitz continuity of the existing contracts. This is closely related to the literature on security design, for example, DeMarzo et al. (2005), which considers the case where a group of bidders can each offer an arbitrary security in an auction. This paper differs in several dimensions: Firstly, here, the contracts are offered by one principal, not multiple agents; Secondly, the agents are endowed with contracts instead of nothing. As a result, there has to be a system of contracts in and out of equilibrium instead of just a bilateral contract. I implicitly assume there's a bankruptcy system that resolves the conflicts among contracts. Lastly, they require the newly offered contracts to be increasing and 1-Lipschitz[55] while I require it to be satisfied by the existing contracts. One can view the primitive contracts in my paper as the solution to a security design problem in theirs.

Some contracts that allow additional contingency may fail 1-Lipschitz Continuity. For instance, the Additional Layer 1 (AT1) bondholders are completely wiped out[56] in the Credit Suisse crisis[57] and these CoCo bonds are not captured by this assumption. Nevertheless, there is also no need to restructure AT1 bonds as they are wiped out in default anyway, so the generality of the model is not hurt much. Indeed, relaxing this 1-Lipschitz continuity would lead to peculiar situations in which a stronger punishment is more credible. When the existing contract holders have a region where the payoff slope is strictly larger than one or has a jump discontinuity, the payoff slope of the principal[58] would inevitably be negative. Thus, a stronger punishment rewards the principal instead of hurting herself, while the agents are hurt more severely. Therefore, relaxing this assumption only makes restructuring easier to solve, leaving little bite to credibility.

[55]This is a different way to formulate the *feasible securities* in DeMarzo et al. (2005), which requires both contracting parties to have an increasing payoff, which implies 1-Lipschitz continuity.

[56]It's a type of contingent convertibles, commonly known as "CoCo" bond, that can be converted into equity or completely wiped out upon certain triggers, e.g., when the Common Equity Tier 1 (CET1) falls below a certain threshold.

[57]See https://www.reuters.com/markets/why-markets-are-uproar-over-risky-bank-bond-known-at1-2023-03-24/ for a discussion of this event.

[58]The shape of the contracts given to the tendered agents do not matter here because the total payment to them, i.e., the punishment, is the exogenous variable.

Contractual Interdependence and Consistency In the restructuring, new contracts are written to replace the old ones, and off path they coexist. Therefore, the general framework must formulate the new contracts' interaction with the old. The challenge is augmented by allowing an arbitrarily large contract space.[59]

It's obvious how the value of a senior debt would affect a junior one, but less so for other general contracts. The principal could write contradictory terms[60] in the newly issued securities. The effect of the new contracts on the old has to be restricted in a meaningful way. In particular, it seems a minimal requirement for the new contracts to respect the internal consistency of the old ones, not altering the relative distribution (i.e., "priorities") among them. To formulate it, I defined a notion of weak consistency. This is a notion weaker than the *consistency* considered in the cooperative game theory literature (Aumann and Maschler, 1985; Moulin, 2000) as it is only required between the new and old sets of contracts, not among themselves.

An origin of the inconsistency comes from contractual incompleteness: the old contracts cannot enumerate all possible ways the new contracts can affect them. But the intrinsic inconsistency among contracts is not only due to the incompleteness: Even if the future states are perfectly foreseeable and there is no cost of writing or reading long, convoluted contractual terms, a set of contracts still cannot necessarily fully specify the payoff of each agent. For example, suppose there's a contract A, which specifies that the payoff to its holder would be one dollar more than what a contract B, either already existing or to be written, gives to its holder. However, B also specifies that the payoff would be one dollar more than whatever contract A gives its holder. No matter how the allocation is, both contracts cannot be satisfied simultaneously.

One way to solve this problem is to specify the set of allowed dependence, such as using Gödel code in Peters and Szentes (2012), which leads to a much smaller space of contracts. Here, we do not explicitly model allowed contracts; Instead, we assume that the contracts need not be consistent literally and that an exogenous rule exists, encoded in R^O and in R, to resolve the conflicts among themselves. This is similar to the convention of looking at the allocations and payoffs in mechanism design. And the only requirement that needs to be explicitly put is between the sets of old and new

[59]A large contractual space is not necessarily a desired property: Other than the issue considered here, it also allows too many possible deviations as articulated in Brzustowski et al. (2023).

[60]For example, P promises to A_1 that he would be paid one dollar more than A_2 and also to A_2 that he would be paid one dollar more than A_1.

contracts.

Deadweight Loss In the model, we do not explicitly allow the principal to create deadweight loss, that is, burning money, as a way to create punishment. This could be easily incorporated by introducing an additional fictive agent, A_{N+1}, Mr. Deadweight-Loss. He has an outside option of 0, i.e., $R^{O}_{N+1}(v,(h,1)) = 0$, and the principal can implement the unanimity by threatening to allocate the entire asset value to Mr. Deadweight-Loss. I.e., $R_{N+1}(v,(h,0)) = v$ whenever $h \neq \mathbf{0}$. Despite being mechanical, the IC for Mr. Deadweight-Loss is superficially satisfied. The credibility constraint will also be the same as any other agent: It is equally painful for the principal to allocate asset value to Mr. Deadweight-Loss as to any other tendering agents. So, the main insight on how initial contracts limit credible punishment is preserved with or without deadweight loss.

Dependence of Value on Contractual Forms Another caveat in this framework is that the asset value is only a function of the asset value but not of the contract form. This simplification captures most papers of interest, such as Grossman and Hart (1980), Bulow et al. (1988), and Gertner and Scharfstein (1991), but not every other paper. In Donaldson et al. (2020), they study the exchange offers where the principal offers senior debt for junior, and the asset value is a direct function of the contractual forms: A higher face value of debt increases the probability of bankruptcy filing ex post, and hence the deadweight loss.

But this is not too much of a concern with the help of Mr. Deadweight-Loss. We can view the value $v(h)$ here as the highest asset value obtained with a contract that implements h when the value depends on the contract form. For any other contract that leads to a lower asset value, we can equivalently model it as a combination of the original contract and an allocation to Mr. Deadweight-Loss. The only possible concern is that allocating value to Mr. Deadweight-Loss is a decision while the dependence of value on the contract form is exogenously given, so the former might not be credible. But this does not impose a challenge either, as using a different contract that implements h but leads to a lower asset value is also voluntary and is subject, to the same extent, to the credibility constraint as allocating value to Mr. Deadweight-Loss.

Types of Externality In this paper, I focus on the design of externality. I want to distinguish two types of externalities here: *Contractual externality* refers to the case where the payoffs of some securities can be affected by others. For example, junior debts get diluted by senior. And the extent to which the dilution affects the existing contract holder depends on the new contracts. On the contrary, there are property rights unaffected by contractual externalities. For example, the oil company that wants to acquire a block of land can affect the land owners by reducing their available amenities if they hold out, but the owners can nevertheless stick to their own houses, which the oil company cannot feasibly dilute. In debt restructuring, the secured debt holders' interest is protected by the underlying collateral, which, according to Ayotte and Bolton (2011), is a right against all other parties instead of the counter-party in the contract. However, they can nonetheless be affected by *physical externality*. In the oil drilling case, the ability to drill through the adjacent land, whose owners sold to the drilling company, exhibits such a physical externality that contracts cannot directly alter.

However, readers could interpret the discussion on the design of contractual externalities as answers to institutional questions on how laws as social contracts affect the reallocation of interest when the physical externality is present. For example, one can interpret model implication on how the law should split the proceeds of the oil drilled from a common pool as a social contract design that affects the dilutability of these protections, and similarly, whether secured debt should be diluted by super-senior debts in DIP financing or debt exchange offers.

Existing Securities and Ex Ante Contracting The paper assumes the existing securities are exogenously given and uses them as primitives to characterize the optimal exchange offers. I do not discuss the optimality of the existing contracts since it would unnecessarily complicate the model and divert the attention away from the focus of the paper. There is a large strand of literature studying the optimal design of securities, but often, they do not yield the optimal outcomes in reality: The real-world securities may not come from an optimal design; instead, it's the accumulation of multiple issuances over time; The errors in the calibration could lead to substantial ex-post suboptimality in practice (e.g., Piskorski and Seru, 2018); Not all future contingencies, for example, Covid shock, can be captured by ex-ante design. The reality also calls for a necessity for the interim discussion as debt restructurings and takeovers do occur, and the literature has overlooked why ex-ante optimal design does not preclude ex-post complication. Lastly, a better understanding would help us enormously to understand the optimal design ex ante. Typically, this requires modeling the friction in the initial stage: Generally, there is a trade-off between ex-ante debt capacity and ex-post efficiency (e.g., Bolton and Jeanne, 2007, 2009); However, Donaldson et al. (2020) shows that the ex-ante optimal policy could coincide with the ex-post optimal policy because an efficient restructuring also benefits the creditors. Moreover, the optimal ex ante contracts may involve renegotiation on path as in Watson et al. (2020) and Kostadinov (2021).

Binding Voting Mechanisms One crucial restriction the model assumes is that no agents are subject to a binding decision made by the majority or supermajority, which is the essence of the holdout problems. This, in many ways, reflects the reality: Typically, such non-consensual decisions are illegal in the US legal environment as they violate the Trust Indenture Actio 316 (b). For takeovers, even though such binding decisions can be made by the board or via the shareholders' voting, the dissenting shareholders still have the option to litigate against the board in violation of their fiduciary duty. In sovereign debt markets, there use of the collective action clauses does not always solve the issue. It is not obvious whether such provisions are desirable as they might infringe on the rights of some minorities when there is substantial heterogeneity among the agents. The sovereign world started with a two-limbed procedure: only allowing binding decisions within each class of the bonds, and they failed to address the holdout issues. A sweeping one-limbed aggregation mechanism could help to facilitate cramming down the dissenting shareholders but faces a bigger risk of being abused. For example, the Pacman strategy and redesignation[61] has been used to achieve a coercive restructuring in practice.

8.2 Discussion of Renegotiation Protocol

A Naïve Formulation A natural response for the principal without commitment to a handful of holdouts would be to advance as if no holdout occurs, i.e., do off path whatever she has promised on path. For instance, in the example of the unanimity rule in takeovers, the principal promises to buy each share at a price of P if and only

[61]See https://theemergingfrontier.com/home/re-designing-pacman and the article on Financial Times https://www.ft.com/content/2b523aa2-402e-4060-8461-969a2132c483.

if everyone tenders. Upon seeing any holdouts, she may choose to continue buying the tendering shares at the initially proposed price of P. (Note: This is not what was promised initially. What was promised initially was not to buy any shares at all if anyone held out.) But such an idea does not generalize as i) this may not always be feasible: As the total size of the pie is smaller when one agent holds out, the principal may not be able to afford the off-path compensation on path; Also, ii) even though the initial offering is incentive compatible for the agents on path, it may no longer be so off path when other agent deviates and iii) there is no guarantee whether this is optimal for the principal. Therefore, we cannot just naïvely assume that off path, the principal offers exactly what she promised on path. Instead, the principal is "free" to propose some other offers. Even if the principal continues to do what she promised on path, it should be understood as the optimal alternative offer the principal can devise.

Sequential Renegotiation An alternative way to model multilateral bargaining is to specify a sequential protocol. There are multiple ways to specify an extensive game in which bargaining or renegotiation occurs sequentially: i) Shaked's unanimity game, where players propose in order, and any players can veto. The problem with this is that it has many perfect equilibria, and any feasible agreement can be implemented; ii) Legislative Bargaining models where proposers are randomly selected and a binding decision can be confirmed by a less-than-unanimous consent. This approach is plagued with impossibility results like the Condorcet paradox and that the majority core can be empty (Eraslan and Evdokimov, 2019); iii) The exit games considered in Lensberg (1988) where any agent satisfied with his share can leave the bargaining table. This approach requires the consistency axiom I employed in this paper. Krishna and Serrano (1996) showed that the equivalence between Nash's axiomatic solution and Rubinstein's alternating bargaining model extends to the multilateral case given this consistency axiom.

Given the empirical observations that in the holdout problems, there is usually a single entity with the exclusive right to propose and the theoretical consideration that dynamic games either cannot provide a sharp prediction or are equivalent to a static axiomatic one, I adopt the static approach with a possible dynamic game embedded in the credibility condition for simplicity.

The reduced form renegotiation protocol I employed in Definition 6 is similar to the one considered in Stole and Zwiebel (1996): A principal negotiates with a group of agents, and the negotiation outcome between the principal and any agent depends on the potential subsequent renegotiation outcome between the principal and the remaining agents, and recursively so. The differences, though, are that i) Stole and Zwiebel (1996) does not have contracts in place, and the principal is only allowed to offer cash payment, i.e., non-contingent contracts; ii) They do allocate some bargaining power to the agents; iii) I also consider an additional agent joining the bargaining table as a trivial deviation; iv) not only the payoff of the principal but also that of the agent depends on the subsequent renegotiation in mine. In spite of the differences, if the existing contracts' payoffs are independent of the asset value (equivalent to agents' outside option in their model) and if the principal is only allowed to offer non-contingent contracts, the result would be largely the same. Their solution resembles the well-known Shapley value in cooperative games.

Bargaining Power In the paper, I only allow the principal to propose, and as a consequence, she has the so-called "formateur advantage" in political science.[62] This assumption is made to contrast the limited commitment case: Even if only the principal can propose, lack of commitment can fully undermine her ability to restructure the existing contracts, as I show in Proposition 3. As a result, she does not have full bargaining power due to limited commitment, even when agents cannot propose counteroffers.

Regenotiation-Proofness In order to define renegotiation-proof contracts, we need to specify what contracts are reasonable deviations to consider in renegotiation. There is no standard notion of renegotiation-proofness. The most commonly used notion is the two-sided renegotiation-proofness: That is, the principal cannot propose an alternative contract that Pareto dominates the current one, i.e., nobody objects to the alternative offer, and some agent or the principal is strictly better off under this new hypothetical offer. This is only feasible when the principal can bring the holdouts back to the table to increase the size of the pie. But such a requirement would be too strong as it can be difficult to achieve in reality for various reasons. For example, i) the holdouts typically

[62]This is different from the usual "take-it-or-leave-it" offer, which gives the proposer "full bargaining power" to extract all the surplus because commitment is implied in the "take-it-or-leave-it" offer.

are tough to handle, and they usually are not negotiated away, and ii) some laws may prohibit preferential treatment of the holdouts. E.g., in takeovers, the *best-price rule*, or sometimes called *all-holders rule* or *Rule 14D-10*.[63] Moreover, Anderlini and Felli (2001) points out that an agreement may never be reached if there is a possibility of renegotiating out of the inefficient punishment. Therefore, I confine the alternative proposals to the contracts that are incentive compatible with the deviation profile, i.e., that the tendering agents still have an incentive to tender under the potential alternative proposal, and the holdouts are not enticed to tender.

Put differently, similar to Hart and Tirole (1988); Hart (1995) I am implicitly assuming that the principal can unilaterally renege on the proposed offer whenever any agents deviate, and no agent can hold her accountable. Otherwise, the principal can credibly threaten to give the entire firm to a tendering agent, and this agent would block any alternative offer. In this regard, the full-value extraction in Proposition 2 would be credible if we were to impose this stronger condition. The reader can view this renegotiation as if the principal calls off the entire deal and re-proposes an entirely new deal to the tendering agents so that the old proposal doesn't constrain her. This differs from the two-sided renegotiation-proofness because the principal can create no deadweight loss. If the deadweight loss can be explicitly created, then the principal could implement the threat initially by destroying the value to punish the holdout instead of giving the value to some agents, and then no agent would want to block a renegotiation that makes them better off. In this sense, it's more similar in spirit to the reconsideration-proofness in Kocherlakota (1996) or revision-proofness in Asheim (1997).

Off-Path Belief Empirical facts aside, there is also a long-standing theoretical complication of specifying off-path belief in renegotiation. After observing an off-path behavior, i.e., a hold-out, if the principal proposes the exact same offer, would it be accepted by everyone? If so, why would the holdout reject it the first time but accept it the second time? This is a classic *backward induction paradox* in game theory and in philosophy. Binmore (2007) offers a nice discussion of many attempts to reconcile it.

[63] This is Code of Federal Regulations §240.14d-10, which can be traced back to the 1968 Williams Act Betton et al. (2008). See `https://www.law.cornell.edu/cfr/text/17/240.14d-10`. SEC also provides a detailed discussion of this rule and possible exemptions in *17 CFR PARTS 200 and 240*. See `https://www.sec.gov/rules/final/2006/34-54684.pdf`

The latest development to my knowledge is to Asheim and Brunnschweiler (2023), who propose an epistemic foundation using non-Archimedean probabilities. In bargaining, a workaround is to introduce an additional restriction that one agent cannot agree to an offer that he has rejected before as in Fershtman and Seidmann (1993).

One classic rationale to the puzzle is to approximate it with an environment of multiple types, as in Kreps et al. (1982). Suppose there is a small probability that the agent is irrational and always holds out with a small probability.[64] Rejection of an incentive-compatible offer sends signals about the type of holdouts, and all other players update their preference in the subsequent renegotiation. Equilibrium in such a dynamic game would converge to my reduced-form game in the baseline model. The literature on ratification and mechanism design with veto constraints (e.g., Cramton and Palfrey, 1995) generally takes this signaling game approach.

Notion of Credibility The notion of credible contracts borrows a lot of insights from the literature on credible equilibria in dynamic games. The closet solution concept is *internally renegotiation-proof equilibrium sets* in Ray (1994) in the context of infinitely repeated games: The set of renegotiation-payoff is required to coincide with the set of all payoffs that can be supported as equilibria by all continuation payoffs that are restricted to be renegotiation-proof. This is a natural extension of the corresponding concept in the finite horizon and sorts out the technical difficulty in several previously developed notions of *Weakly/Strongly Renegotiation-Proof Equilibrium* in Farrell and Maskin (1989), which are not fully recursive, *Strong Perfect Equilibrium* in Rubinstein (1980), which sometimes fails to exist, and *Internal/External/Minimal/Simple Consistency* in Bernheim and Ray (1989), which could rule out some attractive and not rule out some unattractive equilibria. Pearce (1987) proposes another version of renegotiation-proofness that captures the intertemporal consistency for the infinitely repeated games. Despite the fact that our game is one-shot, there might be infinitely repeated negotiations. Other related notions include *Simple/Optimal Penal Code* in Abreu (1988), *Recursive Efficiency* in Bergin and MacLeod (1993) in the setting of repeated games. Pearce (1991) provides a survey. Kletzer and Wright (2000) and Bulow and Rogoff (1989) consider a repeated lending, borrowing, and recontracting model where the sovereign can repeatedly

[64]Note this is very different from a trembling-hand argument. If the mistake is caused by a trembling hand, the principal will offer the same contracts.

renegotiate with the lender using the Rubinstein bargaining protocol, which doesn't work in our multilateral setting.

I do not explicitly model the form of renegotiation but take a reduced-form approach as in Maskin and Moore (1999) except that here the renegotiation outcome is endogenous. The reader can think of it as an extensive-form game where the principal can propose a new contract to replace old ones whenever there's a deviation.

Costly Renegotiation The possibility of renegotiation generally limits the set of implementable outcomes, but not always. Evans (2012) finds that if renegotiation involves a small cost, then any Pareto-efficient, bounded social choice function can be implemented in SPNE. When the outcome is inefficient, contracting parties may want to renegotiate out of it. But they would not want to do so if renegotiation itself is a punishment. Anderlini and Felli (2001) points out that when renegotiation involves a cost, it is possible that the unique equilibrium is one in which an agreement is never reached unless an inefficient punishment cannot be renegotiated out of. Rubinstein and Wolinsky (1992) shows that if the renegotiation involves a delay, then the set of implementable outcomes is generally larger. However, the exact knowledge of time preference may not play a role. This paper confirms the general insight that more costly renegotiation reduces the incentive to renegotiation and can allow the principal to implement a better outcome but also points out that the effect can be locally non-monotone. In addition, Proposition 22 also documents an irrelevance of renegotiation cost and the discontinuity in the discount rate similar to Rubinstein and Wolinsky (1992, Corollary on p.611).

Role of Discounting One way I interpret the parameter δ is the principal's discount factor, a proxy for commitment. One would naturally expect the discount factor of the agents would have the opposite effect. But the role of discounting can be quite nuanced here: Intuitively, if the agents are more impatient, then they are more willing to accept the offer the principal proposed since the delay caused by holdout and renegotiation is costly. And it gives the principal advantages in bargaining, which probably alleviates the holdout problem. In the extreme case, if the agents have a discount rate of zero, they would accept any offer instead of entering renegotiation because the discounted payoff from holding out is zero. However, the holdout problem can also be easier to solve for an impatient principal: She can stick to some punishment that hurts herself instead of renegotiation since renegotiation also destroys some value for her. In either case, the incentive to renegotiate is diminished by impatience, and it benefits the principal.

Moreover, one may suspect that if the agents, or both parties, have a discount factor of zero, the game should revert to the static setting plagued by the holdout problem. This is not necessarily the case! Even though a discount factor of zero makes future payoff irrelevant, it gives the principal a credible threat to destroy the value through renegotiation, which may not be available in the static setting. Put differently, a holdout receives the payoff from the existing claims in the static setting; in contrast, a holdout receives nothing if he holds out when the discount factor is zero, and the principal can commit to renegotiation.

Renegotiation with Tendering Agents In the potential renegotiation and the formal definition of credibility in Section 4, the renegotiation protocol I laid out on possible punishments via "dilution" is effectively a renegotiation with the tendering agents instead of with the holdouts. It's meant to capture the principal's lack of commitment to the punishment.[65]

Empirically, holdouts are usually not easily renegotiated away and they extract significant value from sticking to their initial contracts. As mentioned above, holdouts in Greek debt restructuring are paid in full. In *Elliott Associates, L.P. v. Banco de la Nacion and The Republic of Peru*, the holdout creditor purchased bonds with a total face value of 21 million for 11 million and received 58 million in settlement for the principal and accrued interests (Alfaro and Vogel, 2006). Moreover, renegotiation with the holdouts could be illegal. In applications like takeovers, providing additional compensation to the holdouts would violate the best-price rule (Exchange Act Rule 10d-10, see 17 CFR § 240.14d-10 - Equal treatment of security holders.). Therefore, we focus on renegotiating the deal with the tendering agents instead of with the holdouts.

Would renegotiation with the holdouts alter the outcome? Unlikely, under the recursively defined credibility. Since no new information is present, the renegotiation would not be very different from the initial offer: The principal could offer whatever she would be willing to offer in renegotiation. And indeed, how credibly the principal can punish the holdout is determined by the renegotiation with the tendering agents,

[65] There is a deeper theoretical issue which I will discuss in Section 4.

not with the holdout. Thus, whether we allow for an explicit renegotiation with the holdout would not alter the outcome too much.

Side Contracting In the model, we do not allow collusion among agents. One might worry that agents may engage in side contracting to undermine the principal's punishment. This is an intentional choice, as the essence of the holdout problems lies in the lack of coordination. The holdout problem would vanish if agents could coordinate. But there are obstacles to it. Asymmetric information and a lack of commitment to fulfill the side contracts could all lead to the failure of a coalition. There is a huge literature in IO on why cartels fail. In general, side contracting does not always lead to efficient outcomes, even for the agents. The inefficiency arises from the side contracting stage and is analyzed more generally in Jackson and Wilkie (2005).

8.3 Discussions of Empirical Relevance

Existent Policies and Relevance of the Holdout Problems Despite many attempts to solve the holdout problems at the institutional level, they remain of first-order concern in all aspects of the economy. In the sovereign bond restructuring case, the IMF proposed adding Collective Action Clauses (CACs) to the new issuance. It has been proven effective in solving the holdout problems within series but not across series (Gelpern and Heller, 2016; Fang et al., 2021). Also, there is a bulk of existing sovereign debts without it. *Squeeze-out* procedures are adopted for takeovers in both the US and EU, which allow the acquirer to gain the full stake of the target when she obtains a majority stake, thus "squeezing out" the holdouts. But the legitimacy has been contested and the holdout can resort to legal remedies such as *"action of avoidance"* and *"price fairness"*.[66] Similarly, the once-popular two-tier tender offer[67] also received great legal challenges. Moreover, the possibility of litigation also restores the incentive to hold out. In urban development, *eminent domain*, which allows the government to expropriate private property for public use, plays a major role in solving holdout

[66] See more discussion in Yarrow (1985), Müller and Panunzi (2004), Broere and Christmann (2021) and Burkart and Lee (2022).

[67] A two-tier tender offer typically offers a high price to purchase shares until the raider obtains a controlling stake and purchases the remaining shares at a lower price. A similar practice is a partial tender offer where the raider only buys a fraction of outstanding shares. Both create a coercive force for the shareholders to tender. The main form of tender offers now are any-and-all, where the bidder promises to buy any shares of the target firm.

problems but is still controversial and incites a constitutional debate related to the Takings Clauses of the Fifth Amendment,[68] whether a private party can benefit from the infringement of property rights after the Supreme Court extended its use to private companies in *Kelo v. New London* (Miceli and Sirmans, 2007). In other jurisdictions, for example, in Colombia, where the legal system follows a civil law tradition, Holland (2022) documented strong property rights protection worsens the holdout problems and curbs city development. In land acquisition for oil drilling, the "*rule of capture*" allows the oil drilling companies to acquire the land adjacent to the holdout block and utilize the oil extracted from a common pool, weakening the bargaining power of the holdout and strengthening the tendering land owners. Yet, the adoption of these legal theories varies across states. For example, in Texas, the land owner has a *possessory interest* in the substances beneath the land. In *Geo Viking, Inc. v. Tex-Lee Operating CO*, the Supreme Court of Texas has ruled a fracture across the property line, as a result of *fracking*, a subsurface trespass (Kramer and Anderson, 2005). Therefore, a better understanding of the holdout problem and its private solutions would still have first-order relevance in the current state.

Empricial Relevance of Limited Commitment The key assumption, limited commitment, is reflected in a multitude of empirical evidence. It's well-documented that sovereigns lack the commitment to debt repayment, new debt issuance, and, in particular, to the negotiated outcome due to both the doctrine of sovereign immunity and the lack of a statutory regime. For example, Argentina filed with the SEC not to pay anything to the holdout creditors in 2004 and passed the Lock Law not to reopen a new exchange offer in 2005. Yet, Congress suspended the Lock Law in 2009, and the government offered a new exchange offer in 2010. In the Greek debt crisis, Greece opted to pay 435 million euros ($552 million) to the holdout creditors in full in order not to trigger the cross-default clauses and be dragged into litigation, even though it announced in the earlier exchange offer that the holdout would not get anything. Meanwhile, the majority (97%) of the tendering creditors only received cents on the euro.[69] Pitchford and Wright (2012) build a dynamic bargaining model on the idea of lack of

[68]The Takings Clause of the Fifth Amendment to the United States Constitution says, "Nor shall private property be taken for public use, without just compensation."

[69]See https://www.reuters.com/article/us-greece-bond/in-about-face-greece-pays-bond-swap-holdouts-idUSBRE84E0MY20120515

commitment to illustrate the delay in the restructuring. Yet, in theirs, renegotiation and settlement occur one by one, and this lack of commitment to the renegotiated outcome is modeled through the sequential rationality of the offers. In mine, it's modeled as the renegotiation-proofness in the collective bargaining process when agents deviate. Despite the relevance of commitment in the holdout problems, many papers on holdout problems assume full commitment. In Shleifer and Vishny (1986), they show a large shareholder who is able to commit to "return all shares tendered to their owners" if the threshold is not met,[70] solves the holdout problem. Similar assumptions are also made in Hirshleifer and Titman (1990). Thus, understanding the role of limited commitment is crucial in understanding holdout problems.

Legal Environment for Certain Solutions One may wonder if the solutions I mentioned earlier, e.g., the unanimity rule and the consent-payment-like contracts in Proposition 2, are feasible in the current legal environment. Right now, there do not seem to be any laws prohibiting the use of unanimity. In takeovers, typically, the acceptance of the tendered shares is "contingent on the delivery of a certain number of shares" (Cohen, 1990, p.116), which can be set to 100%.[71] Indeed, it's already suggested in the optimal threshold result in Holmström and Nalebuff (1992). In addition, despite that the bidder has an obligation to complete the deal (Afsharipour, 2010), the raider could nonetheless include a *bidder termination provision*[72] which gives the raider a real option to terminate the transaction at a fee to implement the unanimity rule. But we rarely see them being used in practice — indeed, even the bidder termination provisions are only included about 20% to 30% of the time (Chen et al., 2022).

In the extreme gauging result, the principal needs to pay the tendering agents a lot when someone holds out. One practical concern is that it would be considered "fraudulent conveyance" when the firm pays certain creditors too much to avoid paying

[70] They also discuss the credibility issue but about the out-of-equilibrium beliefs.

[71] Grossman and Hart (1980) argues the absence of unanimity is due to the sleepy investor problem. We are not particularly concerned with the issue of inability to find all the agents as most takeover offers are widely publicized (Cohen, 1990) and in other cases, for example, in sovereign debt restructuring, the holdouts are usually big well-known players, such as hedge funds (known as vulture funds), e.g., Elliot Investment Management in the sovereign debt restructuring of Argentina, Peru, Panama; Oppenheimer, Franklin, and Aurelius Capital Management in Puerto Rico's debt crisis; Dart Management in the sovereign debt crisis of Brazil, Argentina, and Greece.

[72] The bidder also has a fiduciary termination right, which allows the raider to terminate when itself receives a takeover offer, and a regulatory termination trigger when it fails to pass the antitrust review, both without recourse.

other known creditors (See 11 U.S. Code § 548). But this only applies i) when there is an imminent bankruptcy and ii) if the payments exceed the face value of the liabilities, not market value. Since in bankruptcy, the firm's asset is not enough to pay off all creditors in full; it is also unlikely to exceed the total debt of the tendering creditors when one holds out. It's generally not a concern in practice for distressed exchange offers. Moreover, this notion is only defined for debt, not other contracts.

Another concern is whether such offers would violate certain covenants, such as the pari-passu clause and fair-dealing/good-faith provisions. Pari-passu clauses are unlikely to be violated as the offers the principal proposed here *is* symmetric: The allocation is only asymmetric after some creditors reject the offer, which is the case for any other offers. Traditionally, the clause is also interpreted in a very narrow sense: Ratable payment, prior to an innovative reading by the Brussels Court of Appeal in *Elliott Associates, L.P. v. Banco de la Nacion* that prevented Chase Manhattan from facilitating the interest payment of Peru's Brady bond.

Typically, in a sophisticated court like the New York court, the judge would interpret any arrangement consistent with the text of the contracts as good faith, even when it looks exploitative to outsiders.[73]

Dilutability of Existing Securities It's also implicitly assumed in the baseline model that all the existing securities are dilutable, e.g., via senior debt. One might argue this is not feasible when the existing contracts are secured by collateral (e.g., secured corporate debt), or when there's no *de jure* seniority structure, for example, in sovereign debt. For the former, secured debt can sometimes be diluted in bankruptcy through *priming lien*, typically in Debtor-In-Possession (DIP) financing to raise new liquidity under Section 364(d). It's a lien on the pre-petition collateral that is senior to all existing liens, and the DIP lenders would be paid ahead of other creditors secured by the same collateral. Moreover, the firm in bankruptcy is also allowed to use *roll-up provisions* to draw the DIP financing to repay some of the creditors' (usually DIP lenders') pre-petition indebtedness, converting these debts to post-petition supersenior debt.[74] For the latter, despite the lack of a formal bankruptcy regime, sovereign debts issued under foreign

[73] I would particularly thank Professor Edward Morrison for informing me of the general knowledge of the law. Any misinterpretation is on me.

[74] *Up-tier exchanges* and *drop-down transactions* are also similar tools commonly used in DIP financing to gain priority.

law sometimes have priority under the judge's discretion. In the *Elliot Management vs. Argentina*, the Southern District of New York court judge Thomas Griesa issued an injunction preventing the Bank of New York Melon from forwarding the payment to the restructured creditors before paying the holdouts. This injunction would also prevent payment to the creditors or the underwriter in case of any new borrowing, creating a *de facto* seniority of the holdout's debt. Currently, New York is considering a bill to rule with sovereign,[75] which effectively lowers the seniority of the holdouts' debt. Even absent foreign law, Bolton and Jeanne (2009) pointed out the possibility of diluting debts that are easy to restructure, such as bank loans, with debt difficult to restructure, e.g., bonds. And Schlegl et al. (2019) finds sovereigns implement a *de facto* seniority by selectively defaulting on certain creditors. Chatterjee and Eyigungor (2015) proposes a modified Absolute Priority Rule in the spirit of Bolton and Skeel Jr (2004).

8.4 Discussion on Optimality of Non-contingent Contracts

In Proposition 3, Condition (27) only depends on the shape of the existing *bilateral* contracts between principal and agent. Neither the underlying bankruptcy rules that address the conflicts among different agents nor any new contracts that can be potentially written have any bearing on this condition.

The power of a contingent contract can be undermined so much by the principal's lack of commitment because she has a very large contracting space: It makes her powerful when she has full commitment but powerless when she doesn't. This effect of large contracting space echoes the theme in Brzustowski et al. (2023), in which they show the Coase Conjecture no longer holds when the monopolist can offer more complicated contracts. A principal with limited commitment could benefit from a smaller set of contracts because it restricts her possible deviations ex post and commits her to the initially proposed contracts. This highlights the importance of relaxing the ad-hoc restriction on the available set of contracts that the literature has assumed when working with limited commitment.

This result is also reminiscent of the result in Rubinstein and Wolinsky (1992) that, in a bilateral trading setting, the only renegotiation-proof implementable price function is not state-contingent when the authors impose a strong requirement that the buyer and

[75] The Assembly Bill A2970 can be found here `https://www.nysenate.gov/legislation/bills/2023/A2970`, and it has received a strong rebuttal from Credit Roundtable, ICMA, IIF, ICI, ACLI, LICONY.

the seller can costlessly renegotiate to an efficient outcome ex post. The irrelevance of the bankruptcy system might seem plausible at first glance. After all, we are considering the case when only *one* agent holds out. However, as we show in the next section, the bankruptcy rule would matter when we calibrate the notion of credibility.

In a different setting, Segal and Whinston (2002) also discusses the optimality of non-contingent contracts. The paper mainly addresses the issue of *holdup* problems when nonverifiability is the main concern. Non-contingent contracts are usually optimal except when it's either too blunt to prescribe all actions or insufficient to provide incentives for actions. Differently, in this paper, non-contingent contracts are optimal only when all contingent contracts involve non-credible punishment and credibility is not an issue in their bilateral setting: One party always gains when the other party loses.

8.5 Discussion on Non-monotonicity of Commitment

Backfiring in renegotiation is a recurrent theme in repeated games. For example, Pearce (1987) also identifies the similar two forces that when players place more weight on the future, it facilitates cooperation because the present gains from contemplated deviation are less important, but the benefit from cooperation also erodes the "deterrence power" available. As a manifestation, Kovrijnykh (2013) also obtains a non-monotonicity result with two players and repeated interactions. But this paper looks at a non-repeated game, and the non-monotonicity would not arise if there were only two agents.

From the analysis in Section 4.2.2, we can also see that non-monotonicity would not arise when only two agents exist. With only two agents A_i and A_j, when one, say A_i, deviates, the maximum possible punishment that can be imposed on A_i is the equilibrium payment to A_j, which is zero, given that he would get nothing if he also deviates. Thus, the principal cannot credibly punish A_i by allocating more value to A_j, and the only credible punishment comes from the loss due to discounting, i.e., $\bar{x}^{\delta}(e_i) = (1-\delta)v(e_i)$. Thus the value of the principal on path is $J(\mathbf{0}) = v(\mathbf{0}) - \delta\alpha_i v(e_i) - \delta\alpha_j v(e_j)$, which is decreasing in δ. It doesn't have non-monotonicity because there's no non-trivial punishment to the second deviator, and the non-monotonicity relies on the renegotiation outcome in the second renegotiation, which determines the credible punishment in the first. This provides a sharp contrast to Kovrijnykh (2013) and other papers in the repeated setting.

9 Conclusion

Economic transactions that take the form of exchange offers are plagued with holdout problems, a phenomenon in which the incentive to free-ride on other agents impedes efficient actions. The holdout problem is pervasive in all aspects of the economy, like takeover, debt restructuring, etc.

Despite being studied for over four decades, it hasn't been widely acknowledged that a meaningful discussion of the holdout problems requires relaxing both the contracting space and the commitment assumption. It is generally understood that the lack of commitment makes solving the holdout problem harder, but the extent to which it has caused the problem has remained a question. Similarly, studying the general mechanism without allowing for limited commitment was unfruitful: The holdout problem is too easy to solve when commitment is not a problem.

The paper looks into the role of commitment in the holdout problems and uncovers two effects: First, it will interact with the shape of the initial set of contracts and determine the credibility of the punishment mechanism and, thus, the optimal exchange offer; second, the commitment has a non-monotone effect through the renegotiation channel.

With full commitment, the holdout problem can be easily solved using a contingent contract that requires unanimous consent. But with limited commitment, the contingency is undermined: When the existing contracts are equity-like, they cannot do any better than a non-contingent contract like cash. The model explains why senior debts, so dominantly used in debt restructuring, are not seen in the takeover.

Moreover, the paper identifies the non-monotonic role of commitment: a small increase in the commitment could make the principal more profitable from renegotiation and harder to commit not to renegotiate, which limits the maximum credible punishment the principal can impose on the holdouts and undermines the exchange offer, exacerbating the holdout problem. This finding reconciles many contradictory evidence in the literature regarding the effects of the collective action clause.

Lastly, following the intuition in this renegotiation channel, greater investor protection through property rights or anti-dilution clauses may not necessarily hinder restructuring: They make the principal's benefit from renegotiation lower and, hence, more committed to the punishment.

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A Proofs for Section 2 (Model Setup)

A.1 Simplication due to Weak Consistency

This weak consistency assumption allows us to write the contractual design problem in a separable form.

Proposition 17 (Separation)**.** *Under Weak Consistency (Definition 2), we can rewrite the original design problem as*

$$\max_{h,R(\cdot,\cdot)} R_0^O\left(v(h) - \sum_{j=1}^{N}(1-h_j)\,R_j(v(h),h),h\right) \tag{63}$$

under the agents' IC condition

$$h_i \in \arg\max_{h_i' \in H_i}(1-h_i')R_i(v(h_{-i},h_i'),(h_{-i},h_i')) \tag{64}$$

$$+h_i'R_i^O\left(v(h_{-i},h_i') - \sum_{j=1}^{N}(1-h_j)\,R_j(v(h_{-i},h_i'),(h_{-i},h_i')),(h_{-i},h_i')\right)\ \forall i \tag{65}$$

Proof. To prove this statement, we only need to show that for any R and $\tilde{R}^O$ satisfying weak consistency (Definition 2), it can be written in a separate form as in the statement. First, under the weak consistency, the payoff to the existing contract when A_i chooses h_i' is

$$\tilde{R}_i^O(v(h_{-i},h_i'),(h_{-i},h_i')) = R_i^O\left(v((h_{-i},h_i')) - \sum_{j=1}^{N}(1-h_j)\cdot R_j(v(h_{-i},h_i'),(h_{-i},h_i')),(h_{-i},h_i')\right). \tag{66}$$

Substituting it to equation (5), we obtain equation (64), so the two ICs coincide. We also need to show that the objective function is identical: again, substituting it to equation (70) and expanding it, we have

$$v(h) - \sum_{i=1}^{N}(1-h_i)\cdot R_i(v(h),h) - \sum_{i=1}^{N} h_i\cdot R_i^O\left(v(h) - \sum_{j=1}^{N}(1-h_j)\cdot R_j(v(h),h),h\right) \tag{67}$$

which by definition is the same as equation (63).

Thus, the two problems are equivalent.

□

One thing to clarify is that the formulation does not mean the new contracts R have priorities over the existing contracts R^O since they could be zero for the realized value v. But this is also more than a notational asymmetry between old and new contracts: The new contract could determine the division of asset value between old and new. There is amount to assuming all old contracts are dilutable by new ones, but not necessarily diluted. I relax this implicit assumption in Section 5.

This proposition allows us to define a simpler concept of exchange offers

Definition 8 (Consistent Exchange Offer)**.** *A consistent exchange offer is a tuple*(H, h, R) *where*

- $H = \prod_{i=1}^{N} H_i$ *is the product space of* A_i*'s action space* H_i *such that* $\{0, 1\} \subset H_i \subset [0, 1]$*;*
- $h = (h_1, h_2, \ldots, h_N) \in H$ *is the (recommended) action profile of the agents;*
- R *is a mapping from* $\mathbb{R}_+ \times H$ *to* $\mathbb{R}_+^N$ *where the ith element* $R_i(v, h)$ *determines the unit payoff of* A_i*'s new contract given the asset value is* v *and the holdout profile* h*;*

such that

- *the allocation is feasible:*

$$\sum_{i=0}^{N} h_i\, R_i^O(v - x, h) + \sum_{i=1}^{N} (1 - h_i)\, R_i(v, h) = v \tag{68}$$

  where $x = \sum_{i=1}^{N} (1 - h_i) R(v, h)$*;*

- *the action* h_i *is incentive compatible:*

$$h_i \in \arg\max_{h_i' \in H_i} u_i(h_i' | h_{-i}, R) \tag{69}$$

  where

$$u_i(h_i | h_{-i}, R) := (1 - h_i)\, R_i(v, h) + h_i\, R_i^O(v - x, h) \tag{70}$$

  is A_i*'s payoff given the action profile* $h = (h_{-i}, h_i)$ *and the corresponding project value* v*.*

It's clear that from the assumption that $v(h)$ is decreasing in h and that $c < v(\mathbf{0}) - v(\mathbf{1})$, the first best is to implement $h = \mathbf{0}$. It is implementable if all agents can coordinate: As per the Coase Theorem, the positive surplus can be split by bilateral bargaining.

A.2 Simplication from Equivalence Exchange Offers

The next result further simplifies the analysis, saying that it is without loss of generality to look at the implementation of $h = \mathbf{0}$, i.e., the equilibrium where everyone tenders. One may argue that it might not be ideal to implement $\mathbf{0}$ when it's too costly to hold in an additional agent, which only slightly improves the asset value. This will not be the case here as the principal can offer the exact same contract as what the agent initially has, and the agent would weakly prefer to exchange. Indeed, the principal would never find it optimal to do so because there are cheaper ways of implementing an exchange offer, as we will show below.

Proposition 18 (Equivalence)**.** *For any consistent exchange offer (H, h^*, R) such that $h^* \neq \mathbf{0}$ is implementable for the principal, there exists an alternative consistent exchange offer, with the same action space H, in which $h = \mathbf{0}$ is implementable, and the principal obtains the same payoff as under the original exchange offer.*

Proof. For a given exchange offer (H, h^*, R) that is incentive compatible, we construct a new exchange offer $(H, \mathbf{0}, \hat{R})$ such that is also incentive compatible. Since the relevant payoff is only "around" the equilibrium payoff, i.e., $\mathbf{0}_{-i} \times H_i$, we only need to specify the payoff on these action profiles.

For the payoff on path, let

$$\hat{R}_i(v(\mathbf{0}), \mathbf{0}) = (1 - h_i) R_i(v(h^*), h^*) + h_i \, R_i^O(h_i^* | h_{-i}^*, R) + \frac{v(\mathbf{0}) - v(h^*)}{N}. \tag{71}$$

Let's check that the principal obtains the same payoff. Under the new exchange offer, the principal's payoff is

$$v(\mathbf{0}) - \sum_{i=1}^{N} \hat{R}_i(v(\mathbf{0}), \mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \left[(1-h_i) R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^* | h_{-i}^*, R) + \frac{v(\mathbf{0}) - v(h^*)}{N} \right] \tag{72}$$

$$= v(h^*) - \sum_{i=1}^{N} \left[(1-h_i) R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^* | h_{-i}^*, R) \right] \tag{73}$$

which is the principal's payoff under (H, h^*, R). So this suggests it is feasible on path and that the principal obtains exactly the same payoff.

Now we proceed to specify the off-path payoffs and show it's feasible and incentive compatible.

For agent A_i let

$$\hat{R}_i(v(\mathbf{0}_{-i}, h_i), (\mathbf{0}_{-i}, h_i)) = \begin{cases} (1-h_i) R_i(v(h^*), h^*) + h_i\, R_i^O(h_i^* | h_{-i}^*, R) + \frac{v(\mathbf{0}) - v(h^*)}{N} & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{74}$$

and

$$\hat{R}_j(v(\mathbf{0}_{-i}, h_i), (\mathbf{0}_{-i}, h_i)) = \begin{cases} (1-h_j^*) R_j(v(h^*), h^*) + h_j\, R_j^O(v(h^*) - x(h^*; R), h^*) + \frac{v(\mathbf{0}) - v(h^*)}{N} & \text{if } h_i = 0 \\ \frac{v(\mathbf{0}_{-i}^*, h_i)}{N-1} & \text{otherwise} \end{cases} \tag{75}$$

where

$$x(h^* | R) = \sum_{i=1}^{N} h_i^* R_i(v(h^*), h^*) \tag{76}$$

It is easy to see that the new contract is feasible: when $h_i = 0$, the payoff coincides with the on-path payoff specified; when $h_i \neq 0$, the total payoff is the total asset value available $v(\mathbf{0}_{-i}^*, h_i)$. Also, since deviation leads to zero payoff, every agent has an incentive to play 0 whenever others do. Thus, this new exchange offer is incentive compatible and delivers exactly the same payoff to the principal.

□

The proof builds on a simple idea: If it is optimal for an agent to retain some of its original shares, then the principal could simply offer the existing contracts through the new contracts. However, the complication comes from our setting that the asset value depends on the actions but not the form of the new contracts. Thus, the asset value is artificially inflated[76] when offering the existing contracts through new contracts. This formulation makes the holdout problem more acute: If the principal offers the same value to each tendering agent, the value of the outside option is higher due to the inflated asset value, so the IC may no longer hold. We solve this issue by giving away the artificially inflated asset value to the agents through the new contracts. Moreover, even if the value that can be distributed to the holders of initial contracts is made the same, the holdout might obtain a higher value through the initial contracts, as fewer agents hold initial contracts. This problem can also be handled by allocating more value to the contracts.

A.3 Derivation of the Bond Buyback Model

Continuous Limit Their main result is a characterization when buying back debt is beneficial to the principal in the limit $N \to +\infty$. Since the existing contracts are fully symmetric, we can use $H = h^\top \mathbf{1}/N$ to denote the fraction of debts that hold out as the state variable in lieu of h. Using the new state variable H, the value of the aggregate debt is

$$\mathbb{E}[\min\{\theta v(H), HD\}] = \int_0^{\hat{X}} \theta(x + W(H))\mathrm{d}F(x) + HD(1 - F(\hat{X})) \tag{77}$$

where $\hat{X} = HD/\theta - W(H)$ is the default threshold.[77] And the marginal value of the debt is[78]

$$\frac{\mathrm{d}}{\mathrm{d}H}\mathbb{E}[\min\{\theta v(H), HD\}] = \theta W'(H)F(\hat{X}) + D(1 - F(\hat{X})) \tag{78}$$

where the second term is the repayment when the firm is not in default, and the first term accounts for the effect on the internal cash reserve through the transaction.

To retire a fraction $\mathrm{d}H$ of the total debt D, the creditors must be compensated at least

[76]This formulation also encompasses the more realistic case when the asset value is not enhanced when the exact same contract is offered.

[77]Firm defaults whenever $\theta(X + W(H)) < HD$.

[78]Using Leibniz rule, the derivative of the debt value is $\theta W'(H)F(\hat{X}) + \theta(\hat{X} + W(H))\frac{\mathrm{d}\hat{X}}{\mathrm{d}H}f(\hat{X}) + D(1 - F(\hat{X})) - HDf(\hat{X})\frac{\mathrm{d}\hat{X}}{\mathrm{d}H}$ where the second and the fourth term cancels out at $\hat{X}$.

the average debt value. Equating it to the marginal cost yields

$$\frac{\mathbb{E}[\min\{\theta v(H), HD\}]}{HD} D\mathrm{d}H = W'(H)\mathrm{d}H \implies W'(H) = H^{-1}\mathbb{E}[\min\{\theta v(H), HD\}] \quad (79)$$

The total value accrued to the principal is the difference between the asset value and the debt value

$$\mathbb{E}[v(H)] - \mathbb{E}[\min\{\theta H^{-1} v(H), HD\}]. \quad (80)$$

whose first order derivative w.r.t. H is

$$W'(H) - \frac{\mathrm{d}}{\mathrm{d}H}\mathbb{E}[\min\{\theta v(H), HD\}] = (1 - \theta F(\hat{X}))W'(H) - D(1 - F(\hat{X})) \quad (81)$$

which is positive (i.e., retiring debt hurts the principal) if and only if

$$1 - \theta F(\hat{X}) \geq \frac{HD}{\mathbb{E}[\min\{\theta v(H), HD\}]}(1 - F(\hat{X})) \quad (82)$$

after substituting the expression from equation (79). This is analogous to Equation (6) in . When this condition holds, the principal benefits from increasing the leverage as the cost of default is also borne by the creditors, and she has no incentive to deleverage, which generates the ratchet effect.

Finite Agent Now, let's try to derive the finite-agent counterpart. Since all agents are symmetric, I let $h^k = (1, \ldots, 1, 0, \ldots, 0)$ be the vector whose first k elements are ones and the rest zero. The number of holdouts is $(h^k)^\top \mathbb{1} = k$. Under action profile h^k, the aggregate debt value is

$$\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \int_0^{\hat{X}^k} \theta(x + W(h^k))\mathrm{d}F(x) + \frac{kD}{N}(1 - F(\hat{X}^k)) \quad (83)$$

where $\hat{X}^k = \frac{kD}{N\theta} - W(h^k)$ is the default threshold when k creditors hold out. Using integration by parts and substituting the value of $\hat{X}^k$, we can write the debt value as

$$\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \theta F(\hat{X}^k)\hat{X}^k - \theta \int_0^{\hat{X}^k} F(x)\mathrm{d}x + \theta W(h^k)F(\hat{X}^k) + \frac{kD}{N}(1 - F(\hat{X}^k)) \tag{84}$$

$$= \frac{kD}{N} - \theta \int_0^{\hat{X}^k} F(x)\mathrm{d}x \tag{85}$$

And the change in the total debt value when one additional creditor holds out is

$$\mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \frac{D}{N} + \theta \int_{\hat{X}^{k+1}}^{\hat{X}^k} F(x)\mathrm{d}x \tag{86}$$

To retire the debt from an additional agent, the debtor has to pay out the average debt value from the internal cash reserve, and thus, the internal cash reserve changes by

$$W(h^{k+1}) - W(h^k) = \frac{1}{k}\mathbb{E}[\min\{\theta v(h^k), kD/N\}] = \frac{D}{N} - \frac{\theta}{k}\int_0^{\hat{X}^k} F(x)\mathrm{d}x \tag{87}$$

The value to the principal at h^k is

$$\mathbb{E}[v(h^k)] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}] \tag{88}$$

The change in the principal's value from h^{k+1} to h^k, if we write completely analogously, is

$$W(h^{k+1}) - W(h^k) - \left\{\mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}] - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]\right\} \tag{89}$$

$$=W(h^{k+1}) - W(h^k) - \left[\int_0^{\hat{X}^{k+1}} \theta x \mathrm{d}F(x) + \theta W(h^{k+1})F(\hat{X}^{k+1}) + \frac{(k+1)D}{N}(1 - F(\hat{X}^{k+1}))\right. \tag{90}$$

$$-\left(\int_0^{\hat{X}^k}\theta x\mathrm{d}F(x)+\theta W(h^k)F(\hat{X}^k)+\frac{kD}{N}(1-F(\hat{X}^k))\right)\Bigg] \tag{91}$$

$$=(1-\theta F(\hat{X}^{k+1}))W(h^{k+1})-(1-\theta F(\hat{X}^k))W(h^k)+\int_{\hat{X}^{k+1}}^{\hat{X}^k}\theta x\mathrm{d}F(x) \tag{92}$$

$$+\frac{kD}{N}(1-F(\hat{X}^k))-\frac{(k+1)D}{N}(1-F(\hat{X}^{k+1})) \tag{93}$$

$$=(1-\theta F(\hat{X}^k))[W(h^{k+1})-W(h^k)]+\theta(F(\hat{X}^k)-F(\hat{X}^{k+1}))W(h^{k+1}) \tag{94}$$

$$+\int_{\hat{X}^{k+1}}^{\hat{X}^k}\theta x\mathrm{d}F(x)-\frac{D}{N}(1-F(\hat{X}^k))-\frac{(k+1)D}{N}(F(\hat{X}^k)-F(\hat{X}^{k+1})) \tag{95}$$

$$=(1-\theta F(\hat{X}^k))\frac{1}{k}\mathbb{E}[\min\{\theta v(h^k),kD/N\}]-\frac{D}{N}(1-F(\hat{X}^k)) \tag{96}$$

$$+\int_{\hat{X}^{k+1}}^{\hat{X}^k}\theta\left(x+W(h^{k+1})-\frac{(k+1)D}{N}\right)\mathrm{d}F(x) \tag{97}$$

which is positive if

$$1-\theta F(\hat{X}^k)\geq\frac{kD(1-F(\hat{X}^k))}{N\mathbb{E}[\min\{\theta v(h^k),kD/N\}]}+\frac{k\int_{\hat{X}^k}^{\hat{X}^{k+1}}\theta\left(x+W(h^{k+1})-\frac{(k+1)D}{N}\right)\mathrm{d}F(x)}{\mathbb{E}[\min\{\theta v(h^k),kD/N\}]}. \tag{98}$$

This is still the same as in , but we have one additional term which vanishes in the continuous limit. Since X^k is increasing in k,[79] this term is negative, as $x+W(h^{k+1}-\frac{(k+1D)}{N})$ is zero when evaluated at $x=\hat{X}^{k+1}$. So, the condition is easier to satisfy, as a non-atomic agent partially takes into consideration his own externality.

Alternatively, we could provide a simpler characterization

$$W(h^{k+1})-W(h^k)-\left\{\mathbb{E}[\min\{\theta v(h^{k+1}),(k+1)D/N\}]-\mathbb{E}[\min\{\theta v(h^k),kD/N\}]\right\} \tag{99}$$

$$=\left(\frac{D}{N}-\frac{\theta}{k}\int_0^{\hat{X}^k}F(x)\mathrm{d}x\right)-\left(\frac{D}{N}-\theta\int_{\hat{X}^k}^{\hat{X}^{k+1}}F(x)\mathrm{d}(x)\right) \tag{100}$$

$$=\theta\int_0^{\hat{X}^{k+1}}F(x)\mathrm{d}x-\frac{k+1}{k}\theta\int_0^{\hat{X}^k}F(x)\mathrm{d}x \tag{101}$$

[79] To see this, notice $\hat{X}^{k+1}-\hat{X}^k=\frac{D}{N\theta}-(W(h^{k+1}-W(h^k))$ but the second term is smaller than $\frac{D}{N}$ by equation (87) while the firm term is larger then $\frac{D}{N}$ as $\theta<1$. This simply says that the default threshold is higher when there are more debts outstanding, even when internal cash is used to repurchase debt.

which is positive if

$$\frac{k}{k+1} > \frac{kD/N - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]}{(k+1)D/N - \mathbb{E}[\min\{\theta v(h^{k+1}), (k+1)D/N\}]} \tag{102}$$

having used that the integration of the CDF multiplied by θ is the difference between the nominal value of the debt and the market value of the debt

$$\theta \int_0^{\hat{X}^k} F(x)\mathrm{d}x = \frac{kD}{N} - \mathbb{E}[\min\{\theta v(h^k), kD/N\}]. \tag{103}$$

The characterization is only available in the finite-agent case as both sides of the inequality approach one in the continuous limit.

A.4 Derivation of the Debt Exchange Model

Hypothetically, we assume a fraction β of the short-term debt holders accept the exchange offer and, for simplicity, assume βN is an integer. Let $h^{(1-\beta)N} = (1, \ldots, 1, , 0, \ldots, 0)$ be the action profile where the first $(1-\beta)N$ agents hold out.

Pari-passu Debt Exchange If the principal offer long-term debt pD/N in exchange for the existing short-term debt qD and long-term debt $(1-q)D/N$, and at any profile h, the debt due at the interim date is $h^\top \mathbf{1} \cdot qD/N$ and will be paid off first. The total amount of debt outstanding at date 2 is $(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N$.

The value of the new contract is thus

$$R_i(v,h) = \frac{pD/N}{(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N} \tag{104}$$

$$\min\{v - h^\top \mathbf{1} \cdot qD/N, (1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N\} \tag{105}$$

$$= \min\left\{\frac{pD/N\,(v - h^\top \mathbf{1} \cdot qD/N)}{(1-q)h^\top \mathbf{1} D/N + pD(N - h^\top \mathbf{1})/N}, pD/N\right\} \ \forall i : h_i = 0, \tag{106}$$

In particular, under the action profile $h^{(1-\beta)N}$, using $(h^{(1-\beta)N})^\top \mathbf{1} = (1-\beta)N$

$$R_i(v, h^{(1-\beta)N}) = \frac{1}{N} \min\left\{\frac{p\,(v - (1-\beta)qD)}{(1-q)(1-\beta) + p\beta}, pD\right\}, \forall i > (1-\beta)N \tag{107}$$

and the total payment to the tendered creditors is

$$x = \sum_{i=1}^{N}(1-h_i)R_i(v, h^{(1-\beta)N}) = \min\left\{\frac{\beta p\,(v-(1-\beta)qD)}{(1-q)(1-\beta)+p\beta}, \beta pD\right\} \tag{108}$$

The payoff of the holdouts under the action profile $h^{(1-\beta)N}$ is thus[80]

$$R_i^O(v-x, h^{(1-\beta)N}) = \min\left\{\frac{1}{(1-\beta)N}\left[v - \min\left\{\frac{\beta p\,(v-(1-\beta)qD)}{(1-q)(1-\beta)+p\beta}, \beta pD\right\}\right], \frac{D}{N}\right\} \tag{109}$$

$$= \frac{1}{N}\min\left\{\frac{1}{1-\beta}\max\left\{\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+p\beta}v + \frac{\beta p\,(1-\beta)qD}{(1-q)(1-\beta)+p\beta}, v-\beta pD\right\}, D\right\} \tag{110}$$

Note, it is equivalent to separate the payoff into the short-term part and the long-term part, i.e.,

$$\min\left\{\frac{1}{(1-\beta)N}\left[(v-(1-\beta)qD) - \min\left\{\frac{\beta p\,(v-(1-\beta)qD)}{(1-q)(1-\beta)+p\beta}, \beta pD\right\}\right], \frac{(1-q)D}{N}\right\} + \frac{qD}{N}. \tag{111}$$

At $p=1$, the payoff to the new and old contracts can be simplified to

$$R_i(v, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{v-(1-\beta)qD}{(1-q)(1-\beta)+\beta}, D\right\} \tag{112}$$

$$R_i^O(v-x, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1}{1-\beta}\max\left\{\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+\beta}v + \frac{\beta(1-\beta)qD}{(1-q)(1-\beta)+\beta}, v-\beta D\right\}, D\right\} \tag{113}$$

[80]Technically, we should evaluate the outside option at the action profile $h^{(1-\beta)N+1}$, but the difference is small when N is large, and it complicates the analysis as we see in the bond buyback example. So I omit that difference to reproduce the result in and then comment on the case when the difference exists.

Notice whenever $\frac{1}{(1-q)(1-\beta)+\beta}(v-(1-\beta)qD) < D$, we have $v < D$ and therefore[81]

$$R_i^O(v-x, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1-q}{(1-q)(1-\beta)+\beta}v + \frac{\beta}{(1-q)(1-\beta)+\beta}qD, D\right\}, \forall v < D \tag{114}$$

and taking the difference between the two terms in the min function in $R_i^O(v-x, h^{(1-\beta)N})$ and $R_i(v, h^{(1-\beta)N})$

$$\left[\frac{1-q}{(1-q)(1-\beta)+\beta}v + \frac{\beta}{(1-q)(1-\beta)+\beta}qD\right] - \frac{(v-(1-\beta)qD)}{(1-q)(1-\beta)+\beta} = \frac{q(D-v)}{1-q(1-\beta)} > 0 \tag{115}$$

So, the payoff to the holdouts is always higher than the tendering agents

$$R_i^O(v-x, h^{(1-\beta)N}) \geq R_i(v, h^{(1-\beta)N}) \tag{116}$$

with the inequality being strict when $v < D$. This is equivalent to Proposition 1 in when N approaches infinity.[82]

It will turn out that holding out is not always optimal when N is finite. For the comparison when N is finite, I need to compare the payoff of accepting at $h^{(1-\beta)N}$ with that of holding out at $h^{(1-\beta)N+1}$. When $v > D$,

$$R_i(v, h^{(1-\beta)N}) = \frac{D}{N} = R_i^O(v-x, h^{(1-\beta)N+1}) \tag{117}$$

[81]The difference of the two terms inside the max function in $R_i^O(v-x, h^{(1-\beta)N})$ is $\frac{(1-q)(1-\beta)}{(1-q)(1-\beta)+\beta}v + \frac{\beta(1-\beta)qD}{(1-q)(1-\beta)+\beta} - (v-\beta D) = \frac{\beta(D-v)}{1-q(1-\beta)} > 0$.

[82]Ideally, we want to compare the payoff of the tendering with β to the holdout with $\beta - \frac{1}{N}$, but the difference diminishes as N approaches infinity.

but when $v < D$, comparing the payoffs between tendering and holdout yields

$$N(R_i^O(v-x, h^{(1-\beta)N}+1) - R_i(v, h^{(1-\beta)N})) \tag{118}$$

$$= \left[\frac{1-q}{(1-q)(1-\beta+1/N)+\beta-1/N}v + \frac{\beta-1/N}{(1-q)(1-\beta+1/N)+\beta-1/N}qD\right] \tag{119}$$

$$- \left[\frac{1}{(1-q)(1-\beta)+\beta}(v-(1-\beta)qD)\right] \tag{120}$$

$$= \frac{q(D-v)}{1-q(1-\beta)} \times \frac{N-1-(1-\beta)Nq}{N-q-(1-\beta)Nq} \tag{121}$$

which is positive whenever $N > \frac{1}{1-q(1-\beta)}$ or $N < \frac{q}{1-q(1-\beta)}$. When N goes to infinity, the condition is satisfied, so we obtain the same result. But when the number of agents is finite, in particular, $\frac{q}{1-q(1-\beta)} \leq N \leq \frac{1}{1-q(1-\beta)}$, holding out may not be optimal as the agent bears his own externality. But the second half of the quality puts a lower bound on the number of agents holding out: at least a fraction $1-\beta > \frac{1}{q}\frac{N-1}{N}$ of the agents hold out.

Senior Debt Exchange In contrast, if the principal offers long-term senior debt pD/N in exchange for the short-term debt qD and long-term debt $(1-q)D/N)$, the holdouts' short-term debts totaling $h^\top \mathbf{1} \cdot qD/N$ are paid-off, and the total amount of senior debt outstanding is $pD(N-h^\top \mathbf{1})$. The payoff to the new contract, i.e., each senior debt contract, is thus

$$R_i(v,h) = \frac{pD}{pD(N-h^\top \mathbf{1})} \min\{v - h^\top \mathbf{1} \cdot qD/N, pD(N-h^\top \mathbf{1})/N\} \tag{122}$$

$$= \min\left\{\frac{1}{N-h^\top \mathbf{1}}(v - h^\top \mathbf{1} \cdot qD/N), \frac{pD}{N}\right\} \tag{123}$$

Using $(h^{(1-\beta)N})^\top \mathbf{1} = (1-\beta)N$

$$R_i(v, h^{(1-\beta)N}) = \min\left\{\frac{1}{\beta N}(v-(1-\beta)qD), \frac{pD}{N}\right\} \tag{124}$$

and the total payment to the senior debts is

$$x = \min\{v - (1-\beta)qD, \beta pD\} \tag{125}$$

while that to each holdout is[83]

$$R_i^O(v - x, h^{(1-\beta)N}) = \min\left\{\frac{1}{(1-\beta)N}\left[v - \min\{v - (1-\beta)qD, \beta pD\}\right], \frac{1}{N}D\right\} \tag{126}$$

$$= \min\left\{\max\left\{\frac{qD}{N}, \frac{v - \beta pD}{(1-\beta)N}\right\}, \frac{D}{N}\right\} \tag{127}$$

At $p = 1$, the payoffs to the new and old contracts are

$$R_i(v, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\frac{1}{\beta}(v - (1-\beta)qD), D\right\} \tag{128}$$

$$R_i^O(v - x, h^{(1-\beta)N}) = \frac{1}{N}\min\left\{\max\left\{qD, \frac{v - \beta D}{1-\beta}\right\}, D\right\} \tag{129}$$

Whenever $\frac{1}{\beta}(v - (1-\beta)qD) < D$, we have $v < (q + \beta - \beta q)D$, and therefore $qD > \frac{v - \beta D}{1-\beta}$. Hence

$$R_i^O(v - x, h^{(1-\beta)N}) = \frac{1}{N}\min\{qD, D\} = \frac{qD}{N} < \frac{v - (1-\beta)qD}{\beta N}, \forall v < (q + \beta - \beta q)D \tag{130}$$

So we have

$$R_i(v, h^{(1-\beta)N}) \geq R_i^O(v - x, h^{(1-\beta)N}), \forall v \tag{131}$$

which the inequality being strict when $v < (q + \beta - q\beta)D$. So it's feasible to implement an exchange offer with senior debt, and we confirm Proposition 2 in as N approaches infinity.

Moreover, as

$$R_i^O(v - x, h^{(1-\beta)N+1}) = \frac{qD}{N+1} < \frac{qD}{N} < \frac{v - (1-\beta)qD}{\beta N} \tag{132}$$

the incentive to hold out is even weaker when N is finite.

[83]Again, we should more pedantically single out the short-term payment and the expression would be the same.

B Proofs for Section 3 (Optimal Exchange Offer with Full Commitment)

Proposition 1. *The necessary and sufficient condition for the existence of a cash exchange offer that implements $h = \mathbf{0}$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{17}$$

Moreover, the principal is willing to implement the exchange offer if and only if

$$v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) \geq c. \tag{18}$$

Proof. First, I will show the condition (17) is necessary. Suppose an exchange offer $\{t_i\}_i$ exists. And we denote the sum $T = \sum_{i=1}^{N} t_i(0)$. Simplifying the conditions (16), we obtain

$$T \leq R_0^O(v(0), 0) + W \leq v(0) + W \tag{133}$$

which is independent of F. It says that the borrowing is unconstrained as long as the principal is solvent. Plug in the definition of T and the individual IC of the agents (14), and I obtain the condition (17) in the proposition.

To see why it is sufficient, let's construct an exchange offer as follows

$$t_i(h_i) = \begin{cases} R_i^O(v(e_i), e_i) & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{134}$$

and the principal borrows

$$F = \max\left\{0, \sum_{i=1}^{N} R_i^O(v(e_i), e_i) - W\right\}. \tag{135}$$

It is easy to verify that all the constraints are satisfied when the inequality (17) holds.

With the cost c, the principal can guarantee his own wealth W without implementing the exchange offer. And the payoff to the principal, if the offer is implemented, is

$$W + v(0) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) - c. \tag{136}$$

Comparing the two scenarios, we obtain the condition in the proposition. □

C Alternative Assumptions for Non-contingent Exchange Offers

C.1 Optimal non-contingent offers with access to externally raised funds

To implement the outcome $h = \mathbf{0}$ when the agent has access to the funds raised, the set of necessary conditions is that

- Tendering is better off than holding out for A_i

$$t_i(0) \geq R_i^O\left(v(e_i) + F + W - \sum_{j=1, j\neq i}^{N} t_j(0), e_i\right), \forall i \in \mathcal{N} \tag{137}$$

- The total payment can be financed by new borrowing and internal wealth

$$\sum_{j=1}^{N} t_j(0) \leq F + W \tag{138}$$

- The principal has enough residual claims to payoff the debt

$$F \leq R_0^O\left(v(\mathbf{0}) + F + W - \sum_{j=1}^{N} t_j(0), \mathbf{0}\right) \tag{139}$$

The main difference is that inside $R_i^O(\cdot, e_i)$, the total amount of assets that can be distributed to A_i has an additional non-negative term $F + W - \sum_{j=1,j\neq i}^{N} t_j(0)$ which strengthen the incentive to holdout.

Proposition 19. *Suppose all bilateral contracts are non-decreasing, i.e, the function $R_i^O(\cdot, e_i)$ is non-decreasing for all i. Let $t_i^* = \inf\{t : t \geq R_i^O(v(e_i) + t, e_i)\}$, then a necessary and sufficient condition for the existence of a cash exchange offer that implements $h = 0$ is*

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} t_i^*. \tag{140}$$

Moreover, $\sum_{i=1}^N t_i^*$ is the minimum cost of all feasible cash transfers when $W \leq \sum_{i=1}^N t_i^*$.

Proof. In what follows, we first prove a lemma describing the property of t_i^* defined; then we show the condition is necessary. After that, we show it's also sufficient in two cases depending on the relative magnitude of W and $\sum_{i=1}^N t_i^*$.

Lemma 6. *If $f(\cdot)$ is a weakly increasing function, then* $\inf\{t : t \geq f(x+t)\}$ *is weakly increasing in x.*

Proof. We prove it by contradiction. Suppose that the statement is not true, i.e., there exists $x_1 < x_2$ but

$$t_2 := \inf\{t : t \geq f(x_2 + t)\} < t_1 := \inf\{t : t \geq f(x_1 + t)\} \tag{141}$$

By the definition of t_2, for any $\varepsilon > 0$, there exists $t' \leq t_2 + \varepsilon$ such that $t' \in \{t : t \geq f(x_2 + t)\}$. Thus we have

$$t' \geq f(x_2 + t') \geq f(x_1 + t') \tag{142}$$

where the first inequality comes from that $t' \in \{t : t \geq f(x_2 + t)\}$, and the second from the weak monotonicity of f and that $x_2 > x_1$. This implies that $t' \in \{t : t \geq f(x_1 + t)\}$, i.e., $t' \geq t_1$. Since this holds true for any $\varepsilon > 0$, it must be that $t_2 \geq t_1$, contradicting the assumption that $t_2 < t_1$. Thus, it must be true that f is weakly increasing in x. □

We now show that the condition is necessary. Suppose an exchange offer $\{t_i(\cdot)\}_i$ exists and we let $T = \sum_{j=1}^N t_j(0)$. By the definition of R_0^O, the break-even condition (139) could be written as

$$F \leq R_0^O\left(v(\mathbf{0}) + F + W - \sum_{j=1}^{N} t_j(0), 0\right) = v(\mathbf{0}) + F + W - T - 0 \Leftrightarrow W + v(\mathbf{0}) \geq T \tag{143}$$

which says that financing is possible as long as the principal is solvent. This condition already resembles the condition in our proposition, except that we have to put a bound on T.

We can rewrite the individual IC (137) as

$$t_i(0) \geq R_i^O \left(v(e_i) + F + W - T + t_i(0), e_i\right), \forall i \in \mathcal{N} \tag{144}$$

and from the feasibility condition (138), we know the slack $F + W - T$ is weakly positive. And from Lemma 6, the lowest possible $t_i(0)$ is increasing in $F + W - T$. (Note, T includes $t_i(0)$ but it doesn't affect the reasoning below.) Therefore, a lower bound of the minimum transfer needed to hold in each existing contract holder is given by $t_i^* = \inf\{t : t \geq R_i^O(v(e_i) + t, e_i)\}$. I.e., any offer must be satisfied $t_i(0) \geq t_i^*$ and *a fortiori* $T \geq \sum_{i=1}^N t_i^*$. Thus, we conclude

$$W + v(\mathbf{0}) \geq \sum_{i=1}^{N} t_i^*. \tag{145}$$

is a necessary condition of the existence of a cash exchange offer with borrowing.

Now we proceed to prove that this condition is also sufficient when $W \leq \sum_{i=1}^N t_i^*$ by constructing a cash exchange offer that satisfies all the conditions (137), (138) and (139). Consider the following transfer

$$t_i(h_i) = \begin{cases} t_i^* & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{146}$$

and the principal borrows the minimum $F := \sum_{i=1}^N t_i^* - W$ to finance the cash offer . Clearly, the condition (138) is satisfied by the choice of F, and condition (139) is satisfied given the condition in the proposition. We only need to prove the IC (inequality 137) is satisfied. Plugging in the definition of $\{t_j^*\}_{j \in \mathcal{N}}$ and F, the player i deviate, he would get $R_i^O \left(v(e_i) + F + W - \sum_{j \neq i} t_j^*, e_i\right) = R_i^O(v(e_i) + t_i^*, e_i) \leq t_i^*$ which confirms the IC.

It is easy to see that this is the minimum cost exchange offer as any offer t_i made to agent i must be higher than t_i^* in equilibrium. Thus $\sum_{i=1}^N t_i^*$ achieves the lowest possible cost.

When $W > \sum_{i=1}^{N} t_i^*$, an exchange offer exists if the following system of inequalities has a solution

$$\begin{cases} t_i \geq R_i^O(v(e) + W - T + t_i, e_i), \forall i \in \mathcal{N} \\ T = \sum_{i=1}^{N} t_i \end{cases}. \tag{147}$$

Let $\Delta = W - T$ and define $t_i^{*\delta} := \inf\{t : t \geq R_i^O(v(e) + \Delta + t_i, e_i)\}$, the system of the inequalities has a solution if and only if the equation

$$\sum_{i=1}^{N} t_i^{*\delta} = W - \Delta \tag{148}$$

has a solution. Notice that the LHS is weakly increasing in Δ by Lemma 6 while the RHS is decreasing in Δ. At $\Delta = 0$ the RHS is smaller than the RHS by the case condition $W > \sum_{i=1}^{N} t_i^*$, and the RHS is zero at $\Delta = W$; Therefore, there must exist an $\Delta^* \in (0, W)$ that solves the equation. We can similarly verify that the transfer

$$t_i(h_i) = \begin{cases} t_i^*(\Delta^*) & \text{if } h_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{149}$$

with borrowing $F = 0$ constitute a cash exchange offer, which satisfies all the conditions (137), (138) and (139). The fact that $\Delta^* \in (0, W)$ indicates that the principal will have to pay more than $\sum_{i=1}^{N} t_i^*$ but not her entire internal wealth W.

□

When the existing contracts have recourse to the assets, then any payment to other agents through borrowing will have a "dilution" effect: if the principal increases the payment to one agent, the RHS of Equation (137) would be lower, reducing the payoff from holding out. Of course, one might suspect that the principal would also need to borrow more to implement the repayment so that F is also higher. But it is never in the principal's interest to do so. In the proof, we show that the optimal non-contingent offer can be described by a fixed-point equation, with the optimal borrowing being just to borrow enough to implement the exchange offers.

Example C.1 (Debt)**.** *Suppose the existing contracts are debts. Each agent has an outstanding debt D_i.*

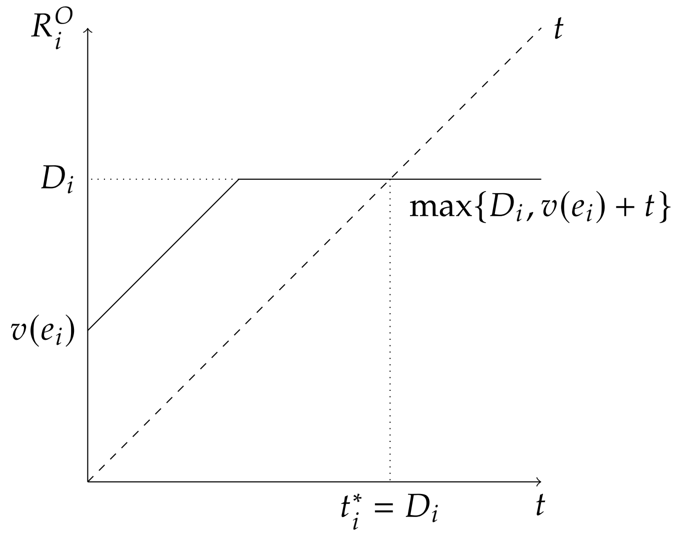

Figure 3: Agents with Debts D_i: $R_i^O(v(e_i) + t_i(0), e_i) = \max\{D_i, v(e_i) + t_i(0)\}$

This example shows that when the existing contracts are debts, the only possible situation in which a cash exchange is feasible is to pay off the debt of the existing contracts.

Example C.2 (Equity)**.** *Now suppose every existing contract holder has an equity claim α_i.*

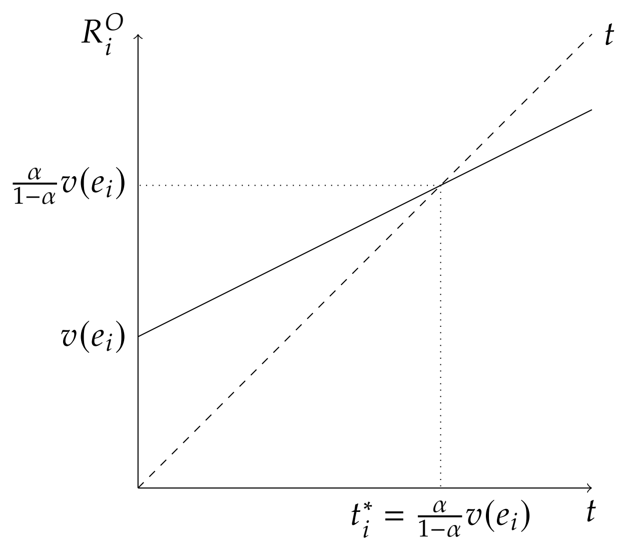

Figure 4: Agent with Equity α_i: $R_i^O(v(e_i) + t_i(0), e_i) = \alpha(v(e_i) + t)$

In contrast, the equity holder would have levered equity: he needs to be compensated by more than his share of the asset when he holdouts because the ex-ante borrowing increases the value of assets that he has recourse to.

Proposition 20 (Asymmetry)**.** *Suppose the value creation is the same when each of the shareholders holds outs, i.e., $v(e_i) = v_1 \forall i \in \mathcal{N}$. Then the cost of a cash exchange offer is larger when the holdings are more asymmetric. That is, if we let compare two sequences of shareholders $\alpha = (\alpha_1, ..., \alpha_N)$ and $\hat{\alpha} = (\hat{\alpha}_1, ..., \hat{\alpha}_N)$ such that there exist $i, j \in \mathcal{N}$*

$$|\alpha_i - \alpha_j| > |\hat{\alpha}_i - \hat{\alpha}_j| \text{ and } \alpha_k \neq \hat{\alpha}_k \forall k \neq i, j, \tag{150}$$

then the cost of the exchange offer is higher with the holding profile α than $\hat{\alpha}$.

Proof. We define $A = \sum_{i=1}^{N} \frac{\alpha_i}{1-\alpha_i}$. First, we will show A is higher when α is more asymmetric, i.e., it is higher under α and $\hat{\alpha}$. And then, we will show the cost higher in both the sufficient-internal-cash region and insufficient-internal-cash region.

Let $\sum_{k \neq i,j} \alpha_k = K$ and $\alpha_i = a$, and hence $\alpha_j = 1 - K - a$. Therefore

$$A = \sum_{k \neq i,j} \frac{\alpha_k}{1-\alpha_k} + \frac{\alpha_i}{1-\alpha_i} + \frac{\alpha_j}{1-\alpha_j} = \sum_{k \neq i,j} \frac{\alpha_k}{1-\alpha_k} + \frac{a}{1-a} + \frac{1-K-a}{1-(1-K-a)} \tag{151}$$

Taking the derivative w.r.t. a, we have the first order derivative

$$\frac{1}{(1-a)^2} - \frac{1}{(K+a)^2} \tag{152}$$

which is positive when $\frac{1-K}{2} < a < 1 - K$ and negative when $0 < a < \frac{1-K}{2}$. When $|\alpha_i - \alpha_j| > |\hat{\alpha}_i - \hat{\alpha}_j|$ and $\alpha_i + \alpha_j = \hat{\alpha}_i + \hat{\alpha}_j|$, it must be the case that $\min\{\alpha_i, \alpha_j\} < \min\{\hat{\alpha}_i, \hat{\alpha}_j\}$ and therefore a lower A when the shareholder structure is α than $\hat{\alpha}$.

When the internal cash is insufficient, the total cost is directly given by Av_1, so it is increasing in A. In the sufficient-cash region, for any slack Δ, the minimum cash needed to hold in the last shareholder is given by

$$t_i = \alpha_i(v_1 + \Delta + t_i) \implies t_i = \frac{\alpha}{1-\alpha}(v_1 + \Delta) \tag{153}$$

where Δ solves the equation

$$A(v_1 + \Delta) = W - \Delta \tag{154}$$

which gives

$$\Delta = \frac{W - Av_1}{A+1}. \tag{155}$$

Note that

$$\frac{\mathrm{d}\Delta}{\mathrm{d}A} = -\frac{W + v_1}{(A+1)^2} < 0 \tag{156}$$

and the total cost is given by $W - \Delta$, which is increasing in A. □

C.2 Optimal non-contingent offers with access to externally raised funds and that debt is not borne by the principal

In this extension, we consider another extension where the debt is imposed on the firm's asset, which affects the existing contract holders' payoff.

To implement the outcome $h = \mathbf{0}$ when the agent has access to the funds raised, the set of necessary conditions is that

- The total payment can be financed by new borrowing and internal wealth

$$\sum_{j=1}^{N} t_j(0) \leq F + W \tag{157}$$

- Tendering is better off than holding out for A_i

$$t_i(0) \geq R_i^O\left(v(e_i) + W - \sum_{j=1, j\neq i}^{N} t_j(0), e_i\right), \forall i \in \mathcal{N} \tag{158}$$

Since the new debt F is paid ahead of the existing contract holder, it has no effect other than relaxing the intertemporal constraint. So the optimal solution is independent of the choice of F as any excess borrowing would be undone by the repayment.

Proposition 21. *The optimal non-contingent offer is given by*

$$t_i(0) = t_i^*(\Delta^*) \tag{159}$$

where $t_i^\delta = \inf\{t \geq 0 : t \geq R_i^O(v(e_i) + \Delta + t, e_i)\}$ *and* Δ^* *solve the fixed point equation* $\sum_{i=1}^N t_i^{*\delta} = W - \Delta$.

Proof. Following the same analysis as in the proof of Proposition 19, the solution to the optimal non-contingent exchange offers can be described by the system of equations

$$\begin{cases} t_i = R^O(v(e_i) + W - T + t_i, e_i) & \forall i \in \mathcal{N} \\ T = \sum_{i=1}^{N} t_i \end{cases} \tag{160}$$

But different from Proposition 19, the term $\Delta = W - T$ can be negative here. This can be described by the fixed-point equation

$$\sum_{i=1}^{N} t_i^{*\delta} = W - \Delta \tag{161}$$

where $t_i^{*\delta} = \inf\{t \geq 0 : t \geq R_i^O(v(e_i) + \Delta + t, e_i)\}$. Since the LHS is increasing in Δ and the RHS decreasing, the LHS exceeds the RHS at $\Delta = W$ and $\square$

Proposition 2 (Extreme Gauging)**.** *With fully contingent contracts, the principal can* uniquely *implement the action profile $h = \mathbf{0}$ and guarantees herself a value of $v(\mathbf{0})$.*

Proof. Consider the following offer: Let $\xi(h) = \{i \in \mathcal{N} : h_i = 0\}$ be the set of agents who fully tender in $h = \sum_{i \notin \xi(h)} h_i e_i$. If $\mathcal{D}(h) = \mathcal{N}$, $R_i(v(h), h) = \frac{\varepsilon}{N}$. If instead $\mathcal{D}(h) = \emptyset$, let

$$R_j(v(h), h) = \begin{cases} 0 & \text{if } j \neq 1 \\ v(h) & \text{otherwise} \end{cases} \tag{162}$$

If $\xi(h)$is neither $\emptyset$ nor $\mathcal{N}$, let

$$R_j(v(h), h) = \begin{cases} 0 & \text{if } j \notin \xi(h) \\ \frac{v(h)}{|\xi(h)|} & \text{otherwise} \end{cases} \quad \forall j \in \mathcal{N} \tag{163}$$

To see why $h = 0$ is the unique equilibrium, first, let's check every agent deviating from 0 is not an equilibrium. If in action profile h^a, all agents choosing $h_i^a \neq 0$, then agent i gets the full project value $v(h^a)$ while others get nothing. It's strictly profitable for agent $j \neq 1$ to deviate to $h_j = 0$ since this deviation would result in an increase of j's payoff by $v((h_{-j}^a, 0)) > 0$. Now let's consider an action profile h^p where the nonempty set $\mathcal{D}(h^p) \neq \mathcal{N}$, then for any agent $i \in \mathcal{D}$, he gets 0 in the action profile while deviating to $h_i = 0$ would give him a positive payoff $\frac{v(h^p_{-i},0)}{N-|\mathcal{D}(h^p_{-i},0)|} > 0$. $\square$

D Proofs for Section 4 (Optimal Exchange Offer with Limited Commitment)

Relaxing 1-Lipschitz continuity I have used the Lipschitz condition in two steps: i) calculating the highest alternative payoff the principal can obtain at the deviation node by punishing the holdout on the deviation node; ii) substituting the required off-path payoff to credibility constraint. In both cases, what matters is the maximum punishment at the deviation node instead of that off-deviation node, i.e., on path. But the maximum is no longer guaranteed without the 1-Lipschitz continuity.

I show here relaxing the continuity could lead to the absence of renegotiation-proof contracts using an example. Suppose $v(e_1) = 2$. Consider a strictly increasing càdlàg function

$$R_i^O(v, e_i) = \frac{1}{3}v + \frac{1}{2}\mathbb{1}_{\{v\geq 1\}} \tag{164}$$

the total payment given punishment x is

$$x + R_i^O(v(e_2) - x, e_i) = x + \frac{1}{3}(2-x) + \frac{1}{2}\mathbb{1}_{\{2-x\geq 1\}} \tag{165}$$

$$= \frac{2(1+x)}{3} + \frac{1}{2}\mathbb{1}_{\{x\leq 1\}} \tag{166}$$

and the optimal punishment would be slightly above $x = 1$ but is never attained. Thus, there would be no renegotiation-proof contracts since any contracts can be improved.

Lemma 1. *Suppose $f(\cdot)$ is a weakly increasing 1-Lipschitz function*[84] *and a is a positive*

[84] To see how this Lipschitz condition affects the optimization problem, let's heuristically discuss what happens without it. Since $f(\cdot)$ is a weakly increasing function, it has, at most, a zero-measure set of discontinuous points and is differentiable almost everywhere. It only admits jump discontinuities by Lebesgue's Theorem, which also stipulates the non-differentiable points are either discontinuous, vertical tangent points or kinky points. The optimal solution cannot be just to the right of a jump point. Otherwise, the principal can reduce the total payment by increasing x by a small ϵ and reduce the objective by a lot. The same argument implies it cannot be at a vertical tangent point. So, any interior solution must either satisfy the first-order condition or be at a kinky point. When the first condition is satisfied, it means $1 = f'(a-x)$, i.e., any small increase or decrease in x would just be offset by the response in $f(a-x)$. Put another way, in the context of the model, the claims of the holdouts resemble debt locally at the optimal punishment. Finally, let's discuss the kinky point. One could increase x without violating any constraints at the optimum. This implies the function $x + f(a-x)$ must have a non-negative right derivative at the optimum $\bar{x}$, i.e., $f(\cdot)$ has a left derivative weakly smaller than one at $a - \bar{x}$. To focus on the interesting case and avoid tedious technical

number. The solution to the following problem

$$\min_{x\in[0,a]} g(x) := x + f(a-x) \tag{25}$$

is obtained at $x = 0$ and the minimum value is $f(a)$. Moreover, if $f(\cdot)$ has a left derivative $f'(a) < 1$, the solution is unique. Otherwise, any $x \in [0, \bar{x}]$, where $\bar{x} = \inf\{x : f'(a-x) < 1\}$, solves the problem and any $x > \bar{x}$ does not.

Proof. By Rademacher's theorem, Lipschitz continuity implies that f is absolutely continuous and differentiable almost everywhere in $[0, a]$. We take the first-order derivatives

$$g'(x) = 1 - f'(a-x) \geq 0, a.e., \tag{167}$$

so the function $x + f(a-x)$ is weakly increasing. Therefore, $x = 0$ is one of the optimizers, and the minimum value is $f(a)$.

When $f'(a) < 1$, $g'(x) = 1 - f'(a-x)$ is strictly positive at $x = 0$ so $x = 0$ is the unique solution. To see why, suppose there's also another minimizer $x' > 0$, then $g(x)$ must be flat on $[0, x']$, which means $g'(x)$ can only be non-zero on a set of Lebesgue measure zero. But this is impossible as Darboux's theorem requires that $(g')^{-1}([g'(a)/2, g'(a)])$ is also an interval, which has a positive Lebesgue measure.

When $f'(a) = 1$, for any $x \in [0, \bar{x})$, $1 - f'(a-x) = 0$ so the function $x + f(a-x)$ is flat on $[0, \bar{x}]$, so any $x \in [0, \bar{x}]$ is an optimizer. For any $x > \bar{x}$, $f'(a-x) < 1$, or equivalently $g'(x) > 0$. Fix an $x' > \bar{x}$, by Darboux's theorem, there's an $x'' \in (0, x')$ such that $g'(x'') = \frac{g'(x')}{2}$. Moreover, since f is absolutely continuous, it's derivatives f' is integrable, i.e., $f' \in L^1(0, a)$ and so is g'. Thus we can write

$$\begin{aligned} g(x) =& g(0) + \int_0^x g'(x)\mathrm{d}s && (168)\\ >& g(0) + \int_{(g')^{-1}\left(\left[\frac{g'(x')}{2}, g'(x')\right]\right)} g'(s)\mathrm{d}s && (169)\\ >& g(0) + \frac{g'(x')}{2} m\left((g')^{-1}\left(\left[\frac{g'(x')}{2}, g'(x')\right]\right)\right) && (170)\\ >& g(0). && (171) \end{aligned}$$

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discussions on the unrealistic cases, we assume 1-Lipschitz.

where $m(\cdot)$ is the Lebesgue measure. So, any $x > \bar{x}$ cannot be a minimizer. $\square$

Lemma 2. *Under Assumption A2, the highest payoff the principal can obtain at the deviating profile e_i with an IC contract $\tilde{R} \in \mathcal{I}(e_i)$ is*

$$v(e_i) - R_i^O(v(e_i), e_i). \tag{26}$$

Proof. I first construct an incentive compatible contract $\tilde{R}$ that delivers a payoff of $v(e_i) - R_i^O(v(e_i), e_i)$ to the principal. The construction is similar to that in Proposition 2. Let

$$\tilde{R}_j(v(h), h) = \begin{cases} \varepsilon/N & \text{if } h = e_i \\ 0 & \text{if } h = \mathbf{1} \text{ or } \mathbf{0} \\ \frac{v(h)-\varepsilon}{|\xi(h)|} & \text{if } h \neq e_i, \mathbf{1}, \mathbf{0} \text{ and } j \in \xi(h) \\ 0 & \text{if } h \neq e_i, \mathbf{1}, \mathbf{0} \text{ and } j \notin \xi(h) \end{cases} \tag{172}$$

and

$$\tilde{R}_i(v(h), h) = 0 \quad \forall h. \tag{173}$$

I will now show that with this proposal, for sufficiently small $\varepsilon \geq 0$, e_i is an equilibrium, and when $\varepsilon > 0$ and $R_i^O(\cdot, h)$ has a strictly positive right derivative at 0 for all h, the equilibrium is unique.

- For agent A_i, as long as $h \neq e_i$,$\mathbf{0}$ or $\mathbf{1}$, the total payment to the tendering agents is $v(h) - \varepsilon$ tendering results in a payoff of 0 while holding out yields a payoff of $R_i^O(\varepsilon, h)$, so holding out is strictly better if $\varepsilon > 0$ and $R_i^O(\cdot, h)$ has a strictly positive payoff. When everyone else holds out, holding out yields a payoff of $R_i^O(v(\mathbf{1}), \mathbf{1})$ while tendering gives him nothing.
- For any other agent A_j, non-tendering gives a payoff of zero, and tendering gives a payoff of either ε/N if everyone else other than A_i tenders, or $\frac{v(h)-\varepsilon}{|\xi(h)|}$ otherwise, which is positive for sufficiently small $\varepsilon > 0$.

Thus, we proved $\tilde{R}$ is incentive compatible with e_i.

For any arbitrary contract $\hat{R} \in \mathcal{I}(e_i)$, let $x(e_i; \hat{R}) = \sum_{k \in \xi(e_i)} R_k(v(e_i), e_i)$ be the payment to the tendering agents and thus the total payment is

$$x(e_i; \hat{R}) + R_i^O(v(e_i) - x(e_i; \hat{R}), e_i) \tag{174}$$

Suppose the principal wants to find another contract $\hat{R}$ to minimize the total payment. Under Assumption A2, $R_i^O(\cdot, e_i)$ is weakly increasing and 1-Lipschitz, by Lemma 1, the solution to the minimization problem above is obtained at $x = 0$, which is achieved by $\tilde{R}$ when $\varepsilon = 0$. And the principal obtains a payoff of cannot obtain a higher payoff than $v(e_i) - R_i^O(v(e_i), e_i)$. □

Proposition 3. *When $N \geq 2$, under Assumption A2, the principal cannot obtain a strictly higher value at $h = \mathbf{0}$ with a strongly credible contingent contract than offering cash if and only if for all $i \in \mathcal{N}$*

$$\frac{\partial}{\partial w} R_i^O(w, e_i)\Big|_{w \uparrow v(e_i)} < 1. \tag{27}$$

where $\uparrow$ indicates the limit from the left.[85] *Consequently, if this condition is satisfied, holdout problems cannot be solved with any strongly credible contingent offers under Assumption A1.*

Proof. To prove this result, I first show that when the condition in equation (27) is satisfied for all$i$, the contract R is strongly credible if and only if the off-path punishment at e_i is $x(e_i) := \sum_{j \neq i} R_j(v(e_i), e_i) = 0$. Then, I calculate the value function of the principal and show that it equals the valuing function when offering cash. Lastly, I show that the principal can do strictly better when the condition in equation (27) is violated.

First, from Lemma 2, we know that at the deviation profile e_i the principal was able to obtain $v(e_i) - R_i^O(v(e_i), e_i)$ using an incentive compatible contract. Therefore, the credibility constraint at e_i is

$$v(e_i) - \sum_{k \neq i} R_k(v(e_i), e_i) - R_i^O\left(v(e_i) - \sum_{k \neq i} R_k(v(e_i), e_i), e_i\right) \geq \delta \left[v(e_i) - R_i^O(v(e_i), e_i)\right] \tag{175}$$

Rearranging the terms, we obtain

$$x(e_i; R) + R_i^O(v(e_i) - x(e_i; R), e_i) \leq (1 - \delta)v(e_i) + \delta R_i^O(v(e_i), e_i) \tag{176}$$

where $x(e_i; R) = \sum_{k \neq i} R_k(v(e_i), e_i)$. When $\delta = 1$, using Lemma 1, the unique solution is $x(e_i; R) = 0$ when the first partial derivative $R_i^O(v(e_i), e_i)$ is strictly smaller than 1 at $v(e_i)$. Since any punishment would be renegotiated away and the holdout would be

[85] Since $R_i^O(\cdot, e_i)$ is only meaningful on $[0, v(e_i)]$ and punishment usually reduces the value, we only look at the left derivatives. It always exists given continuity at $v(e_i)$.

paid $R_i^O(v(e_i), e_i)$, in order to persuade the agent to tender, the principal has to pay at least this much to A_i, leaving at most

$$v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i) \tag{177}$$

to the principal, which is equivalent to offering cash. This is lower than c under Assumption A1; therefore, the restructuring plan is infeasible. □

Proposition 4. *When existing securities are debt contracts $D = \{D_i\}_i$, the principal's value function is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} D_i \mathbb{1}_{D_i < v(e_i)} \tag{31}$$

under the strong δ-credibility constraint.

Proof. To prove this, we first show that the maximum possible punishment is $\bar{x}^\delta(e_i) = (1-\delta)(v(e_i) - D_i)$. This is obtained by finding the maximum x such that

$$x + \min\{v(e_i) - x, D_i\} \leq v(e_i) - \delta\left[v(e_i) - \min\{v(e_i), D_i\}\right] \tag{178}$$

When $v(e_i) \leq D_i$, the RHS is simplified to $v(e_i)$, while the LHS is always smaller than $v(e_i)$ as

$$x + \min\{v(e_i) - x, D_i\} \leq x + \min\{v(e_i), D_i + x\} \leq v(e_i) \tag{179}$$

so the maximum punishment is $\bar{x}^\delta(e_i) = v(e_i)$. The holdout A_i doesn't get paid anything.

When $v(e_i) > D_i$, the LHS ranges from D_i to $v(e_i)$ while the RHS $(1-\delta)v(e_i) + \delta D_i$ is a value strictly in between. So the maximum possible value is given by $\bar{x}^\delta(e_i) = (1-\delta)(v(e_i) - D_i)$. And the holdout is paid $\min\{v(e_i) - \bar{x}^\delta(e_i), D_i\} = D_i$.

Thus, at $h = \mathbf{0}$, the principal has to pay D_i to any agent A_i such that $D_i < v(e_i)$ since he could otherwise hold out and get paid in full. So the value function of the principal is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} D_i \mathbb{1}_{D_i < v(e_i)} \tag{180}$$

under the strong δ-credibility constraint. □

Proposition 5. *When existing securities are equities $\alpha = \{\alpha_i\}_i$, the principal's value function on the set of strongly δ-credible contracts is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \delta \sum_{i=1}^{N} \alpha_i v(e_i) \tag{34}$$

which is higher when the commitment is higher (δ is smaller).

Proof. To prove this, I calculate the principal's value function when the cost is sunk.

First, I show that the maximum possible punishment at e_i is $x(e_i; R) = (1-\delta)v(e_i)$. This is obtained by substituting the functional form $R_i^O(x, e_i) = \alpha_i x$ into Equation (176), which becomes

$$x(e_i; R) + \alpha_i(v(e_i) - x(e_i; R)) \leq (1-\delta)v(e_i) + \delta\alpha_i v(e_i) \tag{181}$$

which gives $x(e_i; R) \leq (1-\delta)v(e_i)$ so the maximum punishment that can be imposed on A_i is $x(e_i) = (1-\delta)v(e_i)$.

Therefore, the principal has to pay at least $\alpha_i(v(e_i) - x(e_i)) = \alpha_i \delta v(e_i)$ on path to A_i. The firm's value function is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \delta\alpha_i v(e_i) \tag{182}$$

which is decreasing in δ. □

Proposition 6. *The principal's value function $J(\mathbf{0})$ on the set of strongly δ-credible contracts is weakly decreasing in δ for any existing contracts R^O.*

Proof. I first prove the maximum punishment $x^\delta(e_i)$, given by finding the largest x subject to the inequality

$$x + R_i^O(v(e_i) - x, e_i) \leq v(e_i) - \delta(v(e_i) - R_i^O(v(e_i), e_i)) \tag{183}$$

is decreasing in δ for any e_i. I prove this auxiliary statement by contradiction. Suppose there exists $\delta_1 < \delta_2$ and $x^{\delta_1}(e_i) < x^{\delta_2}(e_i)$ for some e_i. Then we have

$$x^{\delta_2}(e_i)+R_i^O(v(e_i)-x^{\delta_2}(e_i),e_i) \leq v(e_i)-\delta_2(v(e_i)-R_i^O(v(e_i),e_i)) < v(e_i)-\delta_1(v(e_i)-R_i^O(v(e_i),e_i)) \tag{184}$$

where the first inequality is given by the definition of $x^{\delta_2}(e_i)$ and the second is by $\delta_2 > \delta_1$. Thus $x^{\delta_2}(e_i)$ is a feasible value of x when $\delta = \delta_1$ in equation (183). This contradicts the optimality of $x^{\delta_1}(e_i)$! Thus, it must be $x^{\delta_1}(e_i) \geq x^{\delta_2}(e_i)$.

The principal's value function

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i) - x^{\delta}(e_i), e_i) \tag{185}$$

is increasing in $x^{\delta}(e_i)$ for each e_i since $R_i^O(\cdot, e_i)$ is increasing for each e_i.

Combining these two facts, we arrive at the conclusion that $J(\mathbf{0})$ is weakly decreasing in δ for any R^O.

□

Proposition 7 (Existence and Uniqueness). *The set of δ−credible contracts $\{C(h)\}_h$ exists, it is non-empty and unique.*

Proof Overview To tackle this problem, we decompose the problem into two subproblems. First, for each h, we assign a number $J(h)$, and define the sets of contracts that are i) IC at each h and ii) allow the principal to guarantee a payoff of at least $J(h)$:

$$C^{\delta}(h|J) := \left\{R \in \mathcal{I} : v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) \geq \delta J(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\right\} \tag{SP1}$$

The set of contract $C^{\delta}(\cdot|J)$ is no longer recursively defined, so we can easily see that the set is unique, despite that it might be empty for some values of J. Indeed, we will show in the proof that the $C^{\delta}(h|J)$ is non-empty if $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$ for $\delta > 0$. Second, for any sets of contracts $\mathcal{R}$ available at h, we define an upper bound of the value attainable by the principal using contracts within $\mathcal{R}$ to be

$$J(h|\mathcal{R}) := \sup_{\tilde{R} \in \mathcal{R}(h)} v(h) - \sum_{i=1}^{N} u_i(h_i|h_{-i}, \tilde{R}) \tag{SP2}$$

We follow the convention and define the supremum to be $-\infty$ if the set $\mathcal{R}(h)$ is empty. The supremum need not be attainable if the contract space $\mathcal{R}$ is not compact or the objective function is not continuous in $\tilde{R}$. But regardless, we have the following *Let J^* be the vector that solves the fixed-point equation*

$$J(h) = J(h|C^{\delta}(h|J)) \quad \forall h \in H, \tag{186}$$

then $C^{\delta}(\cdot|J^*)$ satisfies the definition of credible contracts in Definition 6. On the other hand, for any credible contracts $C^{\delta *}$ defined in Definition 6, whenever it exists, the value function $J(h|C^{\delta *})$, as defined in Equation (SP2) solves the fixed-point equation (186).

Note the first J on the RHS of equation (186) is part of the conditional operator $J(\cdot|\cdot)$ and hence not part of the solution J^*, which is a vector. The proof is largely standard and formalizes the idea that the recursive definition can be characterized by a fixed-point equation. Here, we look at the fixed point of the value function instead of the sets to circumvent technical issues with the mapping between sets of contracts.

This approach is very similar to the classic dynamic contracting problems starting from the seminal paper where they reduce the dynamic contract problem to a static one given the continuation value. The main difference is that here, the recursion is over the action space instead of time so that there is no order of dependence. Here, the value functions of two different action profiles can mutually depend on each other, which brings up the issue of existence and uniqueness. The next result says it is not a concern.

Proof. Proof. I first prove that $C^{\delta}(\cdot|J^*)$ satisfies the definition of credible contracts. For any contract $R \in C^{\delta}(h|J^*)$, the IC at h is satisfied automatically, so we only need to check that at any deviation node $\hat{h} \in \mathcal{B}(h)$, it dominates any contract $\tilde{R} \in C^{\delta}(\hat{h}|J^*)$. From the definition of J^* and thus $C^{\delta}(h|J^*)$, we know that for any $R \in C^{\delta}(h|J^*)$, we have

$$v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) \geq \delta J^*(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \tag{187}$$

and that

$$J^{*}(\hat{h}) = J(\hat{h}|C^{\delta}(\hat{h}|J^{*})) = \sup_{\tilde{R}\in C^{\delta}(\hat{h}|J^{*})} v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, \tilde{R}). \tag{188}$$

Passing the inequality from the supremum to each contract in $C^{\delta}(\hat{h}|J^{*})$, we arrive at

$$v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) \geq \delta\left[v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, \tilde{R})\right] \quad \forall \tilde{R} \in C^{\delta}(\hat{h}|J^{*}) \quad \forall \hat{h} \in \mathcal{B}(h) \tag{189}$$

which proves that $C^{\delta}(\cdot|J^{*})$ is a set of credible contracts.

Now I prove the other direction by showing that $J(h|C^{\delta*})$ solves the fixed-point equation (186), i.e., $J(h|C^{\delta*}) = J(h|C^{\delta}(h|J(h|C^{\delta*})))$. For any h, by definition of $C^{\delta*}$ and $J(\cdot|\cdot)$, we have

$$J(h|C^{\delta*}) = \sup_{\tilde{R}\in C^{\delta*}(h)} v(h) - \sum_{i=1}^{N} u_i(h_i|h_{-i}, \tilde{R}) \tag{190}$$

and that

$$C^{\delta*}(h) = \left\{ R : \begin{array}{l} u_i(h_i|h_{-i}, R) \geq u_i(h_i'|h_{-i}, R) \quad \forall h_i' \in H_i \quad \forall i \in \mathcal{N} \quad \& \\ v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) \geq \delta\left[v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, \tilde{R})\right] \quad \forall \tilde{R} \in C^{\delta*}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \end{array} \right\}. \tag{191}$$

Substitute in the definition $C^{\delta*}(\hat{h})$ and passing inequality to the supremum, we can write

$$C^{\delta*}(h) = \left\{ R : \begin{array}{l} u_i(h_i|h_{-i}, R) \geq u_i(h_i'|h_{-i}, R) \quad \forall h_i' \in H_i \quad \forall i \in \mathcal{N} \quad \& \\ v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) \geq \delta J(\hat{h}|C^{\delta*}) \quad \forall \hat{h} \in \mathcal{B}(h) \end{array} \right\} = C^{\delta}(h|J(h|C^{\delta*})). \tag{192}$$

To see this, suppose instead $v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) < \delta J(\hat{h}|C^{\delta*})$, then by definition of sup, there exists a $\tilde{R}$ such that $v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R) < \delta\left[v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, \tilde{R})\right]$, contradicting the definition of $C^{\delta*}(h)$. Finally, applying the J operator on the identity $C^{\delta*}(h) = C^{\delta}(h|J(h|C^{\delta*}))$, we get $J(h|C^{\delta*}) = J(h|C^{\delta}(h|J(h|C^{\delta*})))$.

Thus, we established the equivalence of the recursive definition 6 and the fixed-point characterization (186). $\square$

The key step in the proof is that the constraint the credibility puts is asymmetric for agents who deviate from holdout to tendering and for those who deviate from tendering to holdout. In the former case, to deter tendering, we must reduce the payoff from tendering for the deviating agent. This can be easily achieved by reducing his payoff from tendering to 0. Doing so would not affect the credibility constraint as it weakly reduces the total payoffs to all agents under the 1-Lipschitz condition, which is weakly beneficial for the principal. However, to discourage an agent from holding out, the principal must try to minimize his payoff off-path. However, there is a limit to what the principal can achieve by imposing externalities on him. In other words, the principal can only punish deviating agents by granting higher payoff to other tendering agents, but doing so would weakly lower the principal's payoff. There will be no renegotiation as long as it's still below the principal's value function at the deviation profile. So, the maximum punishment the principal can credibly impose on the deviator on the deviation node is the one that makes her payoff equivalent to her value function at the deviation node.

This asymmetry in constraints reveals an asymmetric inter-dependence of the value functions that the value of $J(h|C^{\delta}(h|J))$ only depends on the values of $J(\hat{h}|J)$ for the profiles $\hat{h}$ where there are more deviating agents than h, i.e., $\xi(\hat{h}) \subset \xi(h)$. Thus, we can prove the existence by constructing a vector J^* that solves the fixed-point equation (186) in finite steps. We start from an arbitrary vector J^0 in the feasible space (specified in the proof) and calculate the value function on the action profile $\mathbf{1}$ on which everyone holds out. It turns out that, as expected, the value function $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$ is independent of the choice of J^0. Then we replace the value of $J(\mathbf{1})$ by $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$, and use that vector, renamed J^1, for the next iteration, i.e., calculating the value function $J(h|C^{\delta}(h|J^1))$ on the action profiles where exactly one agent tenders. Again, it turns out the value function is independent of the initial choice J^0: it only depends on the value $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$. We update the vector and continue the process by calculating the value functions on all the profiles where one more agent tenders. This process ends after we calculate the value function on the node $\mathbf{0}$ on which everyone tenders and set the vector J^{N+1} to be J^*. Finally, we conclude that the vector found J^* is indeed the solution to the fixed-point equation by noticing $J^*(h) = J(h|C^{\delta}(h|J^{k+1})) = J(h|C^{\delta}(h|J^*))$ for any h such that $|\xi(h)| = k$.

The uniqueness can be obtained by noticing that in the construction above, the fixed point found is independent of the choice of the initial J^0. In the proof, I give a more formal proof by contradiction, showing that there's no other solution than the one found using the procedure above.

Solving for fixed-point To show a fixed point J^* exists and is unique, I first prove that the set $C^\delta(h|J)$ is non-empty for all $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$; then I display the asymmetry mentioned by solving the problem SP2 over the sets $C^\delta(h|J)$ for any vector $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, i.e., I want to calculate $J(h|C^\delta(h|J))$.

Non-emptiness of $C^\delta(h|J)$ I first show that $C^\delta(h|J)$ is non-empty for any $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$. To do so, I only need to give one example of a contract, and an obvious one would be this "no-punishment contract".

- At h, the principal pays to whoever holds out 0 through new contracts, and to whoever tenders what he would otherwise obtain, he holds out, i.e.,

$$R_i(v(h), h) = \begin{cases} 0 & \forall i \notin \xi(h) \\ R_i^O(v(h + e_i), h) & \forall i \in \xi(h) \end{cases} \tag{193}$$

  The IC, the first constraint in the definition (Equation SP1), is clearly satisfied.

- At $\hat{h} \in \mathcal{B}(h)$, the principal pays nothing to the tendering agents and any arbitrary amount, e.g., 0, to those who hold out in the new contract, which they don't accept. Then the total payout to all agents is

$$0 + \hat{h} \cdot R^O(v(\hat{h}) - 0, \hat{h}) \tag{194}$$

  which is no larger than $v(\hat{h}) - \delta J(\hat{h})$ so the second constraint in the definition (Equation SP1) is also satisfied.

- It takes any arbitrary values on any other action profiles.

Since at least one contract exists in $C^\delta(h|J)$ when $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, it's non-empty.

Now, I prove another auxiliary lemma that would be used in the main proof.

Lemma 8. *Let $f(\cdot)$ and $g(\cdot)$ be two weakly increasing 1-Lipschitz functions and so is their sum. Given two constant $a, b > 0$, the solution to the problem*

$$\inf_{x \in [0,a]} g(a-x) \tag{195}$$

subject to

$$g(a-x) + f(a-x) + x \leq b \tag{196}$$

exists if and only of $f(a) + g(a) \leq b$ and one solution is given by

$$\bar{x} = \max\{x \in [0,a] : g(a-x) + f(a-x) + x = b\}. \tag{197}$$

Proof. Invoking Lemma 1, the fact that $f(\cdot)+g(\cdot)$ is 1-Lipschitz implies that $g(a-x)+f(a-x)+x$ is a weakly increasing function and its minimum can always be attained at $x = 0$, so the feasible set is non-empty if and only if $f(a) + g(a) \leq b$. Moreover, the continuity of $f(\cdot)$ and $g(\cdot)$ also implies the feasible set $\{x \in [0,a], g(a-x) + f(a-x) + x \leq b\}$ is compact so the infimum can be attained whenever it is non-empty.

Since $g(a-x)$ is a weakly decreasing function of x, its minimum can be achieved at the largest x in which the constraint is satisfied. Since $g(a-x)+f(a-x)+x$ is a weakly increasing, an obvious one is simply $\bar{x} = \max\{x \in [0,a] : g(a-x)+f(a-x)+x = b\}$. □

Asymmetry in ICs We want to show the value of J only affects the credibility constraints at the deviation node $\hat{h}$, which in turn affects the IC constraint through the off-path threat $u_i(h_i'|h_{-i}, R)$. To be more specific, let's say, at h, the agent A_j deviates, i.e., $\hat{h} = (h_{-j}, 1-h_j)$, which includes two cases:

- Agent j deviates from 1 to 0, i.e., $h_j = 1$ and $\hat{h}_j = 0$: In this case, the on-path IC for agent j is

$$u_j(h_j|h_{-j}, R) = R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h\right) \geq R_j(v(\hat{h}), \hat{h}) \tag{198}$$

  using the fact that after taking the deviation $h_j' = 1 - h_j$, the action profile arrives at $\hat{h}$. The credibility constraint at $\hat{h}$ can be written as

$$R_j(v(\hat{h}), \hat{h}) + \sum_{i \in \xi(h)} R_i(v(\hat{h}), \hat{h}) + \sum_{i \notin \xi(\hat{h})} R_i^O\left(v(\hat{h}) - \sum_{k \in \xi(h)} R_k(v(\hat{h}), \hat{h}) - R_j(v(\hat{h}), \hat{h}), \hat{h}\right)$$
$$\leq v(\hat{h}) - \delta J(\hat{h}). \tag{199}$$

I used the fact that $\xi(\hat{h}) = \xi(h) \coprod \{j\}$ and consequently $\{j\} \coprod \xi(\hat{h})^c \coprod \xi(h) = \mathcal{N}$, which allows me to write the total payoff on the left-hand side to all agents in three parts.

In order to maximize the principal's payoff at h, I want to set the total payoff to the agents $\sum_{i=1}^N u_i(h_i|h_{-i}, R)$ as small as possible. The problem is only relaxed if $R_j(v(\hat{h}), \hat{h})$ is made smaller.

I want to ask what's the smallest possible value for $R_j(v(\hat{h}), \hat{h})$. Under the assumption that $h \cdot R^O(\cdot, \hat{h}) = \sum_{i=1}^N h_i \cdot R_i^O(\cdot, \hat{h})$ is 1-Lipschitz, *without additional constraints,* the minimum of the left-hand side can be achieved by setting $R_j(v(\hat{h}), \hat{h}) = 0$ and $\sum_{i \in \xi(h)} R_i(v(\hat{h}), \hat{h}) = 0$ simultaneously by Lemma 1. This constitutes a solution to the minimization problem if and only if $\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h})$. Moreover, the IC is reduced from $R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h\right) \geq R_j(v(\hat{h}), \hat{h})$ to $R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h\right) \geq 0$ by setting $R_j(v(\hat{h}), \hat{h})$ to 0, which always holds as R_j^O is assumed to be non-negative.

This tells us that in order to prevent the agent A_j from deviating to tendering, I could just set the payoff of tendering to zero for agent j without affecting any other constraints, so neither the IC nor the credibility constraint has a bite as long as $\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h})$.

- Agent j deviates from 0 to 1, i.e., $h_j = 0$ and $\hat{h}_j = 1$. The on-path IC for agent j is

$$u_j(h_j|h_{-j}, R) = R_j(v(h), h) \geq R_j^O\left(v(\hat{h}) - \sum_{i \in \xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h}\right) \tag{200}$$

Again, the problem can be relaxed if we can make $R_j^O\left(v(\hat{h}) - \sum_{i:\hat{h}_i=0} R_i(v(\hat{h}), \hat{h}), \hat{h}\right)$

smaller, if unimpeded by the credibility constraint at $\hat{h}$. The credibility constraint now is

$$\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}) + R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right) \tag{201}$$

$$+\sum_{k\notin\xi(h)} R_k^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right) \leq v(\hat{h}) - \delta J(\hat{h}) \tag{202}$$

Again, we are using the fact that $\{j\}\coprod\xi(\hat{h})\coprod\xi(h)^c = \mathcal{N}$. And again, the left-hand side could be minimized by setting $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$ to zero without affecting other constraints under 1-Lipschitz condition using Lemma 1. So the condition for the existence of the solution is again $\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h}\cdot R^O(v(\hat{h}),\hat{h})$.

However, setting $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$ to zero,[86] despite of minimizing the total payoff to $\{j\}\coprod\xi(h)^c$, doesn't necessarily minimize $R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right)$ as the value of it is $R_j^O(v(\hat{h}),\hat{h})$ instead of zero. I could further increase the value of $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$, the value of the LHS might also increase until the constraint is binding, *without additional constraints.* Using Lemma 8, we know that $R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right)$ is minimized at

$$\bar{x}^\delta(J(\hat{h});\hat{h}) := \max\left\{x\in[0,v(\hat{h})]:\ \ R_j^O\left(v(\hat{h})-x,\hat{h}\right) + x + \right. \tag{203}$$

$$\left.\sum_{k\notin\xi(h)} R_k^O\left(v(\hat{h})-x,\hat{h}\right) = v(\hat{h}) - \delta J(\hat{h})\right\} \tag{204}$$

$$= \max\left\{x\in[0,v(\hat{h})]:\ \ \hat{h}\cdot R^O\left(v(\hat{h})-x,\hat{h}\right) + x = v(\hat{h}) - \delta J(\hat{h})\right\}, \tag{205}$$

using the fact $\{j\}\coprod\xi(h)^c = \xi(\hat{h})^c$. Note the solution exists because $0 + \hat{h}\cdot R^O\left(v(\hat{h}),\hat{h}\right) = v(\hat{h}) - J(\hat{h}) \leq v(\hat{v}) - \delta J(\hat{h}) \leq v(\hat{h}) + \hat{h}\cdot R^O\left(0,\hat{h}\right)$ and the LHS is a continuous function of x . The maximum is attainable because the zeros of a Lipschitz function on a closed interval are a compact set.

In what follows, we will call this the maximum possible punishment (or threat)

[86] Note, I do not require R to be IC at $\hat{h}$ so it can be set to 0. The alternative contract $\tilde{R}$ that can be proposed needs to be IC, but it's captured in the $J(\hat{h})$.

at $\hat{h}$, and write it as $\bar{x}^{\delta}(\hat{h})$ when the value function $J(\hat{h})$ is plugged in recursively. Again, we will drop δ from the notation when it equals 1.

The minimum possible value of $R^O_j\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}), \hat{h}), \hat{h}\right)$ is $R^O_j\left(v(\hat{h}) - \bar{x}(J(\hat{h}); \hat{h}), \hat{h}\right)$. This tells us that the credibility constraint does have a bite in order to persuade agent j to tender.

Note that the function $\bar{x}^{\delta}(\cdot; \hat{h})$ is not necessarily continuous. It admits a jump whenever $\hat{h} \cdot R^O(\cdot, \hat{h})$ has a flat region.

Remark: One caveat though is that the contract R need not be the same one when $\hat{h}$ is considered from other profiles. Readers might notice that in the first case when we set $R_j(v(\hat{h}), \hat{h}) = 0$ and $\sum_{i\in\xi(h)} R_i(v(\hat{h}), \hat{h}) = 0$, we only consider the deviation from h to $\hat{h} = h - e_j$, but the value of the contract R on $\hat{h}$ would matter if we view $\hat{h}$ as deviation from other action profiles. In particular, we can divide them into two categories:

- Profile $\hat{h}$ as deviation from $\hat{h} + e_i$ for some $i \in \xi(\hat{h})$ and $i \neq j$. Since $\xi(\hat{h}) = \xi(h)\coprod\{j\} = \xi(\hat{h} + e_i)\coprod\{i\}$, setting $R_k(v(\hat{h}), \hat{h})$ to zero for all $k \in \xi(\hat{h})$ coincides with minimization of agent i's payoff when preventing agent i deviating from $\hat{h} + e_i$ to $\hat{h}$. Therefore, we can set them to zero without worrying about other deviations.
- Profile $\hat{h}$ as deviation from $\hat{h} - e_i$ for some $i \notin \xi(\hat{h})$. According to the analysis in the second case, in order to prevent this type of deviation while minimizing the outside option of the deviator, we need to have $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}), \hat{h}) = \bar{x}(J(\hat{h}); \hat{h})$. This condition cannot be satisfied simultaneously if $\bar{x}^{\delta}(J(\hat{h}); \hat{h}) > 0$.

Similarly, the same issue occurs when we set $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}), \hat{h}) = \bar{x}^{\delta}(J(\hat{h}); \hat{h})$ the in second deviation case. However, this is not an issue as we are calculating the contracts in $C^{\delta}(h|J)$ that maximize the principal's payoff at h, it need not be the same contract with the one in $C^{\delta}(h - e_j + e_i|J)$ that maximizes principal's payoff at $h - e_j + e_i$, i.e.,

$$\forall\, R \in \arg\max_{C^{\delta}(h|J)} v(h) - \sum_{i=1}^{N} u_i(h_i|h_{-i}, \tilde{R}) \tag{206}$$

$$\not\Longrightarrow\; R \in \arg\max_{C^{\delta}(\tilde{h}|J)} v(h) - \sum_{i=1}^{N} u_i(\tilde{h}_i|\tilde{h}_{-i}, \tilde{R}) \text{ for } \tilde{h} = h - e_j + e_i : i \neq j \tag{207}$$

Summary of existence and uniqueness In summary, the condition for a credible to exist is that for any deviation $\hat{h} \in \mathcal{B}(h)$, the highest value $J(\hat{h})$ that can be alternatively obtained using a credible contract at $\hat{h}$ is smaller than the difference between the asset value $v(\hat{h})$ and the collective holdout payout $\hat{h} \cdot R^O(v(\hat{h}), \hat{h})$, i.e.,

$$\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h). \tag{208}$$

Moreover, the analysis above shows that, for any $J \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$, the value $J(h|C^\delta(h|J))$ depends only on $J(h + e_i)$ for some $i \in \xi(h)$, and recursively on any $h' \geq h$. On the contrary, the value $J(h - e_j)$ for some $j \notin \xi(h)$ does not affect the value of $J(h|C^\delta(h|J))$ and recursively so does any h' not in the upper contour set of h: $\{h' : h' \geq h\}$.

Construction of the fixed point: The discussion above allows us to calculate the J^* via the following procedure for $\delta > 0$. In particular, we want to emphasize that we are not calculating $J(h|C^\delta(h|J))$ for a specific J.

1. First we decompose $H = \{0, 1\}^N$ into $N + 1$ disjoint sets

$$H^k = \{h : \xi(h) = k\} \text{ for } k = 0, ..., N \tag{209}$$

   on which exactly k agents tender.

2. We calculate the $J(h|C^\delta(h|J^0))$ on $H^0 = \{\mathbf{1}\}$ for any fixed $J^0 \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$. Since at $h = \mathbf{1}$, none of the credibility constraints matter, and the ICs are simply

$$R_i^O(1|\mathbf{1}_{-i}, \mathbf{1}) = R_i^O(v(\mathbf{1}, \mathbf{1})) \geq 0 \quad \forall i \in \mathcal{N} \tag{210}$$

   so we can calculate the value function

$$J^*(\mathbf{1}) := J(\mathbf{1}|C^\delta(\mathbf{1}|J^0)) = v(\mathbf{1}) - \sum_{i=1}^N R_i^O(v(\mathbf{1}, \mathbf{1})) = R_0^O(v(\mathbf{1}), \mathbf{1}) \tag{211}$$

   and the maximum possible punishment

$$\bar{x}^\delta(\mathbf{1}) := \bar{x}^\delta(J^*(\mathbf{1}), \mathbf{1}) = v(\mathbf{1}) - \delta J^*(\mathbf{1}) = \delta \mathbf{1} \cdot R^O(v(\mathbf{1}, \mathbf{1})) + (1 - \delta) v(\mathbf{1}). \tag{212}$$

Then, we update our J^0 to J^1 as follows

$$J^1(h) = \begin{cases} J^0(h) & \text{if } h \notin H^0 \\ J^*(h) & \text{if } h \in H^0 \end{cases} \tag{213}$$

3. Now instead of calculating $J(h|C^\delta(h|J^0))$ on $H^1 = \{\mathbf{1} - e_i\}_{i\in\mathcal{N}}$, we calculate $J(h|C^\delta(h|J^1))$. Given that, we can calculate the action profile with one fewer 1, i.e., $h = \mathbf{1} - e_i$, the only relevant constraint is

$$R_i(v(h), h) \geq R_i^O(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}) \tag{214}$$

and we obtain the value function

$$J^*(\mathbf{1} - e_i) := v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(J^1(\mathbf{1}); \mathbf{1}), \mathbf{1}\right) \tag{215}$$

$$- \sum_{j\neq i} R_j^O\left(v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(J^1(\mathbf{1}); \mathbf{1}), \mathbf{1}\right), \mathbf{1} - e_i\right) \tag{216}$$

$$= v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right) \tag{217}$$

$$- \langle \mathbf{1} - e_i, R^O\rangle\left(v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right), \mathbf{1} - e_i\right) \tag{218}$$

To calculate $\bar{x}^\delta(J(\mathbf{1} - e_i); \mathbf{1} - e_i)$, we need to find the largest solution to

$$(\mathbf{1} - e_i) \cdot R^O(v(\mathbf{1} - e_i) - x, \mathbf{1} - e_i) + x = v(\mathbf{1} - e_i) - \delta J^*(\mathbf{1} - e_i) \tag{219}$$

where the RHS is

$$(1 - \delta)v(\mathbf{1} - e_i) + \delta R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right) \tag{220}$$

$$+ \delta\langle \mathbf{1} - e_i, R^O\rangle\left(v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right), \mathbf{1} - e_i\right) \tag{221}$$

If we substitute $x = R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right)$ into the expression, the LHS yields

$$R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right) + \langle \mathbf{1} - e_i, R^O\rangle\left(v(\mathbf{1} - e_i) - R_i^O\left(v(\mathbf{1}) - \bar{x}^\delta(\mathbf{1}), \mathbf{1}\right), \mathbf{1} - e_i\right) \tag{222}$$

which is smaller than the RHS as

$$(1-\delta)v(\mathbf{1}-e_i) \geq (1-\delta)R_i^O\left(v(\mathbf{1})-\bar{x}^\delta(\mathbf{1}),\mathbf{1}\right) \tag{223}$$

$$+(1-\delta)\langle \mathbf{1}-e_i, R^O\rangle\left(v(\mathbf{1}-e_i)-R_i^O\left(v(\mathbf{1})-\bar{x}^\delta(\mathbf{1}),\mathbf{1}\right),\mathbf{1}-e_i\right). \tag{224}$$

And if we choose $x=v(\mathbf{1}-e_i)$, then the LHS is $v(\mathbf{1}-e_i)$. By continuity, a solution must exist. The maximum can be attained because, for a continuous function, the pre-image of a singleton is a closed set, which is also bounded here.

In particular, when $\delta=1$, $x=R_i^O\left(v(\mathbf{1})-\bar{x}(\mathbf{1}),\mathbf{1}\right)$ is a solution to the equation above and thus by Lemma 1

$$\bar{x}(\mathbf{1}-e_i)=\bar{x}(J(\mathbf{1}-e_i);\mathbf{1}-e_i)=R_i^O\left(v(\mathbf{1})-\bar{x}(\mathbf{1}),\mathbf{1}\right) \tag{225}$$

$$+\inf\left\{x\geq 0,\frac{\partial}{\partial v}(\mathbf{1}-e_i)\cdot R^O\left(v(\mathbf{1}-e_i)-R_i^O\left(v(\mathbf{1})-\bar{x}(\mathbf{1}),\mathbf{1}\right)-x,\mathbf{1}-e_i\right)<1\right\}. \tag{226}$$

the second term of which is zero if the holdout agents do not collectively take all the asset value and positive otherwise The we update the value of J^1 to J^2 as follows

$$J^2(h)=\begin{cases} J^1(h) & \text{if } h\notin H^1 \\ J^*(h) & \text{if } h\in H^1 \end{cases} \tag{227}$$

$k+3$. Now we carry out the calculation by induction: Suppose $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ are defined on all H^κ for $\kappa=0,1,\dots,k$, and J^{k+1} is also defined. We solve for $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ defined on all H^{k+1} by solving for $J(h|C^\delta(h|J^{k+1}))$ and update J^{k+1} to J^{k+2}. For any $h\in H^{k+1}$, the relevant constraints are

$$R_i(v(h),h)\geq R_i^O(v(h+e_i)-\bar{x}^\delta(h+e_i),h+e_i). \tag{228}$$

The RHS is known as $h+e_i\in H^k$. We re-iterate the principal's problem following the simplification above:

$$\max_R v(h) - \sum_{i \in \xi(h)} R_i(v(h), h) - \sum_{j \notin \xi(h)} R_j^O \left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h \right) \tag{229}$$

subject to the constraints

$$\begin{cases} R_j^O \left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h \right) \geq 0 \quad \forall j \notin \xi(h) \\ R_j(v(h), h) \geq R_j^O \left(v(h + e_j) - \bar{x}^\delta(h + e_j), h + e_j \right) \quad \forall j \in \xi(h) \end{cases} \tag{230}$$

The first set of constraints for the holdouts $\{i \in \mathcal{N} : h_i = 1\}$ are naturally satisfied by the feasibility constraints, so we only have credibility constraints for tendering agents.

Again, under the assumption that $h \cdot R^O(\cdot, h)$ is 1-Lipschitz, the objective is weakly decreasing in each on-path payoff $R_j(v(h), h)$ for each j such that $h_j = 0$.

And we have

$$J^*(h) := J(h|C^\delta(h|J^{k+1})) = v(h) - \sum_{i \in \xi(h)} R_i^O \left(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i \right) - \sum_{j \notin \xi(h)} R_j^O \left(v(h) - \sum_{i \in \xi(h)} R_i^O \left(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i \right), h \right). \tag{231}$$

To calculate the maximum possible punishment at h, we find the largest solution to the equation

$$h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h) \tag{232}$$

Using a similar argument as in Step 3, the maximum solution exists and is unique, and we calculate

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)\}. \tag{233}$$

In particular, when $\delta = 1$, the maximum possible punishment

$$\bar{x}(h) = \sum_{i \in \xi(h)} R_i^O \left(v(h + e_i) - \bar{x}(h + e_i), h + e_i \right) \tag{234}$$

$$+ \inf \left\{ x \geq 0 : \frac{\partial}{\partial v} h \cdot R^O \left(v(h) - \sum_{i \in \xi(h)} R_i^O \left(v(h + e_i) - \bar{x}(h + e_i), h + e_i \right) - x, h \right) < 1 \right\}. \tag{235}$$

We also update J^{k+1} to J^{k+2} as follows

$$J^{k+2}(h) = \begin{cases} J^{k+1}(h) & \text{if } h \notin H^{k+1} \\ J^*(h) & \text{if } h \in H^{k+1} \end{cases} \tag{236}$$

$N + 3$. Finally, after calculating J^* for $k = N - 1$, we obtain $J^* = J^{N+1}$, and we need to verify that it satisfies

$$J^*(h) = J(h|C^\delta(h|J^*)). \tag{237}$$

This could be easily done by observing that $J^*(h) = J(h|C^\delta(h|J^{k+1})) = J(h|C^\delta(h|J^*))$ for any $h \in H^k$ for $k = 0, 1, ..., N - 1$.

Finally, the uniqueness should be obvious, noticing the J^* we calculated in the procedure is independent of the initial choice J^0. But the readers may wonder if there's a fixed point not found through the procedures above. To alleviate this concern, suppose there exists to exist points J and $\tilde{J}$ such that $J \neq \tilde{J}$ and $J(h) = J(h|C^\delta(h|J))$ (resp. $\tilde{J}(h) = J(h|C^\delta(h|\tilde{J}))$) for any h. Since $J(\mathbf{1}|C^\delta(\mathbf{1}|J))$ doesn't depend on any J, it must be that $J(\mathbf{1}) = J(\mathbf{1}|C^\delta(\mathbf{1}|J)) = J(\mathbf{1}|C^\delta(\mathbf{1}|\tilde{J})) = \tilde{J}(\mathbf{b})$. Then there must be an h such that $J(h) = \tilde{J}(h)$. Let

$$\underline{k} = \min\{k \geq 1 : \exists h \in H^k : J(h) \neq \tilde{J}(h)\} \tag{238}$$

Then on all the action profiles $h \in H^{\underline{k}-1}$, $J(h) = \tilde{J}(h)$, then we would have for all $h \in H^{\underline{k}}$

$$J(h) = J(h|C^\delta(h|J)) = J(h|C^\delta(h|\tilde{J})) = \tilde{J}(h), \tag{239}$$

contradicting the definition of $\underline{k}$. Thus, the solution to the fixed point equation (186) is unique.

□

Proposition 8. *The pair of vectors* $\{J^*(h), \bar{x}^\delta(h)\}_{h\in\{0,1\}^N}$ *is the pair of the principal's value function* J^* *and the maximum punishment* $\bar{x}^\delta$ *at each node* h *if and only if they satisfy the following recursive relation*

$$J^*(h) = v(h) - \underline{x}(h) - \sum_{j\notin\xi(h)} R_j^O\left(v(h) - \underline{x}(h), h\right) \tag{37}$$

where

$$\underline{x}(h) := \sum_{i\in\xi(h)} R_i^O\left(v(h+e_i) - \bar{x}^\delta(h+e_i), h+e_i\right) \tag{38}$$

is the minimum punishment to implement h*, and*

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)\} \tag{39}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$.

Proof. The "only if" part is derived in the proof of 7, and the "if" part is by uniqueness. □

Lemma 3. *When* $\{R_i^O\}_i$ *are equity contracts, i.e.,* $R_i^O(v, h) = \alpha_i v$ *for all* h*, the maximum possible punishment on the action profile* h *satisfies the recursive relation*

$$\bar{x}^\delta(h) = (1-\delta)v(h) + \delta \sum_{i\in\xi(h)} \alpha_i(v(h+e_i) - \bar{x}^\delta(h+e_i)) \quad \forall h \neq \mathbf{1} \tag{40}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$ *if either* $\sum_{i=1}^N \alpha_i = 1$ *or* $v(\mathbf{1}) = 0$.

Proof. We first calculate the initial condition at $h = \mathbf{1}$. Since credibility constraint matters at $\mathbf{1}$, the principal obtains her highest value by paying every agent his holdout payoff

$$J(\mathbf{1}) = v(\mathbf{1}) - \mathbf{1} \cdot R^O(v(\mathbf{1}), \mathbf{1}) \tag{240}$$

To solve for $\bar{x}^\delta(\mathbf{1})$, I solve the equation

$$x + \mathbf{1} \cdot R^O(v(\mathbf{1}) - x, \mathbf{1}) = (1-\delta)v(\mathbf{1}) + \delta\mathbf{1} \cdot R^O(v(\mathbf{1}), \mathbf{1}) \tag{241}$$

which, in the equity case, can be written as

$$x + \langle \mathbf{1}, \alpha \rangle (v(\mathbf{1}) - x) = (1 - \delta) v(\mathbf{1}) + \delta \langle \mathbf{1}, \alpha \rangle v(\mathbf{1}) \tag{242}$$

Rearranging terms

$$(1 - \langle \mathbf{1}, \alpha \rangle) x = (1 - \delta)(1 - \langle \mathbf{1}, \alpha \rangle) v(\mathbf{1}) \tag{243}$$

If $\langle \mathbf{1}, \alpha \rangle \neq 1$, the only solution is

$$\bar{x}^{\delta}(\mathbf{1}) = (1 - \delta) v(\mathbf{1}) = 0 \tag{244}$$

using the normalization $v(\mathbf{1}) = 0$.

If instead $\langle \mathbf{1}, \alpha \rangle = 1$, the equation is reduced to an identity that always holds regardless of the choice of x. Thus, the largest possible solution is

$$\bar{x}^{\delta}(\mathbf{1}) = v(\mathbf{1}) = 0 \tag{245}$$

So, in either case, the initial condition is $\bar{x}^{\delta}(\mathbf{1}) = 0$. Note, here $\bar{x}^{\delta}(\mathbf{1}) = 0$ holds either when the asset value is zero or if all agents don't have the full stake of the asset, which in case there's some equity that is either held by the agents outside the game or the principal herself and the principal can create punishment to by allocating assets to this particular guy. Otherwise, there's no feasible threat at $\mathbf{1}$.

Now, I show the iterative relation. When $\bar{x}^{\delta}(h + e_i)$ is known, I can write the value function at h as

$$\begin{aligned}
J^*(h) =& v(h) - \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i) \right) && (246)\\
& - \langle h, \alpha \rangle \left(v(h) - \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i) \right) \right) && (247)\\
=& v(h) - (1 - \langle h, \alpha \rangle) \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i) \right) - \langle h, \alpha \rangle v(h) && (248)
\end{aligned}$$

Then in order to find $\bar{x}^{\delta}(h)$, we solve the equation

$$\langle h, \alpha\rangle(v(h) - x) + x = v(h) - \delta J^*(h). \tag{249}$$

Substitute in $J^*(h)$ and we write the RHS as

$$v(h) - \delta J^*(h) = (1-\delta)v(h) + \delta\langle h, \alpha\rangle v(h) + \delta(1 - \langle h, \alpha\rangle) \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right). \tag{250}$$

Rearranging terms yields

$$(1-\langle h, \alpha\rangle)x = (1-\delta)(1-\langle h, \alpha\rangle)v(h) + \delta(1-\langle h, \alpha\rangle) \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right) \tag{251}$$

Whenever $\langle h, \alpha\rangle \neq 1$, which is true for all $h \neq \mathbf{1}$, there's a unique solution

$$\bar{x}^{\delta}(h) = (1-\delta)v(h) + \delta \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right) \tag{252}$$

Thus, I proved the lemma. □

Proposition 9. *For equity contracts, the maximum possible punishment on action profile h takes the following alternating multi-linear form*

$$\bar{x}(h) = (1-\delta)v(h) + \sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)| - k)!} \sum_{\sigma \in \Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)}\right) \tag{41}$$

where $\xi(h) = \{i : h_i = 0\}$ is the set of tendering agents and $\Sigma(\xi(h))$ is the set of all the permutations on $\xi(h)$. The highest payoff the principal can credibly obtain at $\mathbf{0}$ is

$$J(\mathbf{0}) = v(\mathbf{0}) + \sum_{k=1}^{N} \frac{(-\delta)^{k}}{(N-k)!} \sum_{\sigma \in \Sigma(\mathcal{N})} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(\sum_{s=1}^{k} e_{\sigma(s)}\right). \tag{42}$$

Proof. To prove this result, we need to show that i) the initial condition is satisfied and that ii) the equation (40) is satisfied when we plug the equation (41) in. The initial condition is very easy to verify: at $\mathbf{1}$, there are no tendering agents, so the RHS is non-existent.

Before plugging, we want to state several basic facts about the set of permutations. By definition, $\xi(h) = \xi(h+e_i) \cup \{i\}$, and thus $|\xi(h+e_i)| = |\xi(h)| - 1$. Moreover, consider two sets of permutations $\Sigma(\xi(h+e_i))$ and $\Sigma(\xi(h))$. It's easy to see that $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))|$ but conditional on the kth element being i, the subset $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ is isomorphic to $\Sigma(\xi(h+e_i))$. Moreover the disjoint union of them is isomorphic to $\Sigma(h)$. That is,

$$\coprod_{i\in\xi(h)} \Sigma(\xi(h+e_i)) \cong \coprod_{i\in\xi(h)} \{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\} \cong \Sigma(\xi(h)) \quad \forall\, k = 1, \ldots, |\xi(h)| \tag{253}$$

Now we plug the solution in Equation (41) into the recursive equation (40), the right hand side of the equation (40) is $(1-\delta)v(h) + \delta \sum_{i\in\xi(h)} \alpha_i (v(h+e_i) - \bar{x}(h+e_i))$. The second term is

$$\begin{aligned}
&\delta \sum_{i\in\xi(h)} \alpha_i (v(h+e_i) - \bar{x}(h+e_i)) \\
=&\delta \sum_{i\in\xi(h)} \alpha_i \left(\delta v(h+e_i) - \sum_{k=1}^{|\xi(h+e_i)|} \frac{(-\delta)^{k+1}}{(|\xi(h+e_i)| - k)!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v\left(h + e_i + \sum_{s=1}^{k} e_{\sigma(s)} \right) \right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h + e_{\sigma(1)}\right) \\
&+ \sum_{i\in\xi(h)} \sum_{k'=2}^{|\xi(h)|} \frac{(-\delta)^{k'+1}}{(|\xi(h)| - k')!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\alpha_i \prod_{s=1}^{k'-1} \alpha_{\sigma(s)} \right) v\left(h + e_i + \sum_{s=1}^{k'-1} e_{\sigma(s)} \right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h + e_{\sigma(1)}\right) \\
&+ \sum_{k=2}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)| - k)!} \sum_{i\in\xi(h)} \sum_{\sigma\in\Sigma(\xi(h)):\sigma(k)=i} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)} \right) \\
=&\sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)| - k)!} \sum_{\sigma\in\Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)} \right)
\end{aligned}$$

where the first quality is the result with $\bar{x}(h+e_i)$ directly plugged in; the second equality is the separation of the first term and the rest, with the replacement $k' = k+1$. Note, we have the $\frac{1}{(|\xi(h)|-1)!}$ term because $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))| = |\xi(h)| \cdot (\xi(h)-1)!$ so the term is used to offset the repetitive counting. In the third equality, we switch the indicator to k, and change the order of the summation using the isomorphism between $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ and $\Sigma(\xi(h+e_i))$. The last line combines the two parts, using the isomorphism in equation (253).

This proves that the solution (41) solves the recursive equation (40).

At $h = \mathbf{0}$, the maximum punishment is also $\bar{x}^\delta(\mathbf{0}) = v(\mathbf{0}) - \delta J(\mathbf{0})$, which gives us that

$$J(\mathbf{0}) = \delta^{-1}(v(\mathbf{0}) - \bar{x}^\delta(\mathbf{0})) = v(\mathbf{0}) + \sum_{k=1}^{N} \frac{(-\delta)^k}{(N-k)!} \sum_{\sigma \in \Sigma(\mathcal{N})} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)} \right) v \left(\sum_{s=1}^{k} e_{\sigma(s)} \right) \tag{254}$$

where I have used $\xi(\mathbf{0}) = \mathcal{N}$ and $|\xi(\mathbf{0})| = N$.

This proves the value function at $J(\mathbf{0})$ is the one given in the lemma.

□

D.1 δ-credible contracts with debts

Now, suppose the existing securities are debts. Each agent A_i holds a debt contract with face value D_i. For simplicity, I use the vector $D = \{D_i\}_i$ to denote the profile of existing securities. Given a profile h, the total outstanding debt (not including the potentially newly issued) is given by the inner product $D \cdot h$. Applying the general formulation of the recursive relation of the maximum credible punishment, we can write it for the debt case as follows.

Lemma 9. *For debt contracts $D = \{D_i\}_i$, the maximum possible punishment on the profile $h \neq \mathbf{1}$ is given by the recursive relation*

$$\bar{x}^\delta(h) = \begin{cases} v(h) & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ (1-\delta)(v(h) - D \cdot h) + \delta \underline{x}(h) & \text{otherwise} \end{cases} \tag{255}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$ where

$$\underline{x}(h) := \sum_{i \in \xi(h)} \min \left\{ \frac{D_i}{D \cdot (h+e_i)} \left[v(h+e_i) - \bar{x}^\delta(h+e_i) \right], D_i \right\} \tag{256}$$

is the sum of the minimal payments to hold in the tendering agents.

Proof. When $h = 1$, there is no tendering agents so by definition $\bar{x}^{\delta}(1) = 0$.

Consider any $h \neq 1$, the principal's value function at h is given by

$$J(h) = v(h) - \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\} \tag{257}$$

$$- \min\left\{v(h) - \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\}, D \cdot h\right\} \tag{258}$$

$$= v(h) - \min\left\{v(h), D \cdot h + \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\}\right\} \tag{259}$$

$$= \max\left\{0, v(h) - D \cdot h - \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\}\right\} \tag{260}$$

The corresponding equation for the maximum punishment is

$$x + \min\{v(h) - x, D \cdot h\} \leq v(h) - \delta J(h) = (1 - \delta)v(h) \tag{261}$$

$$+ \delta \min\left\{v(h), D \cdot h + \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\}\right\} \tag{262}$$

When $D \cdot h + \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h+e_i)} \left[v(h + e_i) - \bar{x}^{\delta}(h + e_i)\right], D_i\right\} \geq v(h)$ or $\delta = 0$, the inequality always holds because

- The LHS is at most $v(h)$: $\min\{v(h) - x, D \cdot h\} + x = \min\{v(h), D \cdot h + x\} \leq v(h)$
- The RHS is simply $v(h)$

so the largest punishment is $\bar{x}^{\delta} = v(h)$;

Otherwise, there's an interior solution as the LHS varies from $D \cdot h$ to $v(h)$ while the RHS is a constant in-between:

- It is strictly smaller than $v(h)$ because $v(h) - \delta J(h) < v(h)$ by the positivity of $J(h)$ and δ.

- It is larger than $D \cdot h$ because both $v(h)$ and

$$\min\left\{v(h), D \cdot h + \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)}\left[v(h + e_i) - \bar{x}^\delta(h + e_i)\right], D_i\right\}\right\}$$

are larger than $D \cdot h$.

The interior solution is given by solving $x + D \cdot h = RHS$ when the RHS is strictly smaller than $v(h)$, which yields

$$\bar{x}^\delta(h) = (1 - \delta)(v(h) - D \cdot h) + \delta \sum_{i \in \xi(h)} \min\left\{\frac{D_i}{D \cdot (h + e_i)}\left[v(h + e_i) - \bar{x}^\delta(h + e_i)\right], D_i\right\} \tag{263}$$

Thus we complete the proof. □

This lemma says that when the asset value is low enough, the principal can credibly punish the holdouts by giving all the asset value to the tendering agents since she will not be paid anyway. But, if the asset value is high, then such punishment hurts the principal, and there's a limit on the punishment, which consists of two parts: i) the value exceeding the full payment to the holdouts $v(h) - D \cdot h$ lost due to discounting; ii) the discounted payment to each tendering agents if he were to hold out. The second case is similar to the situation where outstanding securities are equities, but the complication comes from the fact that there are two cases on each profile h depending on the magnitude of the asset value versus the total outstanding debt of the holdouts with an additional recursive component for payment to tendering agents. The formulation that the holdouts' debt is subtracted from the asset value seems to suggest that all holdouts would be paid in full in the second case, and it is indeed true:

Lemma 10. *Each holdout $i \notin \xi(h)$ is either paid nothing or in full at any $h \neq \mathbb{1}$. More specifically, the value that can be distributed to the holdouts and the principal herself is*

$$v(h) - \bar{x}^\delta(h) \begin{cases} = 0 & \text{if } \underline{x}(h) \geq v(h) - D \cdot h \text{ or } \delta = 0 \\ > D \cdot h & \text{otherwise,} \end{cases} \tag{264}$$

where $\underline{x}(h)$ is defined in Lemma 9.

Proof. The proof is obtained by simply calculating $v(h) - \bar{x}^\delta(h)$ using the recursive equation in Lemma 9

$$v(h)-\bar{x}^\delta(h) = \begin{cases} 0 \quad \text{if } \sum_{i\in\xi(h)} \min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\} \geq v(h) - D\cdot h \text{ or } \delta = 0 \\ \delta\left(v(h) - D\cdot h - \sum_{i\in\xi(h)} \min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\}\right) + D\cdot h \end{cases} \tag{265}$$

and the non-negativity of the first term in the second case. Since $v(h) - D\cdot h - \sum_{i\in\xi(h)} \min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\}$ is strictly positive, the second case is always positive. □

Thus, either no holdouts are paid anything, or all of them are paid in full. This allows us to describe the recursive relation using an indicator variable:

Lemma 11. *Let $\eta = \{\eta(h)\}_h \in \{0,1\}^{2^N}$ be a vector of indicator functions such that $\eta(h) = 1$ if and only if $\delta > 0$ and $v(h) - \bar{x}^\delta(h) \geq D\cdot h$. Then the recursive relation in Lemma 9 can be described as*

$$\eta(h) = \begin{cases} 0 & \text{if } \delta = 0 \\ \mathbb{1}_{\{v(h)\geq D\cdot h\}} & \text{if } \delta \neq 0 \text{ and } h = \mathbf{1} \\ \mathbb{1}_{\left\{v(h) > D\cdot h + \sum_{i\in\xi(h)} D_i\eta(h+e_i)\right\}} & \text{otherwise} \end{cases}. \tag{266}$$

Proof. The case when $\delta = 0$ is trivial since $\eta(h) = 0$ for all h by definition.

At $h = \mathbf{1}$, since no punishment can be imposed $\bar{x}^\delta(\mathbf{1}) = 0$, $\eta(\mathbf{1}) = 1$ if and only if $v(h) \geq D\cdot h$ by definition.

At any $h \neq \mathbf{1}$, by Lemma 10 the condition $v(h) - \bar{x}^\delta(h) \geq D\cdot h$ is satisfied if and only if $\sum_{i\in\xi(h)} \min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\} < v(h) - D\cdot h$. Also, whenever $v(h+e_i) - \bar{x}^\delta(h+e_i) > 0$, we have $v(h+e_i) - \bar{x}^\delta \geq D\cdot(h+e_i)$ by Lemma 10 and hence $\eta(h+e_i) = 1$. Therefore, whenever this is the case, $\min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\} = D_i$ as $\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right] \geq \frac{D_i}{D\cdot(h+e_i)} D\cdot(h+e_i) = D_i$. So the condition $\sum_{i\in\xi(h)} \min\left\{\frac{D_i}{D\cdot(h+e_i)}\left[v(h+e_i)-\bar{x}^\delta(h+e_i)\right], D_i\right\} > v(h) - D\cdot h$ can be rewritten as $\sum_{i\in\xi(h)} D_i\eta(h+e_i) > v(h) - D\cdot h$. And we obtain the recursive relation in the lemma. □

Whenever $\eta(h) = 1$, the asset value is more than enough to pay off every creditor, tendering or not; And the principal gets paid something instead of nothing, so she cannot credibly punish the holdouts by diverting asset value to the tendering creditors since any diversion hurts herself. Otherwise, when $\eta(h) = 0$, the principal doesn't get paid anything, and the punishment is credible.

An immediate implication of this result is that the level of commitment δ is almost irrelevant to the success of the exchange offer

Proposition 22 (Almost Irrelevance and Discontinuity of Commitment)**.** *The vector $\boldsymbol{\eta}$ that solves the recursive relation in Lemma 11 is independent of δ for any $\delta > 0$ and $\eta(h) = 0$ for all h if $\delta = 0$. Given the solution $\boldsymbol{\eta}$, the value of the principal is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} D_i \eta(e_i) \tag{267}$$

Proof. Since the recursion in Lemma 11 doesn't depend on δ for any $\delta > 0$, neither would the solution. When $\delta = 0$, by definition $\eta(h) = 0$ by definition.

Given $\boldsymbol{\eta}$, the holdout A_i would be paid in full if $\eta(e_i) = 1$ and nothing otherwise. So the principal has to pay A_i exactly what he would get if he deviates, i.e., D_i. And this gives the principal a value of $v(\mathbf{0}) - \sum_{i=1}^{N} D_i \eta(e_i)$ $\square$

This result, together with the lemma above, reveals a discontinuity at $\delta = 0$: Almost full commitment is very different from full commitment, but the level of the commitment between 0 and 1 affects neither the resolution of the holdout problem nor the principal's value.

This discontinuity in the value function is a result of the discontinuity of the maximum punishment due to the flat region in the payment function illustrated in Figure 5. The RHS of the equation for the maximum punishment is $v(h) - \delta J(h)$ is plotted using the dashed line. The LHS, the total payment to all creditors, as a function of the punishment when the holdout has a debt D is $x + \min\{v - x, D\}$ and is displayed using a solid black line. When the discount rate is $\delta > 0$, the dashed line (in red) is always below the flat region of the payment function, so the maximum punishment is the intersection point $v(h) - D - \delta J(h)$. But when $\delta = 0$, the dashed line (in blue) overlaps with the flat region of the payment function, and the maximum punishment jumps from $v(h) - D$ to v.

Moreover, it is also different from strong credibility. Under strong credibility, the principal needs to pay D_i on path to A_i if $v(e_i) > D_i$ but under $\delta-$credibility, she does

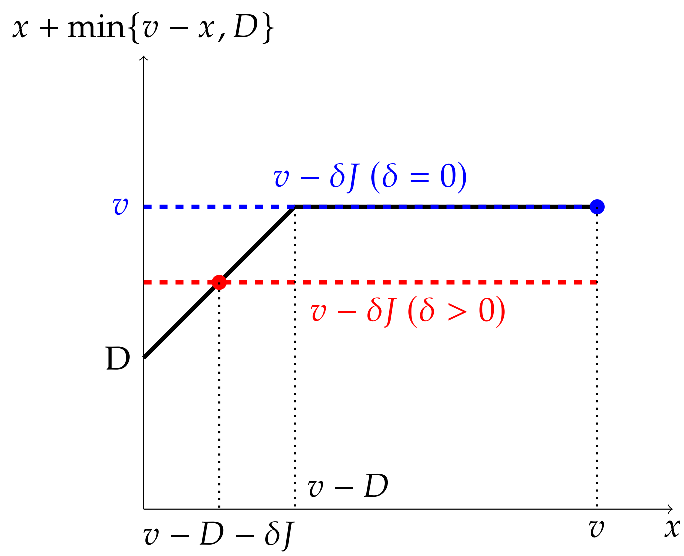

Figure 5: Discontinuity of Punishment

so if $v(e_i) > D_i + \sum_{j \neq i} D_j \eta(e_i + e_j)$. I illustrate this point using the three-agent example below.

A deeper characterization of the relationship between strong δ-credibility and δ-credibility is provided in Section 6

Numerical Example: Three-Agent Case with Debts The principal has 3 creditors A_i for $i \in \{1, 2, 3\}$ and A_i has outstanding debt $D_i = 10i$. And the value of the asset depends on the number of holdouts $v(h) = 40 + 5h^\top \mathbf{1}$. I.e., the value is 55 (resp. $50, 45, 40$) when 0 (resp. $1, 2, 3$) agents hold out.

Under full commitment, the principal can extract full surplus as per Proposition 2, so the principal's value is $J(\mathbf{0}) = v(\mathbf{0}) = 55$.

Under strong δ-credibility for $\delta > 0$, since $v(e_i) > D_i$ for all i, the principal has to repay everyone in full. His value is $55 - 10 - 20 - 30 = -5$. So, he might not even initiate the restructuring. He will only do so if $v(\mathbf{0}) > 60$.

Under δ-credibility for $\delta > 0$, I calculate the η function using backward induction. $\eta(\mathbf{1}) = 0$ as $v(\mathbf{1}) = 40 < D \cdot \mathbf{1} = 60$. At $h = e_1 + e_3$ (resp. $h = e_2 + e_3$), the asset value 45 is larger than the total outstanding debt 40 (resp. 30), so $\eta(e_1 + e_3) = \eta(e_2 + e_3) = 1$. Now I calculate $\eta(e_3)$:

$$v(e_3) = 50 < D_3 + \sum_{j=1,2} D_j \eta(e_3 + e_j) = D_1 + D_2 + D_3 = 60. \tag{268}$$

So, by definition, $\eta(e_3) = 0$. Similarly one can get $\eta(e_2) = 0$ and $\eta(e_1) = 1$. Thus, the principal's value is $v(\mathbf{0}) - D_1 = 45$.

E Proofs for Section 5 (Property Rights)

Proposition 10. *With full commitment, greater property rights protection exacerbates the holdout problem. More specifically, the principal's value at* $\mathbf{0}$ *is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \pi_i \tag{46}$$

which is always decreasing in π_i *for all* i.

Proof. When A_i deviates, the principal could promise to give the entire asset to other tendering agents, and the holdout A_i still enjoys a value of π_i by retaining his property. Thus, to convince A_i to tender, he must be paid π_i on path. Therefore, the value at $\mathbf{0}$ is the asset value minus the sum of property values. □

Proposition 11. *There exists a set of initial contracts such that a locally small increase in property rights protection facilitates restructuring. In particular, let* $\hat{v}_1 = \hat{v}_3 = 1, \hat{v}_2 = 98/100,$ $\pi_1 = \pi_2 = 1/100$ *and* $\pi_3 = 99/100,$ $\alpha_2 = 7/10, \alpha_1 = \alpha_3 = 1/10, \beta_1 = \beta_2 = 1/10, \beta_3 = 7/10.$ *Let* $v(\cdot)$ *be such that* $v(\mathbf{1}) = 0, v(\mathbf{0}) = 3, v(e_i) = 2, v(\mathbf{1} - e_i) = 1$ *for all* i. *The principal's value function* $J(\mathbf{0})$ *is increasing in* π_1 *at the parameters specified above.*

Proof. Since the asset value $v(\mathbf{1}) = 0$, when all three agents hold out, they get nothing more than their property value, so in order to convince one of them, say A_i, to tender, the principal only needs to pay him π_i, that is,

$$X(\mathbf{1} - e_i) = \pi_i \tag{269}$$

and the principal obtains a value

$$J(\mathbf{1}-e_i) = v(\mathbf{1}-e_i) - \pi_i - \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - \pi_i, \mathbf{1}-e_i) \tag{270}$$

Solving for the maximum x such that

$$x + \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - x, \mathbf{1}-e_i) \leq J(\mathbf{1}-e_i) \tag{271}$$

yields

$$\bar{x}(\mathbf{1}-e_i) = X(\mathbf{1}-e_i) = \pi_i \tag{272}$$

given the parametric assumption on the slopes of R_j^O.

Now consider the holdout profile e_i. The principal obtains a value

$$J(e_i) = v(e_i) - X(e_i) - R_i^O(v(e_i) - X(e_i), e_i) \tag{273}$$

where

$$X(e_i) = \sum_{j\neq i} \left[R_j^O(v(e_i+e_j) - \pi_k, e_i+e_j) + \pi_j\right] \quad k \neq i, j. \tag{274}$$

Again, solving for the maximum x such $x + R_i^O(v(e_i) - x, e_i) \leq v(e_i) - J(e_i)$ yields

$$\bar{x}(e_i) = X(e_i) = \sum_{j\neq i} \left[R_j^O(v(e_i+e_j) - \pi_k, e_i+e_j) + \pi_j\right] \quad k \neq i, j. \tag{275}$$

Taking derivatives with respect to π_j gives

$$\frac{\mathrm{d}\bar{x}(e_i)}{\mathrm{d}\pi_j} = 1 - \frac{\partial}{\partial v} R_k^O(v(e_i+e_k) - \pi_j, e_i+e_k) \tag{276}$$

The principal's value at $h = \mathbf{0}$ is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{3} \left[R_i^O(v(e_i) - X(e_i), e_i) + \pi_i\right] \tag{277}$$

Taking derivatives with respect to π_i gives

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} = -1 + \sum_{j \neq i} \frac{\partial}{\partial v} R_j^O(v(e_j) - \bar{x}(e_j), e_j) \frac{\mathrm{d}\bar{x}(e_j)}{\mathrm{d}\pi_i} \tag{278}$$

$$= -1 + \sum_{j \neq i} \frac{\partial}{\partial v} R_j^O(v(e_j) - \bar{x}(e_j), e_j) \left[1 - \frac{\partial}{\partial v} R_k^O(v(e_j + e_k) - \pi_i, e_j + e_k)\right] \tag{279}$$

In particular, given the parameters in the proposition, we have

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_1} = -1 + \frac{\partial}{\partial v} R_2^O(v(e_2) - \bar{x}(e_2), e_2) \left[1 - \frac{\partial}{\partial v} R_3^O(v(e_2 + e_3) - \pi_1, e_2 + e_3)\right] \tag{280}$$

$$+ \frac{\partial}{\partial v} R_3^O(v(e_3) - \bar{x}(e_3), e_3) \left[1 - \frac{\partial}{\partial v} R_2^O(v(e_2 + e_3) - \pi_1, e_2 + e_3)\right] \tag{281}$$

$$= -1 + \alpha_2(1 - \alpha_3) + \beta_3(1 - \beta_2) = \frac{13}{50} > 0 \tag{282}$$

as $\bar{x}(e_2) = 1.1$ and $\bar{x}(e_3) = 0.806$. □

Proposition 12 (Property rights hinder equity restructruring)**.** *For any equity contracts $\{\alpha_i\}_i$, the prinicpal's value $J(\mathbf{0})$ under δ-credibility for any $\delta \in (0, 1]$ is decreasing in π_i for all $i \in \mathcal{N}$.*

Proof. We first show that the maximum punishment satisfies the recursion

$$\bar{x}(h) = \sum_{i \in \xi(h)} \left[\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i\right] \tag{283}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$. This is because given $\bar{x}(h + e_i)$, at h, each tendering agent A_i could have otherwise obtained a value of $\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i$ were he to hold out. Thus, the value function of the principal is

$$J(h) = v(h) - X(h) - \langle h, \alpha \rangle (v(h) - X(h)) \tag{284}$$

where $X(h) = \sum_{i \in \xi(h)} \left[\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i\right]$. And solving for the maximum x such that

$$x + \langle h, \alpha \rangle (v(h) - x) \leq v(h) - J(h) \tag{285}$$

yields

$$(1 - \langle h, \alpha \rangle)x \leq \langle h, \alpha \rangle X(e_i) \tag{286}$$

which gives

$$\bar{x}(h) = \begin{cases} v(h) & \text{if } h = \mathbf{1} \\ X(h) & \text{otherwise} \end{cases} \tag{287}$$

From the recursive relation of $\bar{x}$, we obtain

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = \mathbb{1}_{\{i \in \xi(h)\}} - \sum_{j \in \xi(h)} \alpha_j \frac{\mathrm{d}\bar{x}(h + e_j)}{\mathrm{d}\pi_i} \tag{288}$$

with the initial condition

$$\frac{\mathrm{d}\bar{x}(\mathbf{1})}{\mathrm{d}\pi_i} = 0 \tag{289}$$

since $\bar{x}(\mathbf{1}) = 0$.

To solve $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i}$, we establish two lemmata:

Lemma 12. *For any h and any i such that $i \notin \xi(h)$,*

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 0 \tag{290}$$

Proof. I prove the lemma by induction. For any h such that $|\xi(h)| = 0$, i.e., $h = \mathbf{1}$, we have the obvious case $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 0$.

Now I show that if the statement is true for any h such that $i \notin \xi(h)$ and $|\xi(h)| = n$, it is also true for any h such that $i \notin \xi(h)$ and $|\xi(h)| = n + 1$. First notice that if $i \notin \xi(h)$, then for any $j \in \xi(h)$, $j \notin \xi(h + e_j)$. And $|\xi(h + e_j)| = |\xi(h)| - 1$. Then, we have

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = -\sum_{j \in \xi(h)} \alpha_j \frac{\mathrm{d}\bar{x}(h + e_j)}{\mathrm{d}\pi_i} = 0 \tag{291}$$

where the first equality holds because $i \notin \xi(h)$ and the second holds by induction hypothesis. □

Lemma 13. *For any h and any i such that $i \in \xi(h)$,*

$$0 < \frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} \leq 1 \tag{292}$$

Proof. I prove the lemma by induction. For any h such that $|\xi(h)| = 1$, i.e., $h = \mathbf{1} - e_i$, we have the obvious case $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 1$.

Now I show that if the statement is true for any h such that $i \in \xi(h)$ and $|\xi(h)| = n$, it is also true for any h such that $i \in \xi(h)$ and $|\xi(h)| = n + 1$. First notice that if $i \in \xi(h)$, then for any $j \in \xi(h) : j \neq i$, $j \in \xi(h + e_j)$. And $|\xi(h + e_j)| = |\xi(h)| - 1$. Thus, The recursive relation could be written as

$$\begin{aligned} \frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} =& 1 - \alpha_i \frac{\mathrm{d}\bar{x}(h + e_i)}{\mathrm{d}\pi_i} - \sum_{j \in \xi(h): j \neq i} \alpha_j \frac{\mathrm{d}\bar{x}(h + e_j)}{\mathrm{d}\pi_i} && (293) \\ =& 1 - \sum_{j \in \xi(h): j \neq i} \alpha_j \frac{\mathrm{d}\bar{x}(h + e_j)}{\mathrm{d}\pi_i} && (294) \end{aligned}$$

where the second equality holds because $i \notin \xi(h + e_i)$ so the middle term is zero. Since by induction hypothesis, each $\frac{\mathrm{d}\bar{x}(h+e_j)}{\mathrm{d}\pi_i}$ is in $(0, 1]$, we have

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} < 1 - \sum_{j \in \xi(h): j \neq i} \alpha_j \times 0 = 1 \tag{295}$$

and

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} \geq 1 - \sum_{j \in \xi(h): j \neq i} \alpha_j \times 1 > 0. \tag{296}$$

Thus, it holds for all h such that $i \in \xi(h)$. □

Using

$$\frac{\mathrm{d}J(h)}{\mathrm{d}\pi_i} = -(1 - \langle h, \alpha \rangle) \frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} \tag{297}$$

I obtain

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} = -\frac{\mathrm{d}\bar{x}(\mathbf{0})}{\mathrm{d}\pi_i} \in [-1, 0) \tag{298}$$

Thus, a higher property rights protection always undermines restructuring when the initial set of contracts are equities.

□

Example 5.1 (Property rights hinder equity restructuring: 3-agent example). *With limited commitment, the value function of the principal at* $\mathbf{0}$ *with equities outstanding is decreasing in each* π_i*,*

$$\frac{\partial}{\partial \pi_i} J(\mathbf{0}) = -\left(1 - \sum_{j \neq i} \alpha_i (1 - \alpha_k)\right) < 0 \textit{ for } k \neq j, i \quad \forall i. \tag{48}$$

Proof. We first show that the maximum punishment satisfies the recursion

$$\bar{x}(h) = \sum_{i \in \xi(h)} \left[\alpha_i (v(h + e_i) - \bar{x}(h + e_i)) + \pi_i\right] \tag{299}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$. This is because given $\bar{x}(h + e_i)$, at h, each tendering agent A_i could have otherwise obtained a value of $\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i$ were he to hold out. Thus the value function of the principal is

$$J(h) = v(h) - X(h) - \langle h, \alpha \rangle (v(h) - X(h)) \tag{300}$$

where $X(h) = \sum_{i \in \xi(h)} [\alpha_i (v(h + e_i) - \bar{x}(h + e_i)) + \pi_i]$. And solving for the maximum x such that

$$x + \langle h, \alpha \rangle (v(h) - x) \leq v(h) - J(h) \tag{301}$$

yields

$$(1 - \langle h, \alpha \rangle) x \leq \langle h, \alpha \rangle X(e_i) \tag{302}$$

which gives

$$\bar{x}(h) = \begin{cases} v(h) & \text{if } h = \mathbf{1} \\ X(h) & \text{otherwise} \end{cases} \tag{303}$$

Using this recursion with the parameters specified, we obtain

$$\bar{x}(\mathbf{1} - e_i) = \pi_i \quad \forall i \tag{304}$$

$$\bar{x}(e_i) = \sum_{j \neq i} \left[\alpha_j (v(e_i + e_j) - \pi_k) + \pi_j\right] \quad k \neq i, j \quad \forall i \tag{305}$$

and the value function of the principal is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{3} [\alpha_i(v(e_i) - \bar{x}(e_i)) + \pi_i] \tag{306}$$

$$= v(\mathbf{0}) - \sum_{i=1}^{3} \alpha_i v(e_i) - \sum_{i=1}^{3} \pi_i + \sum_{i=1}^{3} \alpha_i \sum_{j \neq i} \left[\alpha_j(v(e_i + e_j) - \pi_k) + \pi_j\right] \tag{307}$$

$$= v(\mathbf{0}) - \sum_{i=1}^{3} \alpha_i v(e_i) + \sum_{i=1}^{3} \sum_{j \neq i} \alpha_i \alpha_j v(e_i + e_j) - \sum_{i=1}^{3} \left(1 - \sum_{j \neq i} \alpha_i (1 - \alpha_k)\right) \pi_i \tag{308}$$

Taking partial derivatives yields the expression in the proposition.

Without loss of generality, look at the coefficient of π_i. Even if I ignore the constraint that $\langle h, \alpha \rangle = 1$, the coefficient $1 - \alpha_2 - \alpha_3 + 2\alpha_2\alpha_3$ is minimized at $\alpha_2 = \alpha_3 = 1/2$ with a minimum value of $1/2$. Thus, all coefficients of π_i are positive. □

Proposition 13 (Property rights generically hinder debt restructruring)**.** *For any debts contracts $\{D_i\}_i$, the prinicpal's value $J(\mathbf{0})$ under δ-credibility for any $\delta \in (0, 1]$ is generically locally decreasing in π_i for all i. That is,*

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} < 0 \tag{49}$$

at any differentiable points.

Proof. Consider the deviation profile e_i, let $X(e_i)$ be the total payments to the tendering creditors according to one of the optimal δ-credible contracts, which could be a function of $\{\pi_i\}_i$

Then, the principal's value at e_i is

$$J(e_i) = v(e_i) - X(e_i) - \min\{D_i, v(e_i) - X(e_i)\} \tag{309}$$

and the maximum punishment is the largest x such that

$$x + \min\{D_i, v(e_i) - x\} \leq v(e_i) - J(e_i) \tag{310}$$

which yields

$$\bar{x}(e_i) = \begin{cases} v(e_i) & v(e_i) - X(e_i) \leq D_i \\ (1-\delta)(v(e_i) - D_i) + \delta X(e_i) & v(e_i) - X(e_i) \geq D_i \end{cases} \tag{311}$$

Then, the principal's value is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} [\min\{D_i, v(e_i) - \bar{x}(e_i)\} + \pi_i] \tag{312}$$

$$= v(\mathbf{0}) - \sum_{i=1}^{N} \left[D_i \mathbb{1}_{\{v(e_i) \geq X(e_i) + D_i\}} + \pi_i\right] \tag{313}$$

because whenever $v(e_i) - X(e_i) \leq D_i$, $\bar{x}(e_i) = v(e_i)$ and thus $\min\{D_i, v(e_i) - \bar{x}(e_i)\} = 0$; In contrast, when $v(e_i) - X(e_i) > D_i$

$$v(e_i) - \bar{x}(e_i) = \delta(v(e_i) - X(e_i) - D_i) + D_i > D_i \tag{314}$$

so $\min\{D_i, v(e_i) - \bar{x}(e_i)\} = D_i$. In either case, the payment to each tendering agent is independent of the renegotiation off-path.

Thus

$$\frac{\partial J(\mathbf{0})}{\partial \pi_i} = -1 \quad \forall i \tag{315}$$

which implies a locally small increase in property rights protection always hinders restructuring. □

Proposition 14. *With limited commitment, the principal's value in the 2-creditor example is*

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{2} \left[D_i \mathbb{1}_{\{v(e_i) \geq \pi_j + D_i\}} + \pi_i\right] \tag{50}$$

Given the parameters above, the principal's value increases when the property rights of A_j increases from any value $\pi_j \in (1/2, 1)$ to any $\pi_j + \Delta\pi_j \in (1, 3/2)$.

Proof. At every e_i, the principal only needs to compensate A_j at most π_j for him to tender so the principal's value is

$$J(e_i) = v(e_i) - \pi_j - \min\{v(e_i) - \pi_j, D \cdot e_i\}. \tag{316}$$

The maximum credible punishment is given by

$$x + \min\{v(e_i) - x, D \cdot e_i\} \leq v(e_i) - J(e_i) \tag{317}$$

which gives

$$\bar{x}(e_i) = \begin{cases} v(e_i) & \text{if } v(e_i) \leq \pi_j + D \cdot e_i \\ \pi_j & \text{otherwise.} \end{cases} \tag{318}$$

The principal's value at $\mathbf{0}$ is then

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{2} [\min\{D_i, v(e_i) - \bar{x}(e_i)\} + \pi_i] \tag{319}$$

$$= v(\mathbf{0}) - \sum_{i=1}^{2} \left[D_i \mathbb{1}_{\{v(e_i) \geq \pi_j + D_i\}} + \pi_i\right] \tag{320}$$

When $\pi_j \in (1/2, 1)$, given that $D_i = 1$ and $v(e_i) = 2$, we have $v(e_i) > \pi_j + D_i$; In contrast, when $\pi_j \in (1, 3/2)$, we have $v(e_i) \leq \pi_j + D_i$, so the change in the principal's value is

$$D_i - \Delta\pi_j > 0 \tag{321}$$

since $\Delta\pi_j < 3/2 - 1/2 = 1$. □

F Proofs for Section 6 (Unifiying Notions of Credibility)

Lemma 4. *The even subsequence of $\{C_k^\delta(h)\}_k$ is weakly decreasing and the odd subsequence is weakly increasing. That is,*

$$C_{2k}^\delta(h) \subset C_{2k-2}^\delta(h) \text{ and } C_{2k-1}^\delta(h) \subset C_{2k+1}^\delta(h) \quad \forall h\, \forall k = 1, 2, 3, \cdots \tag{52}$$

Proof. For simplicity let $J(\hat{h}; R) := v(\hat{h}) - \sum_{i=1}^N u_i(\hat{h}_i|\hat{h}_{-i}, R)$.

I prove this lemma by induction. First, I prove that it is true for $k = 1$. By definition

$C_2^\delta(h) \subset C_0^\delta(h) = \mathcal{I}(h)$. For any $R \in C_1^\delta(h)$, by definition, for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_0^\delta(\hat{h}) \tag{322}$$

and since $C_2^\delta(\hat{h}) \subset C_0^\delta(\hat{h})$, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_2^\delta(\hat{h}). \tag{323}$$

Thus $R \in C_3^\delta(h)$. I proved the first step of the induction.

Now I proceed to the second step. Suppose this is true for $k \in \{1, 2, \ldots, \kappa\}$, I want to show this is true for $k = \kappa + 1$.

- I show $C_{2\kappa}^\delta(h) \subset C_{2\kappa-2}^\delta(h)$. By definition, for any $R \in C_{2\kappa}^\delta(\hat{h})$, for any $\hat{h} \in \mathcal{B}(h)$

  $$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_{2\kappa-1}^\delta(\hat{h}) \tag{324}$$

  and since $C_{2\kappa-3}^\delta(\hat{h}) \subset C_{2\kappa-1}^\delta(\hat{h})$ by the induction hypothesis, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

  $$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_{2\kappa-3}^\delta(\hat{h}). \tag{325}$$

  Thus $R \in C_{2\kappa-2}^\delta(h)$ given $R \in \mathcal{I}(h)$.

- Now I show $C_{2\kappa-1}^\delta(h) \subset C_{2\kappa+1}^\delta(h)$. By definition, for any $R \in C_{2\kappa-2}^\delta(\hat{h})$, for any $\hat{h} \in \mathcal{B}(h)$

  $$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_{2\kappa-2}^\delta(\hat{h}) \tag{326}$$

  and since $C_{2\kappa}^\delta(\hat{h}) \subset C_{2\kappa-2}^\delta(\hat{h})$ by induction hypothesis, it is also true that for any $\hat{h} \in \mathcal{B}(h)$

  $$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_{2\kappa}^\delta(\hat{h}). \tag{327}$$

  Thus $R \in C_{2\kappa+1}^\delta(h)$ given $R \in \mathcal{I}(h)$.

Therefore, we conclude that the statement is correct. □

Lemma 5. *The odd subsequence never exceeds the even subsequence. That is,*

$$C_{2k+1}^\delta(h) \subset C_{2k}^\delta(h) \quad \forall h \, \forall \, k = 1, 2, 3, \cdots \tag{54}$$

And as a corollary, $\lim_{k\to\infty} C^{\delta}_{2k+1}(h) \subset \lim_{k\to\infty} C^{\delta}_{2k}(h)$.

Proof. For simplicity let $J(\hat{h}; R) := v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R)$.

Fix an h, let

$$\kappa = \inf\{k \geq 1 : C^{\delta}_{2k+1}(h) \not\subset C^{\delta}_{2k}(h)\} \tag{328}$$

which implies both $C^{\delta}_{2\kappa+1}(h) \not\subset C^{\delta}_{2\kappa}(h)$ and, by the minimality of κ, $C^{\delta}_{2\kappa-1}(h) \subset C^{\delta}_{2\kappa-2}(h)$.

Therefore two possibilities between $C^{\delta}_{2\kappa-1}(h)$ and $C^{\delta}_{2\kappa}(h)$ and we prove by contradiction that both are not possible.

- $C^{\delta}_{2\kappa-1}(h) \subset C^{\delta}_{2\kappa}(h)$. In this case, from $C^{\delta}_{2\kappa+1}(h) \not\subset C^{\delta}_{2\kappa}(h)$ we know $\exists R \in C^{\delta}_{2\kappa+1}(h)$ but $R \notin C^{\delta}_{2\kappa}(h)$, which implies for any $\hat{h} \in \mathcal{B}(h)$,

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C^{\delta}_{2\kappa}(h) \tag{329}$$

  while $\exists \tilde{R} \in C^{\delta}_{2\kappa-1}(\hat{h})$

$$J(\hat{h}; R) < \delta J(\hat{h}; \tilde{R}). \tag{330}$$

  This implies $C^{\delta}_{2\kappa-1}(h)$ is not a subset of $C^{\delta}_{2\kappa}(h)$, contradicting the case $C^{\delta}_{2\kappa-1}(h) \subset C^{\delta}_{2\kappa}(h)$.

- $C^{\delta}_{2\kappa-1}(h) \not\subset C^{\delta}_{2\kappa}(h)$. This means $\exists R \in C^{\delta}_{2\kappa-1}(h)$ but $R \notin C^{\delta}_{2\kappa}(h)$, which implies for any $\hat{h} \in \mathcal{B}(h)$,

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C^{\delta}_{2\kappa-2}(h) \tag{331}$$

  while $\exists \tilde{R} \in C^{\delta}_{2\kappa-1}(\hat{h})$

$$J(\hat{h}; R) < \delta J(\hat{h}; \tilde{R}). \tag{332}$$

  This suggests $C^{\delta}_{2\kappa-1}(h)$ is not a subset of $C^{\delta}_{2\kappa-2}(h)$, contradicting the minimality of κ.

Thus we must have $C^{\delta}_{2k+1}(h) \subset C^{\delta}_{2k}(h) \quad \forall h \, \forall k = 1, 2, \ldots$

To prove $\lim_{k\to\infty} C^{\delta}_{2k+1}(h) \subset \lim_{k\to\infty} C^{\delta}_{2k}(h)$, it is enough to show for any k and any $k' > k$, $C^{\delta}_{2k+1}(h) \subset C^{\delta}_{2k'}(h)$. This is true, given

$$C^{\delta}_{2k+1}(h) \subset C^{\delta}_{2k'-1}(h) \subset C^{\delta}_{2k'}(h). \tag{333}$$

The first inclusion holds because the odd subsequence is non-decreasing, and the second holds by the first half of this lemma. ☐

Proposition 15. *The recursively defined $C^{\delta}(h)$ in Definition 6 satisfies*

$$\liminf_{k\to\infty} C_k^{\delta}(h) \subset C^{\delta}(h) \subset \limsup_{k\to\infty} C_k^{\delta}(h) \quad \forall h \tag{58}$$

Proof. For simplicity let $J(\hat{h}; R) := v(\hat{h}) - \sum_{i=1}^{N} u_i(\hat{h}_i|\hat{h}_{-i}, R)$.

I first show that $\liminf_{k\to\infty} C_k^{\delta}(h) \subset C^{\delta}(h)$. For any $R \in \liminf_{k\to\infty} C_k^{\delta}(h)$, there exists a k such that for all $j \geq k$, $R \in C_j^{\delta}(h)$, i.e., $R \in \bigcap_{j\geq k} C_j^{\delta}(h)$ which implies for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in C_{j-1}^{\delta}(\hat{h}) \quad \forall j \geq k \tag{334}$$

This can be equivalently written as

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h}) \quad \exists k \tag{335}$$

which, since $\bigcap_{k\geq 1}\bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h}) \subset \bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h})\, \forall k$, implies

$$J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \bigcap_{k\geq 1}\bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h}) \tag{336}$$

Since $\bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h})$ is a decreasing sequence, $\bigcap_{k\geq 1}\bigcup_{j\geq k} C_{j-1}^{\delta}(\hat{h}) = \bigcap_{k\geq 0}\bigcup_{j\geq k} C_j^{\delta}(\hat{h}) = \limsup_{k\to\infty} C_k^{\delta}(h)$ and therefore

$$\liminf_{k\to\infty} C_k^{\delta}(h) \subset \{R \in \mathcal{I}(h) : J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \limsup_{k\to\infty} C_k^{\delta}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\} \tag{337}$$

$$\subset \{R \in \mathcal{I}(h) : J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} C_k^{\delta}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\} \tag{338}$$

where the second inclusion holds because $\liminf_{k\to\infty} C_k^{\delta}(\hat{h}) \subset \limsup_{k\to\infty} C_k^{\delta}(\hat{h})$. This shows that for any $R \in \liminf_{k\to\infty} C_k^{\delta}(h)$, we have $J(\hat{h}; R) \geq \delta J(\hat{h}; \tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} C_k^{\delta}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)$, which satisfies the definition of $C^{\delta}(h)$, and therefore $R \in C^{\delta}(h)$. Thus, we proved $\liminf_{k\to\infty} C_k^{\delta}(h) \subset C^{\delta}(h)$.

Now I proceed to show that $C^{\delta}(h) \subset \limsup_{k\to\infty} C_k^{\delta}(h)$. For any $R \in$, by definition, we have

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in C^{\delta}(\hat{h}). \tag{339}$$

And since $\liminf_{k\to\infty} C_k^{\delta}(h) \subset C^{\delta}(\hat{h})$, we have

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \liminf_{k\to\infty} C_k^{\delta}(h). \tag{340}$$

Since $\bigcap_{j\geq k} C_j^{\delta}(h)$ is an increasing sequence in k, we have

$$\liminf_{k\to\infty} C_k^{\delta} = \bigcup_{k\geq 0}\bigcap_{j\geq k} C_j^{\delta}(h) = \bigcup_{k\geq 1}\bigcap_{j\geq k} C_{j-1}^{\delta}(h). \tag{341}$$

In order to show $R \in \limsup_{k\to\infty} C_k^{\delta}(h)$, by definition, we need to show $R \in \bigcup_{j\geq k} C_j^{\delta}(h)\,\forall k \geq 1$, which means for any $k \geq 1$, there is a $j(k) \geq k$ such that $R \in C_{j(k)}^{\delta}(h)$, which means for any $\hat{h} \in \mathcal{B}(h)$

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in C_{j(k)-1}^{\delta}(\hat{h})\,\forall k \geq 1. \tag{342}$$

This could be equivalently written as

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1} C_{j(k)-1}^{\delta}(\hat{h}). \tag{343}$$

Now, it remains to show that if

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1}\bigcap_{j\geq k} C_{j-1}^{\delta}(\hat{h}), \tag{344}$$

then there exists a $j(k) \geq k$ such that

$$J(\hat{h};R) \geq \delta J(\hat{h};\tilde{R}) \quad \forall \tilde{R} \in \bigcup_{k\geq 1} C_{j(k)-1}^{\delta}(\hat{h}), \tag{345}$$

which amounts to show that

$$\exists j(k) \geq k : \bigcup_{k\geq 1} C_{j(k)-1}^{\delta}(\hat{h}) \subset \bigcup_{k\geq 1}\bigcap_{j\geq k} C_{j-1}^{\delta}(\hat{h}) = \lim_{k\to\infty} C_{2k+1}^{\delta}(\hat{h}). \tag{346}$$

For any k, there's a $j(k) \geq k$ and is an even number, then by the fact that the odd subsequence is increasing (Lemma 4), we have $C^{\delta}_{j(k)-1}(\hat{h}) \subset \lim_{k\to\infty} C^{\delta}_{2k+1}(\hat{h})$. Thus, we proved $C^{\delta}(h) \subset \limsup_{k\to\infty} C^{\delta}_{k}(h)$.

$\square$

Proposition 16. *There exists a set of initial contracts R^O such that $\liminf_{k\to\infty} C^{\delta}_{k}(h) \subsetneqq \limsup_{k\to\infty} C^{\delta}_{k}(h)$ for some h.*

Proof. To prove this, we show that when the existing securities are equities and $\delta > 0$, $C^{\delta}_{2}(\mathbf{0}) = \mathcal{I}(\mathbf{0}) \subsetneqq C^{\delta}_{1}(\mathbf{0})$ and thus by induction, $C^{\delta}_{2\kappa}(\mathbf{0}) = \mathcal{I}(\mathbf{0}) \subsetneqq C^{\delta}_{2\kappa+1}(\mathbf{0}) = C^{\delta}_{1}(\mathbf{0})$.

We have shown that

$$\sup_{R\in\mathcal{I}(\mathbf{0})} J(\mathbf{0}; R) = v(\mathbf{0}) > \sup_{R\in C^{\delta}_{1}(\mathbf{0})} J(\mathbf{0}; R) = v(\mathbf{0}) - \delta \sum_{i=1}^{N} \alpha_i v(e_i) \tag{347}$$

Thus, it must be the case that $\mathcal{I}(\mathbf{0}) \subsetneqq C^{\delta}_{1}(\mathbf{0})$.

To see why $C^{\delta}_{2}(\mathbf{0}) = \mathcal{I}(\mathbf{0})$, notice we have proven that when the existing securities are equities, no contracts can do better than simply using cash (Proposition 3), and the same is true at any e_i. Formally

$$J(e_i; R) \geq \sup_{R\in C^{\delta}_{1}(e_i)} J(e_i; R) \quad \forall R \in \mathcal{I}(e_i) \tag{348}$$

Therefore, in the definiton of $C^{\delta}_{2}(\mathbf{0})$, the condition

$$R \succeq_{\delta} \tilde{R} \quad \forall \tilde{R} \in C^{\delta}_{1}(e_i) \quad \forall e_i \tag{349}$$

always holds and thus $C^{\delta}_{2}(\mathbf{0}) = \mathcal{I}(\mathbf{0})$. $\square$

References for the Appendix

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