A General Theory of Holdouts*

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Abstract

This paper presents a unified framework for the holdout problem, a pervasive phenomenon in which value creation is hindered by the incentive to free-ride on others' participation. My framework nests classic applications, such as takeovers and debt restructuring, and highlights the role of the commitment: A simple unanimity rule solves all holdout problems if the principal can commit to calling off the deal when anyone holds out. In contrast, a lack of commitment substantially alters the optimal offers depending on the dilution sensitivities of the initial contracts, which explains the absence of the unanimity rule despite its efficacy, and cross-sectional heterogeneity in offers. (E.g., senior debt used in debt restructuring but not in takeovers.) Furthermore, stronger partial commitment can backfire via renegotiation, exacerbating the holdout problem. This non-monotonicity reconciles contradictory findings on the CACs in the sovereign debt and sheds light on various policies. Lastly, the paper shows stronger investor protection could facilitate instead of hinder restructuring under limited commitment. *JEL codes:* G34, G38, C78, D86.

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[The] effectiveness [of punishment] is seen as resulting from its inevitability.

Michel Foucault, *Discipline and Punish*

1 Introduction

Holdout problems are pervasive. They occur whenever a socially beneficial transaction fails because one of the parties in the transaction free-rides on the participation of the other parties, *holding out*, hoping to obtain a larger individual payoff later on.Sovereign debt renegotiations, corporate debt restructuring, and corporate takeovers are some of the situations in which one party proposes new contracts in exchange for old ones held by dispersed agents, and holdout problems arise. The social costs of these problems can be quite sizeable. For instance, in the recent Argentinian sovereign debt restructuring, Elliot Management and five other funds held out on the Argentinian government's proposal to restructure its debt after the country defaulted on its $132 billion debt in 2001, preventing it from accessing world financial markets for fifteen years.[1,2] It has cost Argentina an estimated 30% loss in the equity value of all the Argentine firms listed in the US (Hébert and Schreger, 2017).

Theoretically, holdout problems are somewhat surprising. The reason is that a proposal requiring unanimous consent by all parties in the transaction is enough to address the holdout problem. It eliminates the incentive of any party to free ride by rendering the decision of each pivotal.[3] This easy fix to the holdout problem, unanimity, has almost never been observed except in land assembly.[4] Instead, we see different

[1]The six funds were Aurelius, Bracebridge Capital, Davidson Kempner, EM Ltd. (A hedge fund held by Kenneth Dart, who was dubbed "el enemigo número uno de Argentina"), Montreux Partners, and NML Capital, an off-shore unit of Elliott.

[2]Argentina's exclusion from the international capital market is largely attributed to the lawsuits with the holdout creditors and the legal risk associated (Schumacher et al., 2021). On the other hand, it is argued that the exclusion lasted for 15 years because Argentina "had the economic and political resources to fight distressed debt fund" and had "no urgency to access the international credit market." (Guzman, 2020, p.733)

[3]Indeed, the effectiveness of the unanimity rule has been known for decades and discussed extensively, e.g., in Kalai and Samet (1985) and Segal (1999). Grossman and Hart (1980, fn 3, p.43) argues unanimity is impractical because the holdout would anticipate a secret payment from the raider to bribe him into the offer and there might be sleeping investors. Now, the best-price rule (Exchange Act Rule 10d-10, see 17 CFR § 240.14d-10) forbids such bribes and holdouts are usually financial experts in these high stake transactions so a similar idea of making them pivotal by deploying contingency should address the issue were it the only concern.

[4]In certain jurisdictions such as Pennsylvania, Maine, and some European regions, the raider, whenever

solutions to the same holdout problem being used. For instance, the Argentinian government settled on a cash payment of $4.65 billion with the holdouts. AMC Entertainment, the world's largest movie theater chain, restructured its dispersedly-held bonds and solved the holdout problem by using high-priority debt, reducing its outstanding debt by over $500 million.[5] In contrast, Elon Musk took over Twitter with an all-cash offer. None of these solutions requires unanimity!

Why do they forgo unanimity? Why do we see different solutions to problems with the same economic structures? Why was cash used in the Twitter takeover and senior debt in the AMC debt restructuring?

Overview In this paper, I offer a general theory of holdouts that explains the observed variation in the solutions to the holdout problem and the absence of unanimity rules. This general framework nests classic models such as takeovers (Grossman and Hart, 1980), corporate debt restructuring (Gertner and Scharfstein, 1991), bond buybacks (Bulow et al., 1988) and the leverage ratchet effect (Admati et al., 2018). The model features a principal and multiple agents with contractual claims on an underlying asset.[6] There are gains from the trade if the principal can exchange outstanding claims for new ones, but agents want to free-ride on others' participation and hold out. The principal can impose punishment to discourage holdouts but cannot i) selectively punish a holdout, a condition I call *weak consitency*, or ii) credibly commit to the proposed punishment, a condition I call *renegotion-proofness*. The novel insight on how variation in the set of initial contracts affects the principal's ability and credibility to punish gives rise to the observed heterogeneity in the solutions to the holdout problem.

In the model, the contracts' payoffs are jointly determined i) by the asset value and ii) by the contractual holding structure, that is, who holds what contracts. The principal has a residual claim on the asset, which affects her incentive to propose new contracts. Each agent's payoff depends on his decision to accept the principal's proposal, or to hold out, as well as the asset value and other agents' contracts. Each agent benefits

reaching a controlling stake, is required by the *mandatory bid rule* to proceed with 100% of the shareholders before she can allocate assets away from or losses to the acquired firm, as a protection for the minority shareholders. See Burkart and Panunzi (2003) and Betton et al. (2008).

[5]In AMC's case, the creditors received secured second-lien notes in exchange for their unsecured senior subordinated notes, and the holdouts, which previously had seniority in-between, were promoted to first-lien.

[6]In the Argentinian case, the agents are creditors, the asset is Argentina's tax revenue, and the contracts are the general-obligation government bonds issued under New York law. The principal is the Argentinian government, which would like to commit to never making a second offer to discourage holdouts.

from others' participation even when he holds out. But this benefit can be diluted by other agents' contracts.The principal aims to design a new set of contracts that all agents accept.[7] The holdout problem restricts the set of contracts the principal can offer.

This participation constraint alone is not hard to satisfy: She can threaten to punish the holdouts severely. But on top of it, there is another constraint: The principal cannot commit to implementing the proposal she has offered when any agent holds out, if she finds it optimal not to implement the punishment once the deviation has occurred. The principal would like to commit to not renegotiating with any agent, deviating or not, because renegotiation undermines the credibility of the punishment. She cannot, and that further restricts the set of feasible contracts.

Full-Commitment Benchmark If the principal can commit, then the holdout problem can be easily solved. She can always offer each agent a contract that awards him slightly more than what the initial contract would yield absent the asset value enhancement, but only if the new contracts are unanimously agreed upon by all agents. The reason, as mentioned, is that unanimity renders each agent pivotal. In this case, the principal can always extract the full surplus associated with the value enhancement of the underlying asset.[8] As long as the principal can commit to punishing holdouts, she will require unanimity no matter the setting, be it a take-over or debt restructuring. It does not explain the observed cross-sectional heterogeneity in contracts.

Result 1: Initial Contracts Sensitive to Dilution are Easier to Restructure Heterogeneity in outcomes arises once I relax the principal's commitment to punishing holdouts. To see this, let us look at two canonical examples. Consider first a corporate debt restructuring in which the agents' initial contracts are debt contracts. The principal (the firm in distress) can dilute the payoff of the holdouts by granting priority to the tendering agents (creditors). This is a credible threat as the dilution only hurts the holdouts, not the principal herself: She would only get paid after the holdouts are

[7]Note this is not the unanimity rule, which requires the threat of calling off the entire transaction when anyone holds out. Here, the principal may nevertheless continue the deal when someone holds out.

[8]In fact, she can extract not just the surplus but the full value of the asset. She does this by using a contingent contract that resembles "consent payment" in practice, giving the tendering agents a penny and nothing to the holdouts. A consent payment "effectively bribes bondholders to vote in favor of a restructuring, thereby trapping them in a prisoner's dilemma." (Donaldson et al., 2022, p.2) It survived judiciary scrutiny in the US and is also ruled legal by the English High Court in *Azevedo v. Imcopa (2012)*, provided that it is i) openly disclosed, ii) offered to all creditors, and iii) on an equal basis.

paid in full. Indeed, this is the solution suggested by the literature (e.g., Gertner and Scharfstein, 1991) and used in practice. Consider next a takeover by a raider in which all agents have equity claims. Now, by granting priority to tendering shareholders, the principal (the raider) would hurt herself because she has the same priority as the holdouts. Therefore, she would have an incentive to renegotiate and undo the punishment. The optimal solution turns out to be offering cash, which involves no punishment, albeit at a premium, because the agents need to be compensated for the rent they would obtain if they were to hold out when the rest of the agents tender and this rent cannot be credibly diluted.

Intuitively, the principal needs to offer a new contract with a credible punishment to deter holdouts, but the punishment is credible insofar as it does not hurt the principal herself, which depends on the payoff sensitivity of the holdout's initial contract. Punishing the holdouts requires diluting the payoff of their initial contract. The punishment is credible if the dilution is fully borne by the holdout (e.g., debt in default). The credibility problem arises when the dilution is also partially borne by the principal. Specifically, I show this occurs whenever the payoff of the holdout's initial contract moves less than one-to-one with the underlying value (i.e., a "dilution sensitivity" smaller than one).

This result explains the heterogeneity of solutions across applications and the absence of more sophisticated contractual solutions in takeovers. Unlike corporate debt restructuring, where over 66% of exchange offers involve offering seniority (Bratton and Levitin, 2018), in takeovers, the dominant forms of offers are cash or the acquirers' stocks.[9] Malmendier et al. (2016) find that more than 92% of the successful takeovers use cash or stock offers with an equal split and pay an average premium of about 50% (Also see Betton et al., 2008). My model rationalizes these findings: Dilution is credible in corporate debt restructuring but not in takeovers, as it also hurts the raider. The optimal tool in takeovers is simply cash.

Result 2: Higher Commitment Might Backfire Unsurprisingly, a lack of commitment makes holdout problems harder to solve. There are various policy proposals to strengthen the principal's commitment to punishing holdouts, for example, the

[9]Stock offers are often used in the presence of financial constraints or relative overvaluation (See Rhodes-Kropf and Viswanathan, 2004). In my framework, there is no distinction between acquirer equity and cash offers, as both are non-contingent on the target asset value and capital structure.

introduction of Collective Action Clauses (CACs), which allows the sovereign principal to implement a restructuring using a (super-)majority vote[10] and limits the dissenting creditors' ability to initiate litigations. Full commitment is always optimal: If not, the principal could simply commit to whatever she would do with limited commitment. A naïve generalization would be that higher commitment helps. However, I show it is not always the case: Higher partial commitment can backfire, hindering restructuring.

The reason is that, whereas higher commitment allows the principal to impose more stringent punishment on the holdouts (a direct effect), it also allows the principal to obtain a higher value from renegotiation following a rejection, making the principal more likely to renegotiate, and lowering the punishment that can be credibly imposed on the holdouts (an indirect effect). This indirect effect can outweigh the direct effect, leading to a lower value to the principal, especially when the principal starts with a low level of commitment: Renegotiation is more likely when the commitment is low. This force gives rise to a non-monotone effect of commitment and alerts policymakers that gradual increases in commitment could exacerbate holdout problems.

This result resonates with evidence that policies increasing commitment can either alleviate or exacerbate holdout problems. Indeed, there are seemingly contradictory findings about CACs. Almeida (2020) suggests that the introduction of CACs would give the sovereign too much commitment[11] to punishing the holdouts ex post, leading to a higher borrow cost ex ante. However, Chung and Papaioannou (2021) finds it actually lowers the borrowing cost. The difference is that the latter looks at a partial inclusion of CACs, a small increase in commitment, while the former looks at a full inclusion. The contradictory findings are reconciled in my model: A small increase in commitment can make restructuring harder. Also consistent with this result, Carletti et al. (2021) finds the mandatory replacement of unanimity with supermajority voting lowers the yields of the sovereign bonds, whereas Donaldson et al. (2022) finds making one class of bonds easier to restructure increases the yields. Similarly, in takeovers, Chen et al. (2022) finds that the inclusion of a bidder termination clause, which slightly strengthens the raider's commitment to calling off the deal,[12] increases the offer premium, making

[10]Generally speaking, there are two main types of CACs: Single-limp and multi-limp, most commonly, two-limb. A single-limp CAC requires an aggregated vote across all series of bonds, and the restructuring plan has to reach supermajority approval, while a two-limb CAC would require the plan to get a majority approval within each class of bonds. For more details, see Gelpern and Heller (2016) and Fang et al. (2021).

[11]Their original phrase is that it *weakens* the sovereign's commitment to fulfilling the debt service.

[12]The bidders would nevertheless have the fiduciary or regulatory rights to termination even without the

takeovers more costly.

Extension on Property Rights The solutions to the holdout problems, by and large, are achieved by deploying *dilution*: the principal designs new contracts to exert a *contractual externality* on the holdouts off path, reducing the value of the existing ones and thus the incentive to hold out. There are cases where agents' interests or claims are protected by property rights, which cannot be diluted by contractual externalities,[13] e.g., houses in land assembly and debt secured by collateral.

Usually, property rights protections are perceived to exacerbate the holdout problems.[14] This is true under full commitment: Each agent needs to be compensated more in order for him to tender since the value protected by property rights cannot be diluted by new contracts. However, when the commitment is limited, the relationship can be overturned: Stronger property rights protection also makes renegotiation harder for the principal. Indeed, the incentive to renegotiate is reduced when the principal's benefit from renegotiation is reduced, which is the case when agents' rights are well protected in renegotiation. This allows the principal to commit to imposing stronger punishment initially, which, on the contrary, facilitates restructuring.

Contribution The general framework nests classic works on the holdout problems, such as Grossman and Hart (1980), Bulow et al. (1988) and Gertner and Scharfstein (1991), by including arbitrary existing contracts, and goes beyond in two dimensions: A more general contracting space and a flexible commitment assumption. Without the ad-hoc restriction on the contracting space, the holdout problem no longer exists since the contracts can be contingent upon everyone's action and make them pivotal. Limited commitment is often allowed in the sovereign debt literature, usually to debt repayment and new borrowing, but they typically do not consider optimal contracting. For example, Pitchford and Wright (2012) looks at the delay caused by the negotiation with a cash settlement. The main insight of the paper on how optimal exchange offers depend on the interaction of commitment and existing contracts' payoff sensitivity

provisions.

[13] I adopt the notions of contractual rights and property rights as defined in Ayotte and Bolton (2011) that contractual right is a right against the contracting party whereas property right against everyone.

[14] For example, Demiroglu and James (2015) finds that loans held more by collateralized loan obligations (CLOs) exhibit greater holdout problems and are more difficult to restructure. Holland (2022) shows using survey data in Colombia that greater property rights protection exacerbates holdout problems in real estate development.

can only be obtained when all three elements are considered. Notably, Segal (1999) also provides a general framework for contracting with externalities, but he mainly considers optimal allocation *given* the externalities, while designing externalities is part of the principal's problem in this paper. Most analysis in his paper only concerns non-contingent transfers, except in the general commitment mechanism section, in which the optimality of unanimity is reaffirmed. He also alludes to the inefficiency of limited commitment and shows how it compares with the commitment case with non-contingent transfers[15] but leaves the contractual design in the face of the limited commitment to future research, and that's my focus.

Readers should be alerted that the abovementioned solutions are private solutions that the principal devises to overcome agents' incentive to hold out given the institutional constraints. The optimal institution design needs to have more elements to be in the objective: For example, it has to balance the ex-ante financing and the ex-post restructuring, which could either conflict with (Bolton and Jeanne, 2007, 2009) or complement (Donaldson et al., 2020) each other. The paper nevertheless provides a broader picture for the ex-post consideration.

2 Model Setup

2.1 Baseline Setup

Agents, Asset, and Actions. There are N agents (A_i), indexed by $i \in \mathcal{N} := \{1, 2, \ldots, N\}$ and a principal (P). Each agent is endowed with a *security*, a claim on an asset with endogenous value. The principal can enhance the asset value by restructuring the claims, and she does so via an *exchange offer*: She proposes new securities in exchange, as well as some cash, for the existing ones, and each agent independently chooses to accept it or to *hold out*.[16]

Let $v(h)$ be the value of the asset as a function of the *holdout profile* $h = (h_1, h_2, \ldots, h_N)^\top \in H \equiv \prod_{i=1}^{N} H_i$ where $h_i \in H_i := \{0, 1\}$ is the holdout decision chosen by A_i: $h_i = 1$ if A_i re-

[15] The notion of limited commitment assumed is that the principal cannot commit in public offers but can commit in subsequent private renegotiation. This corresponds to the strong credibility I develop later in Section 4.1. I also relax it and consider the case when the principal cannot commit even in subsequent renegotiation.

[16] Coalition formation and side contracting as in Jackson and Wilkie (2005) is ruled out since the essence of a holdout problem is the lack of collective bargaining.

jects the offer and *h*olds out; and $h_i = 0$ otherwise.[17] I use $e_i = (0, 0, \ldots, 1, \ldots, 0)^\top \in \mathbb{R}^N$ to denote the unit vector of length N, whose ith element is 1 and all others are 0. And I will use $\xi(h) = \{i \in \boldsymbol{N} : h_i = 0\}$ to denote the set of agents who tender at h.

I assume $v(h)$ is a *weakly decreasing* function of h: $v(h^a) \leq v(h^b)$ if and only if $h^a \geq h^b$,[18,19] with equality if and only if $h^a = h^b$. This assumption is intuitive: Holding out destroys the asset value. The value of the asset decreases as more agents hold out.[20]

Securities and Payoffs We have two sets of contracts, the *O*riginal ones $R^O(w, h)$ and the new ones $R(w, h)$, both of which are functions $\mathbb{R}_+ \times [0, 1]^N \to \mathbb{R}_+^N$ that map some value $w \geq 0$ that can be distributed to the respective contract holders, and the agents' holdout profile h to their payoffs, given the new or original securities held by the agents.[21] The payoff function of the original contracts R^O encodes both the original set of claims as well as the underlying system of conflict resolution among securities, such as a bankruptcy code; while that of new contracts R, in addition, implicitly encodes R^O, as they are written when R^O is in place. Cash paid to A_i at h is denoted $t_i(h) \geq 0$.

We write $R_i^O(w, h)$ as the ith entry in the payoff vector and assume that payoffs are, trivially, i) feasible, that is, $h \cdot R^O(w, h) := \sum_{i=1}^N h_i R_i^O(w, h) \leq w$ for all w and ii) non-negative, $R_i^O(w, h) \geq 0$, for all w, h and i. And similarly for $R_i(w, h)$.

[17]We could allow each agent to accept a fraction $1 - h_i \in [0, 1]$ of the offer, but this could be achieved by offering a combination of a fraction $1 - h_i$ of the offered contract and a fraction h_i of the original contract. Similarly, if the principal wants to exclude some agent A_i from the exchange offer, she can simply offer the same old contract A_i has. One caveat is that the function $v(\cdot)$ also needs to be modified. I show this can be done in Section **??** in the Appendix. Another caveat is we do not allow v to depend on the new contracts offered, as it turns out that adding this dependence will affect how a dilution affects the remaining value of the asset, but not the relative allocation between the principal and the holdouts, so it does not interfere with the credibility constraint.

[18]As a standard notation, $h^a \geq h^b$ means $h_i^a \geq h_i^b$ for all i and the inequality is strict for some i's. This assumption captures many scenarios. For example, projects that naturally require the participation of k agents can be encoded as a step function $v(h) = v_0 + \Delta v \mathbb{1}_{\{h^T \mathbf{1} \leq N-k\}}$, with the unanimous participation as a special case of $k = N$. Note that this differs from using a unanimity or majority rule specified by P in the exchange offer. I discuss the microfoundations of this assumption in Section 7.1.

[19]Also notice that since $v(\cdot)$ is only weakly decreasing, it may not necessarily be optimal for everyone to tender. Indeed, having everyone tender is optimal in the full-commitment case as shown in Proposition **??**, and also in the limited-commitment case with additional regularity conditions. I focus on the implementation of $h = \mathbf{0}$ for expositional purpose.

[20]In the baseline model, I assume the asset value $v(h)$ is a deterministic function of the holdout profile h, but the analysis could be extended to the case of random functions and write $v(h)(\omega)$ for the explicit dependence on the state ω. For example, a firm may still have uncertain cash flow after a restructuring and end up in bankruptcy as in Donaldson et al. (2020).

[21]Notice that potentially, $w \neq v(h)$. For instance, it may be the case that the amount to be distributed to initial claimants is only $w = v(h) - x$, with $x > 0$ being the value of the asset that accrues to new claims created in the exchange offer.

Conveniently, we index the principal[22] by 0 and write her payoff from her residual claim as $R^O_0(w, h) := w - h \cdot R^O(w, h)$.

The function R^O does not automatically capture the effect on payoffs resulting from the new securities offered by the principal since it was unknown when it was created. In other words, the original contracts were incomplete because it was impossible to enumerate all possible future exchange offers, which could further depend on the initial contracts and lead to infinite regress. To tackle this issue, I introduce an additional requirement, called the *weak consistency* (defined and discussed in Section 2.2), with which the payoff of the new security is simply $R(v(h), h)$, in the spirit of direct mechanism, and the payoffs of the original security holders can be written as

$$R^O(v(h), h|R) = R^O(v(h) - x(h), h) \tag{1}$$

where $x(h) = (1 - h) \cdot R(v(h), h)$ is the total amount paid to the new security holders and I conveniently call it "dilution".[23] Notice, now, the dependence of R^O on R is only through the first argument in a linear fashion. Table 1 shows how my notation encompasses the classic models.

	$v(h)$	$R_i(v, h)$	$R^O_i(w, h)$
Takeover	Threshold	(Cash t_i)	Equity $\alpha = (\alpha_i)_i$
(Grossman and Hart, 1980)	$v_0 + \Delta v \cdot \mathbb{1}_{\{h^\top \mathbf{1} \leq \bar{h}\}}$	0	$\alpha_i w$
Bond buyback	Decreasing	(Cash t_i)	Debt $D = (D_i)_i$
(Bulow et al., 1988)	$W(h) + X$	0	$\min\left\{\frac{wD_i}{h^\top D}, D_i\right\}$
Debt Restructuring	Constant	Senior Debt D^S	Junior Debt D^J
(Gertner and Scharfstein, 1991)	$X + Y - I$	$\min\left\{\frac{vD^S_i}{(1-h)^\top D^S}, D^S_i\right\}$	$\min\left\{\frac{wD^J_i}{h^\top D^S}, D^J_i\right\}$

Table 1: Classic Applications:In a takeover, the asset value jumps from v_0 to $v_0 + \Delta v$ if the number of holdouts is less than $\bar{h}$. The existing securities are equities and only cash offer is considered so the securities portion is zero. The bond buyback model allows the asset value to have an component $W(h)$ decreasing in the holdout profile and the existing securities are debt. In the debt restructuring case, a senior debt offer is considered while the asset value is fixed. In each, $v = v(h)$ and $w = v(h) - (1 - h) \cdot R(v, h)$.

[22]The principal need not have an explicit claim on the asset as her identity as the residual claimant is determined by the contractual relationship with the agents.

[23]The name was motivated by the offering of senior debt in debt restructuring, which dilutes the payoff of the holdouts who have a junior claim. However, the model, per se, does not have a notion of priority, and the new securities could well have a payoff of zero. (According to Moulin (2000), priority can only be defined with two other axioms: lower and upper composition. We do not have them here.)

Renegotiation The principal cannot commit to (not renegotiating) his initial exchange offer. I will focus on the renegotiation-proof exchange offers (defined later in Section 4). In particular, the principal can always call off the deal; in this case, the payoffs are simply evaluated at $h = 1$, as if everyone holds out.

Cost The principal faces a random cost c, whose value is realized before announcing the exchange offer but incurred only if the plan is carried out, that is, $h \neq 1$. It could be interpreted as the outside option of the principal or the cost of carrying out the plan (e.g., investment, attorney fees, etc). Thus, the principal is willing to carry out the plan if and only if her benefit from the plan exceeds the cost, c. Throughout, I assume the cost is small, $c < v(0) - v(1)$, so it is always socially efficient to carry out the deal. The randomness of this cost is not essential for the analysis but captures unobserved heterogeneity that can potentially be important to explain the variation in outcomes in otherwise similar situations.

Principal's Simplified Problem I directly lay out the principal's simplified problem and relay the full definition and the full problem to Section **??** in the Online Appendix.

With some abuse of notation, I write the payoff associated with the new exchange offer and the original contract in a conditional form: I let $(h_i, h_{-i}) := h = (h_1, \dots, h_i, \dots, h_N)^\top$ and write $R_i(h_i|h_{-i}) := R_i(v(h), (h))$ and $R_i^O(h_i|h_{-i}, R) := R_i^O(v(h) - (1-h) \cdot R(v(h), h), h)$ to highlight the incentives and actions of a particular agent. The payoff of agent A_i is

$$u_i(h_i|h_{-i}, R) := h_i R_i^O(h_i|h_{-i}, R) + (1 - h_i)[R_i(h_i|h_{-i}) + t_i(h_i|h_{-i})] \tag{2}$$

and all incentive-compatible contracts at h is denoted by

$$\mathcal{I}(h) := \left\{ R : [\underline{v}, \bar{v}] \times H \to [0, \bar{v}]^N \,\middle|\, h_i \in \arg\max_{h_i' \in H_i} u_i(h_i'|h_{-i}, R) \; \forall i \in \mathcal{N} \right\}. \tag{3}$$

The principal's value at h from an exchange offer R with cash t is[24]

$$J(h|R) := R_0^O(v(h) - (1-h) \cdot R(v(h), h)) - (1-h) \cdot t(h). \tag{4}$$

[24]For most of our analysis other than Section 3.1, cash portion will be 0 as it is mostly costly for the principal to offer. So we don't explicit write the dependence on t.

Given the feasible set of contracts $\mathcal{R}(h) \subset \mathcal{I}(h)$, her problem is to find $R \in \mathcal{R}$ along with cash transfer $t \geq \mathbf{0}$ to maximize her value $J(h|R)$

$$J(h|\mathcal{R}) := \max_{R \in \mathcal{R},\, t \geq \mathbf{0}} J(h|R). \tag{SP}$$

When $\mathcal{R}$ is clear from the context, I omit it and write $J(h)$. In particular, we will be interested in her value function $J(\mathbf{0})$ when everyone tenders. Below, I consider the case in which the principal lacks commitment, and thus, an additional credibility constraint enters into the optimization problem.

In the absence of commitment, the principal is subject to an additional credibility constraint. This is the focus of the paper, studied in Section 4. In what follows, I discuss the origin, interpretation, and role of weak consistency in the model, and how the complexity in the full model can be reduced without loss of generality.

2.2 Weak Consistency and the Problem Simplification

The generality of the proposed framework makes it difficult to characterize the problem, but it can be simplified as follows. First, I impose a condition that the principal cannot alter the existing contractual relationship among securities using the new securities proposed in the exchange offer. In other words, the relative payment to two holdouts has to stay the same. For instance, the principal may want to write a contract with a tendering agent by which the priority structure between two non-tendering agents is flipped. The assumption, which I refer to as *weak consistency*, excludes this type of exchange offer. Second, without loss of generality, it is enough for the principal to focus on exchange offers in which all agents tender. A formal statement can be found in Section **??** and **??** in the Online Appendix.

Start with weak consistency. It is defined as follows.

Definition 1 (Weak Consistency)**.** *An exchange offer is* weakly consistent *if the payoff of each holdout equals his payoff from the original securities evaluated at the asset value minus the part that accrues to tendering agents. That is, let* $x := \sum_{i=1}^{N} (1 - h_i)\, R_i(v, h)$ *be the part of the asset value that accrues to the tendering agents given* R *and* h*, i.e., the "dilution", the exchange* *offer is* weakly consistent *if each holdout receives*

$$R_i^O(v, h|R) = R_i^O(v - x, h)\ \forall i = 0, 1, .., N. \tag{5}$$

Weak consistency[25] captures the intuition that the principal can create externalities on the holdouts by diluting them through new contracts, but the dilution cannot be selective. For example, if the principal has three creditors: Alice has a senior debt, Bob has a junior debt, and Claire also has a junior debt. Suppose both Alice and Bob hold out. She can offer a claim to Claire that is senior to both Alice and Bob, or junior to both Alice and Bob, or junior to Alice but senior to Bob. What she cannot offer is a claim that is senior to Alice but junior to Bob: It would flip the priority between Alice and Bob. In addition, since she is in the nexus of the old contracts, she cannot sign a new contract with Claire to make herself senior to Alice or Bob. In summary, she, as the residual claimant, cannot make an exchange offer that dilutes holdouts without diluting herself. Without this weak consistency, she could simply offer herself a super-senior claim and solve all holdout problems.[26,27]

The second simplification, that it is enough for the principal to focus on exchange offers in which all agents tender, builds on a simple idea: If it is optimal for an agent to retain a fraction or the entirety of the initial security for a given exchange offer, the principal could equivalently offer the claim the agent has in his hand post-restructuring.

[25]This is a weaker version of the consistency axiom widely used in the study of bankruptcy problems in the cooperative game theory literature, e.g., in Aumann and Maschler (1985) and Moulin (2000). It has also been used in the study of multilateral bargaining games as in, for example, Lensberg (1988) and Krishna and Serrano (1996). It serves a role similar to the more commonly known axiom of Independence of Irrelevant Alternatives. The difference between the consistency axiom and weak consistency is that consistency requires this condition to hold for *any* subset of the securities, while I only require it to hold between the new and old contracts. Informally, given an allocation rule $R^O(\cdot,\cdot,\cdot)$ is a map from the set of N agents $\mathcal{N}$, the total value available $v > 0$, and a vector of claims $\boldsymbol{d} \in \mathbb{R}_+^N$, to an allocation vector $R^O(\mathcal{N}, v, \boldsymbol{d}) \in \mathbb{R}_+^N$ where agent A_i receives $R_i^O(\mathcal{N}, v, \boldsymbol{d})$, the rule is consistent if, for any subset $\mathcal{N}_0 \subset \mathcal{N}$, the allocation among the agents in the subset is identical to the original allocation as long as the total resource available is the total resource allocated to $\mathcal{N}_0$ under the original allocation and the agents in the subset $\mathcal{N}_0$ have the exactly same claim $\boldsymbol{d}|_{\mathcal{N}_0}$. Or in formula, $R^O(\mathcal{N}_0, v - \sum_{j \notin \mathcal{N}_0} R_j^O(\mathcal{N}, v, \boldsymbol{d}), \boldsymbol{d}|_{\mathcal{N}_0}) = R^O(\mathcal{N}, v, \boldsymbol{d})|_{\mathcal{N}_0}$. Thomson (1990) and Maschler (1990) have a comprehensive survey on this topic.

[26]In practice, there are situations where the principal can effectively divert value to herself with the help of a third party. In Müller and Panunzi (2004), they described a procedure called *freezeout merger*, or *bootstrap acquisition*, where the acquirer could use the target as the collateral to raise senior debt from a third-party lender and pocket in the proceeds from borrowing. As they analyzed, doing so could appropriate value from the existing shareholders and facilitate the takeover. The legality of this practice is challenged but not overturned.

[27]The cash offers cannot be viewed as securities because they violate the weak consistency in the other direction: They allow the principal to get less than the holdouts and we do not want to take away this ability.

This way, the agent would at least find it equally optimal to accept the entire exchange offer. There might be two technical issues: i) With the new offers, there might be actions that are not initially available; ii) The asset value is higher when the agent accepts, so the outside option is more valuable. I address them in Section **??** in the Online Appendix.

3 Optimal Exchange Offer with Full Commitment

In this section, I provide two drastically different benchmark results to illustrate the power of a large contract space. First, I reproduce the classic results by showing that holdout problems occur whenever i) the principal is only allowed to offer cash, and ii) the cost of implementing the exchange offer c is not too small.[28] Second, if, instead, the contracts are fully contingent, the principal can uniquely implement an equilibrium that extracts the full value of the assets. These two benchmarks represent the lowest and the highest value the principal can obtain in the transaction.

3.1 Optimal Non-Contingent (Cash) Exchange Offers

Suppose first that the principal can only offer cash, a one-shot payment $t_i(h_i) \geq 0$ to agent A_i, which is only a function of A_i's decision to hold out h_i, independent of v and h_{-i}. Cash offers are widely used and do not have any dilution. For simplicity, I will assume that the principal has a deep pocket[29] and pays the cash when an agent tenders. To focus on the interesting cases, I also assume

Assumption A1 (Moderate Cost)**.** *The cost is neither too small nor too large*

$$v(\mathbf{0}) - v(\mathbf{1}) > c > v(\mathbf{0}) - \sum_{i=1}^{N} R_i^O(v(e_i), e_i). \tag{A1}$$

[28]Of course, since whether the holdout problem occurs depends on the type of new contracts offered, it may not be limited to cash offers. For example, Gertner and Scharfstein (1991) and Donaldson et al. (2020) demonstrate that the holdout problem also arises with pari-passu debt offering. But in most studies, a cash-like payoff is considered, e.g., in takeover (Grossman and Hart, 1980; Bagnoli and Lipman, 1989; Holmström and Nalebuff, 1992) and bond buyback (Bulow et al., 1988; Admati et al., 2018).

[29]In takeovers or similar transactions, the principal often needs to finance her deal by borrowing from a third party, such as a leveraged buyout, or a freezeout merger. Two institutional details might change the feasibility of a cash offer: Whether the original security holders have recourse to the borrowed cash, that is, if the cash is added to the target company's balance sheet; and whether the principal can divert the cash. It turns out the former will not affect the feasibility as analyzed in Section **??** while the latter will because it violates the weak consistency.

The first inequality is there to guarantee that it is socially efficient to implement $h = \mathbf{0}$. The second inequality says if the principal has to give each agent what he obtains under the old contract if he holds out, she would not want to initiate the exchange offer;[30,31] Otherwise, the holdout problem does not impede efficient transactions.

Notice that agent A_i will receive $R^O(v(e_i), e_i)$ if he holds out when everyone else tenders. So the principal will have to pay at least this much to him and almost immediately we have

Proposition 1 (Holdouts with cash offers)**.** *Under Assumption A1, the first best $h = \mathbf{0}$ cannot be implemented via an exchange offer with only a non-contingent cash offer.*

The proposition simply states that under Assumption A1 Moderate Cost the classic holdout problem occurs: A simple cash transfer is not enough to compensate each agent for his reservation value under the deviation, $R_i^O(v(e_i), e_i)$. The key force in a typical holdout problem is that the incentive compatibility constraint of any single agent becomes more difficult to satisfy as more of the rest of the agents tender[32] as illustrated in the example below. This makes addressing the holdout problem using cash prohibitively costly, and an efficient value enhancement cannot be obtained.

Example 3.1. *Suppose a situation with 3 creditors, each with an outstanding debt claim with a face value of $D_i = 6$. Assume that the asset value is $v(h) = 9 + \sum_i (1 - h_i)$. Each creditor would be paid $9/3 = 3$ without asset value improvement and up to $(9+3)/3 = 4$ when all of them tender. If the principal can renegotiate with all creditors collectively, then she could offer any price between 3 and 4 to each claimant and the first best obtains. However, when collective decision is not feasible, if all but one agent tender, the holdout could get paid in full, i.e., 6 out of the asset value 11, and this leaves the principal a residual value of 5, which allows her to pay each tendering agent at most 2.5 to remain profitable, which is worse than their initial value. Of course, each agent thinks of himself as the marginal holdout and demands 6; thus, the holdout*

30 Notice that the RHS of inequality (A1) could be negative, for instance, when outstanding claims are debt (See Example 3.1). In this case, the holdout problems occur even if there is no cost. When the outstanding claims are equity as in Grossman and Hart (1980), it is always non-negative and it converges to zero when the number of agents goes to infinity, and a single holdout does not affect the asset value.

31 Notably, the practice of toeholding would violate the second inequality as more surplus goes to the raider when she accumulates the shares and less value needs to be distributed to dispersed shareholders without any dilution.

32 The argument alludes to the slightly more restrictive assumption A2 that the original securities are increasing in the underlying value. But Proposition 1 holds even without this assumption. It is possible, though almost never seen, that the original contract prescribes that a single holdout gets nothing.

problem cannot be solved with a simple cash offering.[33]

The intuition for this is straightforward. As the number of agents who tender increases, it becomes increasingly more difficult to get other agents to tender. There are two forces at work that induce a form of strategic substitutability amongst agents. First, the asset value is higher when more agents tender; and second, there are few competing claims on the asset. To see this, if three agents hold out, each holdout will get 3 *out of the asset value* 9*, but when two agents hold out, each gets* 5 *out of the asset value* 10*. The value of the outside options grows even faster than the asset value growth as more agents tender.* □

3.2 Optimal Contingent Exchange Offers

In diametric contrast, if the principal can offer *contingent contracts*, i.e., securities whose payoffs depend on both the asset value and the decision of each agent to tender or not, which indirectly depends on the type of contracts other agents end up with, whether old or new, she will not only solve the holdout problem, but also be able to extract the full value of the asset and implement it as a unique equilibrium. We also let $t = 0$ since she can give tendering agent the same at a lower cost by giving a prioritized claim of the asset.

Example 3.2. *Continuing example 3.1, suppose, in exchange for each creditor's junior debt, the principal offers each creditor a cash payment of 1 cent if everyone tenders and an option that can be converted into a senior debt with a face value of* 6 *when at least one holdout exists. Everyone tendering is an equilibrium since any holdout triggers the option and leaves the holdout with nothing. For the same reason, there's no equilibrium of one holdout. If there are two holdouts, each will get* $(10-6)/2 = 2$*, and deviating to tendering gives* $\min\{6, 11/2\} = 5.5$*, so it can not be an equilibrium. Similarly, when everyone holds out and gets* 3*, whoever deviates to tendering gets* 6*. Therefore, everyone tendering is the unique equilibrium that gives the principal a payoff of* $12 - 0.03$ *and the on-path payment* 0.01 *can be made arbitrarily small.*

To see it formmally, recall the definition of unique implementation of Segal (2003) and Halac et al. (2020).

Definition 2 (Unique Implementation)**.** *The principal uniquely implements an action profile* h *and guarantee a value* w *i) if there exists a weakly consistent exchange offer* (H, h, R) *such*

[33] Note the RHS of the Equation (A1) is $12 - 6 \times 3 = -6$, so a positive cost is not needed to generate the holdout problem.

that h is an equilibrium in the subgame played by the agents and ii) for any $\varepsilon > 0$, there exists a weakly consistent exchange offer $(H^\varepsilon, h, R^\varepsilon)$ such that h is the unique equilibrium in the subgame, in which the principal obtains a payoff of at least $w - \varepsilon$.

Introducing this perturbation ε is purely technical as the set of exchange offers that admits a unique equilibrium is not necessarily closed.[34] With this definition, I derive

Proposition 2 (Extreme Gouging)**.** *If $v(\mathbf{1} - e_i) > 0$ for some $i \in \mathcal{N}$, with contingent contracts, the principal* uniquely *implements the action profile $h = \mathbf{0}$ and guarantees a value of $v(\mathbf{0})$.*

To get the gist of the proof,[35] notice the IC facing an agent is that the on-path payoff from tendering must be greater than the off-path payoff from holding out

$$R_i(v(\mathbf{0}), \mathbf{0}) \geq R_i^O\left(v(e_i) - x(e_i), e_i\right). \tag{6}$$

In A_i's off-path payoff, the right-hand side of inequality (6) (the IC constraint in problem (SP)), the total payment to all other agents $x(e_i) = \sum_{j \neq i} R_j(v(e_i), e_i)$ "dilutes" the value that A_i is able to claim. When the principal can commit to paying the tendering agents more, she can punish the holdouts up to the full value of the asset $v(e_i)$. The equilibrium will thus feature the principal offering an arbitrarily small fraction of the asset to each agent. If any one agent deviates and holds out, she will then distribute the entirety of the asset to the tendering agents. This occurs off-equilibrium path. It is the principal's ability to commit that matters here: When the principal assigns the entirety of the asset to tendering agents, she also dilutes her claim. Instead, absent commitment, the principal will be incentivized to renegotiate, rendering this exchange offer non-credible.

4 Optimal Exchange Offer with Limited Commitment

What happens when the principal cannot commit to punishing holdouts off the equilibrium path but is tempted to renegotiate with holdouts? Instead of looking for

[34] In this equilibrium below, if the principal offers $\varepsilon/N > 0$ to every agent, their incentive to accept is a strict inequality so the implementation is unique. But when they are offered nothing, they only weakly prefer to accept, and thus the principal can only get $v(\mathbf{0}) - \varepsilon$ for any $\varepsilon > 0$ in a unique equilibrium but not for $\varepsilon = 0$. For a more detailed discussion, see Section 4 in Segal (2003) (and footnote 9 in particular).

[35] The condition that "$\exists i : v(\mathbf{1} - e_i) > 0$" is simply to ensure the principal has some recourse to maneuver, to induce tendering, so that everyone holding out is not an equilibrium. This condition does imply A_i's tendering contributes to the value $v(\mathbf{1} - e_i) > v(\mathbf{1})$, so it doesn't rule out the case when a significant mass of tendering is required to improve the value of the asset.

what happens in renegotiation, I look for contracts that are renegotiation-proof:[36] The principal prefers just executing the original contracts even if an agent deviates. This strictly shrinks the space of contracts[37] the principal can propose initially and rules out some non-credible threats off path. And the key issue is to specify the feasible contracts that the principal can offer in renegotiation.

Before introducing a formal definition of credibility, I impose a regularity condition on the existing contracts:

Assumption A2 (Increasing and 1-Lipschtiz)**.** *The sum of the payoffs to the agents who do not tender at h, $h \cdot R^O(\cdot, h)$, is increasing and 1-Lipschitz for all h.*

This assumption is common in the security design literature.[38] This condition says the payoff function has a non-negative "slope" weakly less than 1. That is, whenever the underlying increases by one dollar, the incremental payoff to the existing contracts cannot exceed one dollar. Most commonly seen contracts, such as equity, debt, and call options, satisfy this condition.[39] It rules out uninteresting cases in which punishing holdouts rewards the principal. This assumption, coupled with weak consistency, allows us to define a simple measure

Definition 3 (Dilution Sensitivity)**.** *The* dilution sensitivity *of contract $R_i^O(\cdot, h)$ at w_0 is defined to be the limit of the left derivative*

$$S_i(w_0; h) := \frac{\partial}{\partial w}\bigg|_{w \uparrow w_0} R_i^O(w, h) \equiv -\frac{\mathrm{d}}{\mathrm{d}x}\bigg|_{x \downarrow 0} R_i^O(w_0 - x, h) \geq 0 \quad \forall i \in \mathcal{N} \tag{7}$$

$S(w_0; h) = \sum_i h_i S_i(w_0; h)$ for all holdouts collectively, and $S_0(w_0; h) := 1 - S(w_0; h)$ for the principal. The average dilution sensitivity on the interval $[\underline{w}, \bar{w}]$ is $S_i([\underline{w}, \bar{w}]; h) = \frac{1}{\bar{w} - \underline{w}} \int_{\underline{w}}^{\bar{w}} S_i(w; h) \mathrm{d}w$ for $i = \emptyset, 0, 1, \ldots, N$.

Assumption A2 implies that $R_i^O(\cdot, h)$ is differentiable almost everywhere. I take the

[36] Specifying the exact sequence of renegotiation might be convoluted as it might involve infinite rounds of bargaining and an agreement may never be achieved as shown in Anderlini and Felli (2001). Absent private information, if the principal finds ex post optimal to offer something else, she could have already anticipated it and offered it in the original exchange offer.

[37] A large contractual space is not necessarily a desired property: Other than the issue considered here, it also allows too many possible deviations as articulated in Brzustowski et al. (2023).

[38] For example, in the definition of feasible contracts in DeMarzo et al. (2005), they require the payoff to each party to be increasing, which is equivalent to 1-Lipschitz continuity.

[39] A notable exception is contingent securities that admit jumps. For example, the Additional Tier 1 (AT1) bonds in the Credit Suisse crisis were wiped out. Holdout problems are not a concern for these securities as the jumps make a larger punishment more credible.

limit for the non-differentiable points. The *left* derivative captures how much the claim holder (or the residual claimant) will lose if the dilution increases by 1 unit marginally.

Credibility issues arise only when agents deviate. Throughout, I consider only *unilateral deviations*. A profile $\hat{h}$ is a unilateral deviation of h if and only if $\hat{h} = h + e_i$ or $\hat{h} = h - e_i$ for some i, or equivalently, $||\hat{h} - h|| = 1$. I use $\mathcal{B}(h) = \{\hat{h} \in \{0,1\}^N : ||\hat{h} - h|| = 1\}$, the unit "ball" around h, to denote the set of unilateral deviations from h.

Lastly, I introduce the language of δ-domination, which characterizes the principal's incentive to deviate, that is, whether carrying out the exchange offer R yields a payoff higher than a fraction $\delta \in [0,1]$ from proposing a different exchange offer $\tilde{R}$ at h.

Definition 4 (δ-domination). *A contract R (weakly) δ-dominates another contract $\tilde{R}$ ($R \geq_\delta \tilde{R}$) at h, for a number $\delta \in [0,1]$, if $J(h|R) \geq \delta J(h|\tilde{R})$.*

The parameter δ can be thought of either i) as a delay cost equivalent to a discount factor as in Rubinstein and Wolinsky (1992), or ii) as the exogenous probability that the contract is voided and the principal is allowed to re-propose a new offer as in Crawford (1982) and Dovis and Kirpalani (2021).[40] Either way, δ parametrizes the principal's inability to commit: A higher δ, from full commitment ($\delta = 0$) to costless renegotiation $\delta = 1$, makes renegotiation more attractive,. I will focus on the lowest-commitment case ($\delta = 1$) and omit δ except for the the analysis of δ. I also drop "weakly" or "at h" whenever no confusion arises.

4.1 Strongly Credible Contracts

4.1.1 Strongly Credible Contracts and the Principal's Problem

I introduce two definitions of credibility. I introduce first what I refer to as Strongly Credible Contracts and later, in section 4.2, a weaker definition I refer to as just Credible Contracts. Strong credibility illustrates in a simple manner how the lack of commitment interacted with the set of initial securities, producing a variety of solutions to the

[40]To see this explicitly, let $\hat{\delta}$ be the discount rate instead, and the principal is allowed to delay the payoff and re-propose a new contract $\tilde{R}$ with some exogenous probability p, then the current proposed contract is preferred if $J(h|R) \geq (1-p)\hat{\delta}J(h|R) + \hat{\delta}pJ(h|\tilde{R})$. Rearranging the terms, the current proposed contract R δ-dominates contract $\tilde{R}$ at h for $\delta = \frac{\hat{\delta}p}{1-(1-p)\hat{\delta}}$, which is a strictly increasing in p for all $\hat{\delta} \in (0,1)$ since $\frac{\partial}{\partial p}\frac{\hat{\delta}p}{1-(1-p)\hat{\delta}} = \frac{\hat{\delta}(1-\hat{\delta})}{(1-(1-p)\hat{\delta})^2} > 0 \, \forall \hat{\delta} \in (0,1)$. Thus, for a fixed $\hat{\delta}$, a higher probability of renegotiation corresponds to a higher δ.

holdout problem. Instead, Credible Contracts illustrate why more commitment is not always good for the principal. It is important to emphasize that strong credibility is not needed to show the two main results: Credibility is enough, and strong credibility is introduced just for clarity, tractability and intuition. Finally, all strongly credible contracts are also credible contracts.

Definition 5 (Strong δ-credibility)**.** *A contract* $R : [\underline{v}, \bar{v}] \times H \to [0, \bar{v}^N]$ *is* strongly δ-credible at h *if*

(a) It is incentive compatible at h*, that is,* $R \in \mathcal{I}(h)$ *(see expression* (3) *above).*

(b) Upon any unilateral deviation $\hat{h} \in \mathcal{B}(h)$*, it weakly* δ*-dominates any IC contracts* $\tilde{R}$ *at* $\hat{h}$*.*

Condition (a) means intuitively that Strongly Credible Contracts must be incentive-compatible. Condition (b) means that even when one agent deviates, the principal will find in her interest "to stick with" the initial offer R rather than any other incentive-compatible contract $\tilde{R}$. I denote the set of strongly δ-credible contracts by

$$\mathcal{S}^{\delta}(h) := \left\{ R \in \mathcal{I}(h) : \;\; R \geq_{\delta} \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in \mathcal{I}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\}. \tag{8}$$

Again, I drop δ and simply call it a strongly credible contract when δ equals 1.

The principal's value function on the set $\mathcal{S}^{\delta}(h)$ is defined by

$$J(h|\mathcal{S}^{\delta}(h)) := \sup_{R \in \mathcal{S}^{\delta}(h)} J(h|R). \tag{9}$$

Notice that $\mathcal{S}^{\delta}(h) \subset \mathcal{I}(h)$, so the problem is more restrictive than the full-commitment case, on account of the principal's credibility constraint (see (SP) in Section 2.2).

4.1.2 Commitment and Diversity of Exchange Offers: A Characterization

To characterize the solution to the principal's problem in Equation (9),[41] I first show that it can be equivalently expressed using a single-dimensional optimization problem: The principal wants to minimize the total payoff to all agents upon the deviation of a single holdout while maximizing the possible punishment to the holdout. Then, as a

[41] Note that while it is convenient to look at the implementation of $h = 0$, the Proposition **??** in the Appendix does not guarantee renegotiation-proofness. It is also optimal under some additional mild conditions on R^O and v, so for expositional purposes, I will focus on the implementation of $h = 0$.

second step, I show that the extent to which the punishment can be credibly increased depends on the dilution sensitivity of the holdout (Lemma 1 below). This, coupled with Lemma 2 on the disagreement point in renegotiation, where strong credibility is used, gives rise to the diversity of exchange offers in Proposition 3.

When agent A_i deviates, the principal wants to lower his payoff to the greatest extent. She does so by imposing a penalty of x that accrues to the tendering agents, but she is not worse off renegotiating. That is, she solves a problem of the form

$$\min_{x \geq \underline{x}(e_i)} R_i^O(v(e_i) - x, e_i) \quad \text{s.t.} \quad v(e_i) - x - R_i^O(v(e_i) - x, e_i) \geq \delta J(e_i | \mathcal{S}^\delta(e_i)) \tag{10}$$

where $\underline{x}(e_i)$ is the minimum punishment at e_i, the minimum amount that needs to be paid to other agents so that they don't deviate from e_i.[42] The two terms $x + R_i^O(v(e_i) - x, e_i)$ in the constraint illustrate the principal's trade-off: A larger punishment x would lower the payment to the holdout A_i, but it would also increase the payoff to the tendering agents. This can potentially lower the principal's payoff, too.

The solution may not be unique as $R_i^O(\cdot, e_i)$ could be flat when x varies, but it is without loss of generality to look for the largest x subject to the same constraint. Moreover, the feasible set is not empty due to the IC constraint in the definition of $\mathcal{S}^\delta(e_i)$: The RHS of (10) must be weakly lower when the LHS is evaluated at $\underline{x}(e_i)$. Moreover, since $R_i^O(\cdot, e_i)$ is 1-Lipschitz, the LHS of Inequality (10) is decreasing in x so there exists a $\bar{x}(e_i)$ such that Inequality (10) holds if and only if $x \leq \bar{x}(e_i)$ and it is given by

$$\bar{x}(e_i) := \max\{x \geq \underline{x}(e_i) : v(e_i) - x - R_i^O(v(e_i) - x, e_i) \geq \delta J(e_i | \mathcal{S}^\delta(e_i))\}. \tag{11}$$

The following lemma presents the structure of the optimizer of a 1-Lipschitz function more generally with a generic set of deviation $\mathcal{R}$ instead of $\mathcal{S}^\delta(e_i)$.

Lemma 1. *Given $h \in H$ and $\mathcal{R} \subset \mathcal{I}(h)$ a non-empty set of feasible IC contracts, let $x(R) := (1 - h) \cdot R(v(h), h)$ be the dilution imposed at h by contract $R \in \mathcal{R}$, $g(x) := v(h) - x - h \cdot R^O(v(h) - x, h)$ be a continuous function of x under Assumpton A2, and $\mathcal{X} := \{x \geq 0 : g(x) = J(h|\mathcal{R})\}$ the set of dilutions that attains the optimum. I claim*

- *$\mathcal{X}$ is a closed interval or a singleton and $S(v(h) - x; h) = 1$ for all $x \in \mathcal{X}$.*

[42] This might further depend on the credible punishment that can be imposed on them when we switch to a weaker notion of credibility. But it's the maximum punishment, not the minimum, that determines the outside option of A_i.

- *Let* $\underline{x}(h;\mathcal{R}) \geq 0$ *be the minimum element in* $\mathcal{X}$*. The set of credible dilutions* $\mathfrak{X} = \{x \geq \underline{x}(h) : g(x) \geq \delta J(h|\mathcal{R})\}$ *admits an attainable supremum* $\bar{x}(h;\mathcal{R}) := \sup \mathfrak{X}$*.*
- *Moreover, the maximum additional dilution beyond the minimum* $\bar{x}(h;\mathcal{R}) - \underline{x}(h;\mathcal{R})$ *satisfies (argument* $(h;\mathcal{R})$ *omitted for brevity)*

$$(\bar{x} - \underline{x}) \cdot S_0([v(h) - \bar{x}, v(h) - \underline{x}]; h) = (1 - \delta) J(h|\mathcal{R}) \tag{12}$$

- *In particular, when* $\delta = 1$*, either* $\bar{x} = \underline{x}$ *or* $S_0([v(h) - \bar{x}, v(h) - \underline{x}]; h) = 0$*, and thus the maximum credible dilution can be characterized as*

$$\bar{x}(h;\mathcal{R}) = \inf\{x \geq \underline{x}(h;\mathcal{R}) : S(v(h) - x; h) < 1\}. \tag{13}$$

This lemma links the commitment to the dilution sensitivity: Even though the sets of contracts immune to deviations or optimal in deviation are hard to characterize, they are easily characterized in terms of the dilution they impose: i) The sets of dilutions are connected and compact;[43] and ii) the dilution sensitivity is merely a constant for optimal contracts in deviation and for renegotiation-proof contracts without any commitment.[44] Moreover, we have a simple relationship the additional dilution must satisfy:

Size of dilution × P's Average dilution sensitivity = Loss from renegotiation.

It usually does not admit a closed-form solution except in the lowest-commitment case: the dilution gets non-credible once the principal has a positive dilution sensitivity.

The next Lemma characterizes the maximum payoff the principal can obtain *upon deviation*. Given that the principal can only renegotiate with the tendering agents and not with the holdout, the best she can do is to offer nothing to the tendering agents (recall that, under strong credibility, she can commit in subsequent renegotiations) and obtain what is left of the asset after the holdout has been paid given his initial contract.

Lemma 2. *Under Assumption A2, the highest payoff the principal can obtain at the deviating profile* e_i *with an IC contract* $\tilde{R} \in \mathcal{I}(e_i)$ *is* $v(e_i) - R_i^O(v(e_i), e_i)$*.*

[43] Without 1-Lipschitz continuity, it might not be connected. For example, the maximizer of $1 - x - \sqrt{1-x}$ on $[0, 1]$ is $\{0\} \cup \{1\}$ and the disconnectedness would make subsequent analysis much more convoluted.

[44] This is because dilution sensitivities, as derivatives of functions, are well-behaved. They cannot have jump or removable discontinuities so we can turn many "almost everywhere" arguments into "everywhere".

Does Lemma 2 imply there is no additional credible dilution under the deviation? No. As shown in Lemma 1, this will depend on the dilution sensitivity of the holdout's original contract. The next proposition combines both lemmata to show the conditions under which dilutions are credible and how they affect the optimal exchange offer.

Proposition 3. *When $N \geq 2$, under Assumption A2, the principal cannot obtain a strictly higher value at $h = \mathbf{0}$ with a strongly credible contingent contract than offering cash if and only if for all $i \in \mathcal{N}$, $S_i(v(e_i); e_i) < 1$. Consequently, if this condition is satisfied, holdout problems cannot be solved with any strongly credible contingent offers under Assumption A1.*

Recall that in the full commitment case and under assumption A1, Proposition 1 shows that cash can never implement the first best. But if she can propose any exchange offer, she can extract the full value of the asset from the agents. Proposition 3 says instead that if she cannot commit, she cannot do better than cash, even when she can use any arbitrary exchange offer. This occurs whenever agents are not perfectly sensitive to dilutions. The next section illustrates two practical examples of when the principal can and cannot do better than cash.

4.1.3 Commitment and Diversity of Exchange Offers: Debt vs. Equity

Consider two canonical examples. In a restructuring case, the agents' contract is debt. In a takeover case, the agents' initial contract is the equity of the target. Consider debt contracts first. The payoff sensitivity of the debt contract is one if the company is in default, in which case the value of the debt of the holdout moves one to one with the value of the asset (recall that we are only considering bilateral deviations). It is zero if it's not getting paid at all or if it's already getting paid in full. As for equity, the payoff sensitivity is one only if the holdout owns the entire equity stake after all debt and other senior claims have been paid in full. It's zero when the company cannot repay its maturing debt, and it is strictly between zero and one if there are other equity holders. It is because these contracts have different payoff sensitivities that they induce, in turn, different problems of commitment for the principal, which in turn result in different solutions to the holdout problem in debt restructuring or takeovers. The following corollary now follows immediately from Proposition 3.

Corollary 1. *When each agent's initial contract is debt, the principal can obtain a higher value* *than offering cash using a contingent contract; when his initial contract is equity, no contingent contracts give a higher value to the principal than simply offering cash.*

Since it's simply an application of the Proposition 3 and is of empirical interest in itself, we lay out the proofs directly in the following examples.

Example 1: Debt. Let's consider the case when the holdout A_i has debt $D_i \geq 0$. His payoff function is $R_i^O(w, e_i) = \min\{w, D_i\}$ and the maximum credible threat is $\bar{x}(e_i) = \mathbb{1}_{\{v(e_i) \leq D\}} v(e_i)$.

The next proposition shows how the different sizes of the agents' claims change the nature of the holdout problem when the principal cannot commit not to renegotiate with the holdouts.

Proposition 4. *When existing securities are debt contracts $D = \{D_i\}_i$, the principal's value function is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^N D_i \mathbb{1}_{D_i < v(e_i)}$ under the strong δ-credibility constraint.*

The comparison with the case of full commitment illustrates the mechanism at work. Under full commitment, the principal will extract the full value of the asset: The principal can always punish the holdout by transferring the full value of the asset to the tendering agents. Instead, with limited commitment, the principal cannot credibly commit to punishing the holdout when doing so results in a lower payoff for the principal herself. This occurs whenever the holdout has a "small" debt claim on the asset, $D_i < v(e_i)$. In this case, given that the holdout gets paid in full,[45] any punishment can only be at the expense of the principal, and thus the commitment problem arises. Indeed, as shown in Definition 3, the payoff sensitivity of the principal is one: Any punishment results in a one-to-one drop in the value of her payoff. As a result of this commitment problem, the principal's payoff is reduced precisely by the quantity $\sum_{i=1}^N D_i \mathbb{1}_{D_i < v(e_i)}$. If, instead, the holdout is a "large" debt holder, $D_i > v(e_i)$, he will not be paid off in full (his payoff sensitivity is one). Now, the principal can credibly commit to punishing him precisely because her payoff is not affected by the punishment (the payoff sensitivity of the principal in Definition 3 is 0).

[45]To see why, remember that the principal wants to punish the holdout, but the only way to punish the holdout is to give more values to the tendering agents. However, doing so is even more costly given the 1-Lipschitz condition of the holdout's payoff. In addition, the principal is committed in renegotiation, so she can use the Extreme Gouging technique as in 2 and pay the tendering agents nothing. This is relaxed in the next section when we use a weaker requirement for credibility. Indeed, the principal may not need to pay a small creditor in full even if she can under the weaker credibility constraint.

The comparison illustrates the different treatments of bank debts versus public bonds in a typical restructuring evidenced in, say, James (1995). Small creditors (bondholders) often have stronger incentives to hold out and are more difficult to punish, so they typically receive preferential treatment, whereas large creditors (banks) internalize their pivotality and can be more credibly punished, so they often make a compromise. Earlier work explains the difference by focusing on the pivotality of large vs small creditors but this paper shows the credibility to punish is also a key determinant.

Example 2: Equity. Suppose now that the holdout A_i has an equity claim of share $\alpha_i < 1$. His payoff function is $R_i^O(w, e_i) = \alpha_i w$ with a dilution sensitivity of α_i everywhere. Thus, the maximum credible dilution is $\bar{x}(e_i) = 0$. No punishment is strongly credible! Any punishment imposed on the holdout would result in a loss for the principal. The reason is that punishing the holdout reduces his payoff only by $\alpha_i < 1$ whereas the payoff of the principal is instead reduced by $1 - \alpha_i > 0$ (see Def. 3). Therefore, the principal always wants to renegotiate in the presence of holdouts. Thus, a contingent offer cannot be better than using only cash. This rationalizes the absence of senior debt offering in takeovers despite the persistent high premium attached to many of them:[46] Contingent contracts cannot do better than cash.[47]

The next result illustrates how the principal's payoff varies with commitment under strong credibility when agents are endowed with equity.

Proposition 5. *When existing securities are equities $\alpha = \{\alpha_i\}_i$, the principal's value function on the set of strongly δ-credible contracts is $J(\mathbf{0}) = v(\mathbf{0}) - \delta \sum_{i=1}^{N} \alpha_i v(e_i)$, which is higher when the commitment is higher (δ is smaller).*

Start with the full commitment case, $\delta = 0$. In this case, the principal can extract the full value of the asset (see Proposition 2). Consider now the case of no commitment at all, $\delta = 1$. Then the principal has to give the holdout the share of the asset that he owns under the deviation, $\alpha_i v(e_i)$. Anywhere in between, the principal is able to capture $1 - \delta$ of the value of the agent's share of the asset. The reason discounting matters is because in effect, the more the principal cares about the future, which is when renegotiation occurs, the less she is committed to the present exchange offer. As a result the exchange

[46] Malmendier et al. (2016) finds that more 92% successful takeovers offer non-contingent contracts such as cash or the stock of the acquirer firm, with an average premium of 46.24%.

[47] Non-contingent contracts are also optimal in Rubinstein and Wolinsky (1992) and Segal and Whinston (2002) albeit for different reasons.

offer today needs to leave more to the agents the more the principal cares about the next round of renegotiation.

In fact the result that the payoff of the principal is decreasing in δ is more general than Proposition 5 may suggest. Under strong credibility, we can show the following

Proposition 6. *The principal's value function $J(\mathbf{0})$ on the set of strongly δ-credible contracts is weakly decreasing in δ for any existing contracts R^O.*

It is only "weakly decreasing" since in some cases, as when agents are endowed with debt, the value function is a constant function of δ as in Proposition 4.

A feature of the notion of strong credibility is that it assumes that the principal has little commitment in the initial proposal but is able to commit to the alternative proposal in the renegotiation stage. Empirically, it may be plausible to assume that the laws governing the on-path negotiation and off-path renegotiation are different, that the agents might not be sophisticated enough to anticipate a series of renegotiations, or that the principal may only be able to propose exchange offers during an exclusive window, as under the US bankruptcy code. Or renegotiation is in private as in Segal (1999).[48] Still, in cases such as sovereign debt restructuring, the ability of the principal to commit is the same irrespective of the renegotiation stage. In the next section, I consider a definition of credibility that considers this weaker form of credibility.

4.2 Credibility: A recursive definition

4.2.1 Credible Contracts: Definition, Existence, and Uniqueness

In this section, I refine the notion of a *credible* contract to be such that the principal can propose alternative contracts to replace the initially proposed one, but only if they are also *credible*. Its rationale and connection to the literature are discussed in Section 7.2. I begin by modifying the previously defined notion of strongly credible contracts as follows, which yields a bargaining protocol resembling Stole and Zwiebel (1996).

[48]Slightly differently, Segal (1999) assumes the principal cannot commit in the public offer, but when she deviates to privately renegotiate with a single agent, she can commit. Secrecy is not the main concern, as private renegotiation can be anticipated absent private information. The key difference is that the principal only wants to renegotiate the offer after some agents hold out in my model. I cannot preclude the incentive for her to deviate to a bilateral negotiation with a single agent: She always wants to do so given her ability to create super seniority at the expense of others. But this is usually forbidden by law.

Definition 6 (δ-Credible Contracts). *A contract R is a δ-credible contract for some $\delta \in [0,1]$ at an action profile h if and only if*

(a) it is incentive compatible for the agents at the action profile h, and

(b) at any unilateral deviation profile $\hat{h}$, it weakly δ-dominates all δ-credible contracts at $\hat{h}$

Similarly, $C^{\delta}(h)$, the set of δ-credible contracts at h, can be denoted by

$$C^{\delta}(h) := \left\{ R \in \mathcal{I}(h) : R \succeq_{\delta} \tilde{R} \text{ at } \hat{h} \quad \forall \tilde{R} \in C^{\delta}(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h) \right\}. \tag{14}$$

In the definition of strongly credible contracts, we considered renegotiation offers that are only incentive compatible, that is, $\tilde{R} \in \mathcal{I}(\hat{h})$. Now, instead, the renegotiation offers also have to "credible going forward", $\tilde{R} \in C^{\delta}(\hat{h})$.[49] Notice then that the set of strongly credible contracts is contained in the set of $\delta-$credible contracts, that is, $S^{\delta}(h) \subset C^{\delta}(h)$.

The set of $\delta-$credible contracts is defined recursively, and thus, issues of existence and uniqueness need to be addressed before continuing with the characterization of the problem. The next proposition confirms the existence and uniqueness of $C^{\delta}(h)$.

Proposition 7 (Existence and Uniqueness). *The set of δ-credible contracts $\{C(h)\}_h$ exists, it is non-empty and unique.*

An important result is that δ-credibility introduces an interesting non-monotonicity in the payoff of the principal as a function of the degree of commitment, δ. The intuition for this important result can be readily grasped in an example while the general characterization of δ-credible contracts is postponed until section 4.2.3..

4.2.2 Non-monotonicity of Commitment: A Numerical Example

In contrast to Proposition 6, when the principal can further deviate (under $\delta-$credibility), more commitment (lower δ) does not always benefit the principal. Two competing forces are at play. First, *conditional* on a fixed continuation payoff "tomorrow," stronger commitment improves the principal's payoff today. But, of course, tomorrow is not fixed: Stronger commitment also improves the principal's position in renegotiation tomorrow, which increases her payoff then, making her more likely to renegotiate

[49] It is shown in the Section **??** in the appendix that this notion is the limiting case when the number of rounds of renegotiation extends to infinite.

tomorrow. This, in turn, makes her less committed to punishing the holdout today *unconditionally*. More commitment can backfire if the second effect dominates. This curse of commitment is a recurrent theme in repeated games. For example, the same force also appears in the bilateral games in Pearce (1987) and Kovrijnykh (2013). This suggests the non-repeated multilateral interaction features similar dynamics to the repeated environment.

Example 4.1. *Consider a three-agent case, all endowed with equity claims. Let the equity share of A_i be $\alpha_i = 1/3$ for $i \in \{1, 2, 3\}$. The asset value is $5 + k$ if k agents tender for $k = 1, 2, 3$, and normalized to 0 when all of them hold out.*

We calculate the principal's value function using backward induction. When two agents, say A_1 and A_2, hold out, the principal can credibly give $6(1 - \delta)$ to the tendering agent A_3: She does not need to give anything to A_3 to tender because A_3 obtains nothing when he holds out; yet she can still give him some value $x_3 > 0$ through senior debt purely as a punishment on A_1 and A_2 without hurting herself. Why? The principal obtains $\frac{1}{3} \times (6 - 0) \times \delta = 2\delta$ if she renegotiates and offers 0 to A_3. (Recall that no punishment is always optimal.) Without renegotiation, she obtains $\frac{1}{3}(6 - x_3)$. Comparing the payoff in the two scenarios, the principal is not willing to negotiate if $x_3 \leq \bar{x}_3 := 6(1 - \delta)$. By symmetry, this is the maximum punishment the principal can impose on any two holdouts, and each holdout obtains $\frac{1}{3} \times (6 - \bar{x}_3) = 2\delta$.

Now, consider the case when only one agent, say A_1, holds out. The principal has to give A_2 and A_3 at least 2δ each. Suppose the principal initially promised to give A_2 and A_3 a total value of $x > 4\delta$. By renegotiating down to 4δ, she obtains $\frac{2}{3} \times \delta \times (7 - 2\delta \times 2)$. Without renegotiation, she obtains a value $\frac{2}{3}(7 - x)$. Comparing the two scenarios, the principal would not renegotiate if $x \leq 7 - 7\delta + 4\delta^2 \equiv (1 - \delta) \times 7 + \delta \times 2 \times 2\delta$. And the holdout would obtain a value of $(7 - (7 - 7\delta + 4\delta^2))/3 = (7\delta - 4\delta^2)/3$. Therefore, the principal can initially promise only to pay each agent $(7\delta - 4\delta^2)/3$ since this is the maximum payoff they each would obtain were they to hold out. The principal's value is thus $8 - 3 \times \frac{1}{3}(7\delta - 4\delta^2) = 8 - 7\delta + 4\delta^2$. Plotted out in Figure 1, the principal's value is decreasing in commitment (increasing in δ) when $\delta > 7/8$.

4.2.3 Credible Contracts: General Characterization

I derive, along with the proof of the existence and uniqueness, a recursive characterization of the solutions using the principal's value function and the maximum credible punishment. The challenge with directly solving the problem is that the dimensionality

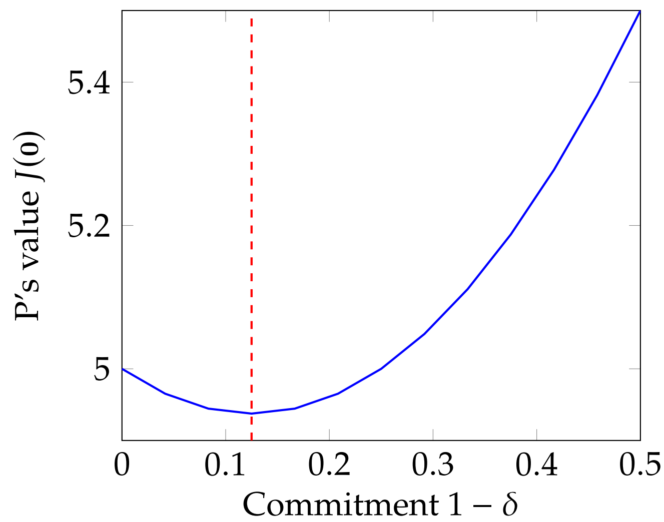

Figure 1: Principal's value function $J(\mathbf{0}) = v(\mathbf{0}) - \delta v_1 + \frac{2}{3}\delta^2 v_2$ when $v(\mathbf{0}) = 8, v_1 = 7, v_2 = 6$

of the contracting space is too large. This problem can be overcome by reducing the problem into a single-dimensional optimization problem, the maximum credible punishment on each holdout profile h.

Proposition 8. *The pair of vectors $\{J^*(h), \bar{x}^\delta(h)\}_{h\in\{0,1\}^N}$ is the pair of the principal's value function J^* and the maximum punishment $\bar{x}^\delta$ at each node h if and only if they satisfy the following recursive relation*

$$J^*(h) = v(h) - \underline{x}(h) - h \cdot R^O\left(v(h) - \underline{x}(h), h\right) \tag{15}$$

where

$$\underline{x}(h) := \sum_{i\in\xi(h)} R_i^O\left(v(h+e_i) - \bar{x}^\delta(h+e_i), h+e_i\right) \tag{16}$$

is the minimum punishment to implement h, and

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)\} \tag{17}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$.

The general characterization allows me to explicitly solve the takeover case for any initial holding structure. I first present a recursive characterization of the amount of credible punishment the principal can impose on each action profile. Then, I will provide a closed-form solution to this recursive equation, which provides an explicit formula for the amount of credible punishment using a contingent contract.

Lemma 3. *When $\{R_i^O\}_i$ are equity contracts, i.e., $R_i^O(w, h) = \alpha_i w$ for all h, the maximum credible dilution on the action profile h satisfies the recursive relation*

$$\bar{x}^\delta(h) = (1-\delta)v(h) + \delta \sum_{i \in \xi(h)} \alpha_i(v(h+e_i) - \bar{x}^\delta(h+e_i)) \quad \forall h \neq \mathbf{1} \tag{18}$$

with the initial condition $\bar{x}^\delta(\mathbf{1}) = 0$ if either $\sum_{i=1}^N \alpha_i = 1$ or $v(\mathbf{1}) = 0$.

The maximum credible dilution the principal can credibly impose at h, i.e., $\bar{x}^\delta(h)$, is a convex combination of the payoff she can credibly give to the tendering agents at h and the total asset value, weighted by the discount rate.

(a) The first term $(1-\delta)v(h)$ is the deadweight loss due to renegotiation: the size of the pie shrinks by $(1-\delta)v(h)$ whenever she wants to renegotiate, so she could impose at least that much to the holdout by paying the tendering agents.

(b) The second term is the sum of the discounted payoff to each tendering agent, which is as much as his holdout payoff. Since the principal has to pay at least what each tendering agent would receive if he holds out, she is not willing to renegotiate with them if the promised value is less than the discounted value of what the principal would otherwise have to pay each.

The initial condition says no punishment is feasible when everyone holds out i) if all agents hold all equity or ii) if the asset value is zero. Otherwise, if there's some third-party who holds a fraction of the firm and the asset is not worthless, then the principal is able to create some punishment when all agents hold out by diverting some asset value to this third-party agent.

When $\delta = 1$, i.e., the principal has the least commitment and can renegotiate at no additional cost, for each of the tendering agents, the maximum payoff that can be credibly promised to him is his contractual payoff from the asset value available to him when he deviates: $v(h+e_i) - \bar{x}^\delta(h+e_i)$.

The lemma gives a hint on the alternating structure of the punishment: A severer punishment upon further deviation would reduce the maximum credible punishment on path because each tendering agent A_i, if otherwise holding out, would receive a lower payoff due to a higher threat. This makes promising a higher payoff to A_i at h

less credible as the principal has a higher incentive to renegotiate. On the contrary, a higher asset value $v(h + e_i)$ on deviation profile $h + e_i$ would increase the maximum punishment at h as the tendering agents would get more if they hold out and hence must be compensated more at h.

Proposition 9. *For equity contracts, the maximum credible punishment on action profile h takes the following alternating form*

$$\bar{x}(h) = (1-\delta)v(h) + \sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{\sigma\in\Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)}\right) \tag{19}$$

where $\Sigma(\xi(h))$ is the set of all the permutations on $\xi(h)$. The highest payoff the principal can credibly obtain at $\mathbf{0}$ is

$$J(\mathbf{0}) = v(\mathbf{0}) + \sum_{k=1}^{N} \frac{(-\delta)^{k}}{(N-k)!} \sum_{\sigma\in\Sigma(\mathcal{N})} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(\sum_{s=1}^{k} e_{\sigma(s)}\right). \tag{20}$$

This result shows how contractual structure and the asset value at each k-step deviation profile $h + \sum_{s=1}^{k} e_{\sigma(s)}$ affects the maximum possible credible punishment at h. The first component $(-\delta)^{k+1}$ captures the alternating structure. Since we only to count the k-step deviation path from h once, the sum over all the permutations on $\xi(h)$ over-count the number of paths since it also includes all the paths further deviating from the k-step deviation profile, and the term $\frac{1}{(|\xi(h)|-k)!}$ is used to offset the repeated counting.[50]

I also derive the more complicated δ-credible contracts when debts are outstanding in Section **??** in the Online Appendix, which exhibits discontinuity and non-responsiveness. The special case illustrates even the non-monotonicity also depends crucially on the set of initial contracts.

[50] This is similar to the factorial in the Shapley value where all possible paths of length N are summed over. Differently, here we sum over all possible paths of length $N-k$ starting at a particular node with k tendering agents.

5 Property Rights

5.1 Modeling Property Rights

The previous analysis assumes full dilutability of all existing contracts. In reality, property rights protection[51] insulates them from being diluted: Secured debts are protected by the property rights of the collateral from subordination.[52] Holdouts in the land acquisition can always stick to the value of his house if he does not accept the offer.[53] Contractual rights only provide protection against the contracting party (the principal), whereas property rights also provide protection against everyone else (Ayotte and Bolton, 2011). This section aims to answer how the ability to solve holdout problems is affected by property rights protection. The main results are that higher property rights protection always makes restructuring harder under full commitment, but it can make restructuring easier under limited commitment. Nonetheless, for commonly used securities such as debt and equities, a small increase in protection always leads to a more difficult situation.

The simplest way to model property rights protection is to introduce an additional term $\pi_i \geq 0$ in each agent A_i's payoff, called the "property value", if he holds out. This term is independent of other agents' action and does not come from the value creation of the project.[54] That is, the utility at h, when the value distributed among holdouts is $v - x$, is $R_i^O(v - x, h) + \pi_i$.[55] And consequently the problem to implement h can be

[51] I do not discuss the optimal allocation of property rights here. Readers can resort to Segal and Whinston (2013) for classic references and Dworczak and Muir (2024) for recent works.

[52] Secured interest, though, can be diluted in DIP financing, for example, via *uptier transactions*. It is usually subject to court scrutiny, and obtaining approval is hard, albeit not impossible. In the milestone case LCM XXII Ltd. v. Serta Simmons Bedding, LLC, the debtor issued two tranches senior to its existing first-lien debts and the court confirmed its legality.

[53] There are subtle differences between the two types of property rights: In the latter case, the "house" is destroyed once the land owner accepts the offer, and the surplus is generated by allowing the developer to utilize a bigger chunk of the land; In the former, the "collateral" is released once the secured creditor accepts the offer. (Whether the new offer is secured by collateral doesn't matter since the value distribution is immediate.) However, they can be unified in modeling by viewing the unencumbered collateral as the value created from the exchange offer instead of the value of the old collateral. I will treat the properties as if they are "houses" in the general definition and show that it can include "collaterals" by normalizing the asset value.

[54] By assuming this, I exclude another layer of coordination problem when the property is owned collectively among the agents; or more complicated cases where a piece of collateral has multiple liens over it. Moreover, this formulation may not cover other types of investor protections that are state-contingent. For example, creditors insured by credit default swaps would get the additional payment only when the borrower defaults. Bolton and Oehmke (2011) identified an "empty creditor problem" when such protection.

[55] Note if the property is collateral and the value goes back to the firm when the creditor accepts the offer and is available to be paid to other agents, we could define an alternative value $\tilde{v}(h) := v(h) + (1 - h_i)\pi_i$ and replace the occurrence of v by $\tilde{v}$ in the formulation of the problem. We model this way because the notation

written as[56]

$$\max_{R\in C^{\delta}(h)} J(h|R) \quad \text{s.t.} \quad h_i \in \arg\max_{h_i'\in H_i} \left\{u(h_i'|h_{-i},R) + h_i'\pi_i\right\} \quad \forall i \in \mathcal{N} \tag{21}$$

I assume participation is efficient even when the properties are destroyed:[57]

Assumption A3 (Monotonicity with Property)**.** $v(h_{-i},0) > v(h_{-i},1) + \pi_i, \forall h_{-i}, \forall i \in \mathcal{N}$.

Similar to Proposition 2, the principal is extremely powerful by deploying contingency: She can extract all the value unprotected by the property rights by creating contractual externalities. Thus, higher property rights protection hinders restructuring.

Proposition 10. *With full commitment, greater property rights protection exacerbates the holdout problem. More specifically, the principal's value at* $\mathbf{0}$ *is* $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N}\pi_i$*, which is always decreasing in* π_i *for all* i.

Intuition is simple: The principal only needs to compensate each claim holder the amount of the property; the remaining claims can be diluted by the contractual externalities. Thus, more protection implies more compensation for the existing contract holder and lower value for the principal.

5.2 Greater Protection Facilities Restructuring: A Negative Example

I first construct an example showing that a higher property right protection could increase the principal's value, facilitating restructuring. Let there be 3 agents, each with a property value π_i and a claim that resembles a "kinked equity" (or debt if $\beta_i = 0$)

$$R_i^O(v,h) = \alpha_i v + (\beta_i - \alpha_i)(v - \hat{v}_i)\mathbb{1}_{v\geq\hat{v}_i} \quad \forall h : i \notin \xi(h) \tag{22}$$

for some parameters $\{\alpha_i, \beta_i, \pi_i, \hat{v}_i\}_i$. I find a set of parameters such that greater property rights protection facilitates restructuring in the next proposition.

Example 5.1 (Example: Property Rights Facilitates Restructuring)**.** *There exists a set of initial contracts such that a locally small increase in property rights protection facilitates*

is simpler.

[56]The definition of the credible contracts is the same except the additional term π_i in the agent's payoff of holding out in the set of incentive-compatible contracts. The existence and uniqueness of credible contracts with property rights protection can be proved, *mutatis mutandis*, similarly to Proposition 7.

[57]Note if we use the other notation as in footnote 55, this is simply monotonicity of $\tilde{v}$: $\tilde{v}(h_{-i},0) > \tilde{v}(h_{-i},1)$.

restructuring. In particular, let $\hat{v}_1 = \hat{v}_3 = 1, \hat{v}_2 = 98/100$, $\pi_1 = \pi_2 = 1/100$ *and* $\pi_3 = 99/100$, $\alpha_2 = 7/10, \alpha_1 = \alpha_3 = 1/10$, $\beta_1 = \beta_2 = 1/10, \beta_3 = 7/10$. *Let* $v(\cdot)$ *be such that* $v(\mathbf{1}) = 0, v(\mathbf{0}) = 3, v(e_i) = 2, v(\mathbf{1} - e_i) = 1$ *for all* i. *The principal's value function* $J(\mathbf{0})$ *is increasing in* π_1 *at the parameters specified above.*

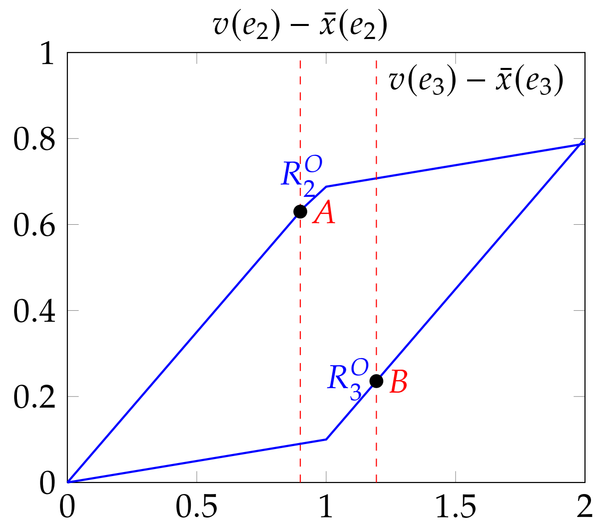

Figure 2: Example of Property Rights Facilitating Restructuring: Initial Contracts with Kinks

This example shows how property rights protection could facilitate the restructuring by giving the principal less bargaining power in renegotiation and, thus, more commitment to the punishment. The protection still undermines the principal's bargaining power initially, so the compensation off-path must exceed this direct effect. For this to be the case, the structure has to be made asymmetric as it's restricted by the 1-Lipschitz continuity. In my example, R_2^O (resp. R_3^O) has a large payoff sensitivity when the asset value accrues to the holdout is small (resp. large). When A_2 holds out, since π_3 is large, a one-dollar increase in π_1 would reduce A_2's payoff by α_2, i.e., the payoff sensitivity evaluated at point A. This is multiplied by a discount factor $1 - \alpha_3$, reflecting the renegotiation when A_3 also holds out. Similarly, when A_3 holds out, a one-dollar increase in π_1 would reduce A_3's payoff by $\beta_3(1 - \beta_2)$ since it is evaluated at the point B.

Despite the quirky example shown above, when the existing securities are the more commonly seen contracts, such as debts or equities, a locally small increase in property rights protection usually exacerbates the holdout problem even in the limited commitment case. Indeed, for equity holdouts, this is even true for large increases in property rights; whereas for debt holdouts, it can be reversed.

5.3 Effect of Property Rights with Equity Holdouts

In contrast, when existing contracts are equities, no matter the structure, a higher property rights protection never leads to an easier resolution of the holdout problem.

Proposition 11 (Property rights hinder equity restructruring). *For any equity contracts $\{\alpha_i\}_i$, the prinicpal's value $J(\mathbf{0})$ under δ-credibility for any $\delta \in (0, 1]$ is decreasing in π_i for all $i \in \mathcal{N}$.*

The result says that despite the countervailing forces that greater property rights protection bolsters her commitment, this indirect force will nonetheless not exceed the direct force that makes restructuring harder. The reason is that each indirect effect is weighted by the equity dilution sensitivities $\{\alpha_j\}_{j \neq i}$ which also sum up less than 1.

To see the force more clearly, let's look at a specific example. Let the existing contracts be equities $\alpha = \{\alpha_i\}_{i=1}^{3}$ such that $\langle \alpha, \mathbf{1} \rangle = 1$. And to simplify the exposition, I assume $\delta = 1, v(\mathbf{1}) = 0$.

Example 5.2 (Property rights hinder equity restructuring). *With limited commitment, the value function of the principal at $\mathbf{0}$ with equities outstanding is decreasing in each π_i,*

$$\frac{\partial}{\partial \pi_i} J(\mathbf{0}) = -\left(1 - \sum_{j \neq i, k \neq j, i} \alpha_i (1 - \alpha_k)\right) < 0 \quad \forall i. \tag{23}$$

The closed-form solution for the sensitivity of the principal's value to property rights protection illustrates the trade-off of the two forces. The direct effect is a one-to-one reduction in P's value and the indirect effect is summarized in the other term. This renegotiation channel is shadowed when equities are in place because higher protection of A_1 also makes punishing A_2 easier, but only at a rate smaller than 1: It is the equity sensitivity to the asset, α_2. Similarly, the effect of punishing A_3 is also dampened by the equity sensitivity α_3. Since the sum of all equity shares adds up to 1, the indirect effect is always smaller than one.

5.4 Effect of Property Rights with Debt Holdouts

The effect of property rights is more nuanced when existing securities are debt contracts. Any locally small increase in property rights protection always exacerbates the holdout problem, but a large increase could facilitate restructuring. I show the two effects in the next two propositions.

Proposition 12 (Property rights generically hinder debt restructruring)**.** *For any debts contracts $\{D_i\}_i$, the prinicpal's value $J(\mathbf{0})$ under δ-credibility for any $\delta \in (0,1]$ is generically locally decreasing in π_i for all i. That is, $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} < 0$ at any differentiable points.*

When creditors are protected by property rights, the force that makes renegotiation harder for the principal does not get transmitted to the initial bargaining due to the fact that a holdout creditor is either repaid in full or not at all. Thus, the effect of a small change in the protection that increases the punishment does not get transmitted from the off path renegotiation since the maximum credible punishment has a discontinuity and is flat in each region. Notice, however, that this effect only applies to a small increase in π_i away from the boundary.

Now, I show that when the existing contracts are debt, a non-locally-small increase in property rights protection could indeed facilitate debt restructuring. Let there be two agents: agent A_i has a debt value of $D_i = 1$ for all $i \in \{1,2\}$. The asset value is $v(\mathbf{1}) = 0$, $v(e_i) = 2$ for all i and $v(\mathbf{0}) = 3$. And for simplicity, we assume $\delta = 1$. For the property value, we focus on the region where $\pi_i \in [1/2, 3/2]$ for all i.

Proposition 13. *With limited commitment, the principal's value in the 2-creditor example is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{2}\left[D_i \mathbb{1}_{\{v(e_i) \geq \pi_j + D_i\}} + \pi_i\right]$. Given the parameters above, the principal's value increases when the property right of A_j increases from any value $\pi_j \in (1/2, 1)$ to any $\pi_j + \Delta\pi_j \in (1, 3/2)$.*

This result says the effect is different when a change in property rights is large enough to "switch the regime". When π_j is small, the principal needs to pay A_i in full if he holds out because she cannot credibly pay more to A_j to punish A_i. But when π_j is slightly larger, above the threshold, she can more credibly pay A_j to punish A_i, which reduces her initial compensation to A_i.

These results echo the finding that higher creditor protection could facilitate or hinder restructuring in Donaldson et al. (2020). Both non-monotonicity stems from the principal's lack of commitment: She cannot commit to a renegotiation policy here and to a bankruptcy filing policy in theirs. Here, higher property rights protection of the creditors has a direct effect of making the restructuring harder but an indirect effect of making the principal more credible when punishing other creditors. In theirs, a more creditor-friendly policy has a direct effect of making priority more attractive but an indirect effect of making a bankruptcy filing less likely, reducing the appeal of priority.

6 Literature

The extensive literature on the holdout problems largely focuses on specific contractual forms of both existing and newly offered contracts and usually evades the commitment issue. Grossman and Hart (1980) first studies the holdout problem in the takeover case where a raider offers cash to buy equity shares from a continuum of shareholders. The holdout problem exists in this context because the atomic shareholders do not internalize the externality created by their free-riding. This assumption was relaxed by Holmström and Nalebuff (1992) and Bagnoli and Lipman (1988), who paid more attention to non-atomic shareholders in mixed strategy, which made solving holdout possible. Other papers also try to solve the problems by relaxing some constraints in the original setting. Shleifer and Vishny (1986) consider the case with a large shareholder and show it significantly alters the outcome because the large shareholder can split the gain from the takeover between its shares and the raider's. But it works only because commitment is implicitly assumed. Similar assumptions are also made in Hirshleifer and Titman (1990). Burkart et al. (2014) studies how legal protection affects the bidding strategy in takeovers. Burkart and Lee (2022) compares free-riding à la Jensen–Meckling in activism vs. free-riding à la Grossman–Hart in takeovers. Gertner and Scharfstein (1991), Bernardo and Talley (1996) and Donaldson et al. (2020) demonstrate that offering priority in exchange offers via senior debt could offset the incentive to free-ride as priority dilutes existing creditors' payoff. Yet, none of the existing papers consider the credibility of offering priority. Sovereign restructuring differs from corporate as there's no formal seniority structure, and there is a greater commitment issue.[58] Bulow et al. (1988); Bulow and Rogoff (1989) study the limit of sovereign bond buyback using cash due to the holdout problems. Kletzer (2003) finds that in a dynamic setting, the principal benefits from a collective action clause as it facilitates bargaining, while a unanimity rule leads to a war of attrition and inefficient outcomes. The difference is that each lender can propose to the borrower in their model. Pitchford and Wright

[58]Here, I omit many macro models on sovereign debt that do not address the holdout problem.

(2012) also studies the case when a sovereign can renegotiate with each creditor one by one and has no commitment. Bolton and Scharfstein (1996); Bolton and Jeanne (2007, 2009) discuss the ex-ante vs. ex-post trade-off of making some classes of bonds difficult to restructure. There are also extensive discussions in land assembly, e.g., Kominers and Weyl (2012).

The paper falls broadly in the literature of mechanism design with limited commitment, with two notable distinctions. Most papers studying the limited commitment of the principal in mechanism design, such as Bester and Strausz (2000), Bisin and Rampini (2006) and Doval and Skreta (2022), focus on the issue of information leakage: The principal cannot commit not to use the information the agents reported, and hence the revelation principle might no longer hold when the principal lacks commitment. But my model is a complete information setting. Another feature of the model is that the outside option is endogenous and depends on the Principal's contracts. Similar feature is also presented in literature on the dissolution of partnership (Cramton et al., 1987), ratifiable mechanism (Cramton and Palfrey, 1995), contracting with externality literature (Jehiel et al., 1996) and partial mechanism design (Loertscher and Muir, 2022).

To the best of my knowledge, there is no paper associating the credibility of contracts with its dilution sensitivity. The non-monotonicity of commitment was derived in a bilateral repeated lending with a similar intuition: "just as commitment increases the lender's payoff in an optimal equilibrium, it increases his payoff from the most profitable deviation."(Kovrijnykh, 2013, p.2850) Higher commitment can backfire for a different reason: Donaldson et al. (2020a,b) proxies commitment with pledgeability (Proposition 1) and collateralizability (Proposition 4) and shows that both might lead to lower ex-ante payoff: higher pledgeability might hurt borrowers due to excessive power to dilute initial creditor at the interim financing stage, leading to the impossibility of lending ex ante; higher collateralizability could harm borrowers by over-collateralization, which leads to impossible interim financing.

7 Discussions

7.1 Discussions of Assumptions

Asset Value Microfoundation The paper assumes the asset value is decreasing in the holdout profile but is silent about why. I present several canonical ways of microfounding this assumption here, based on agency theory, costly default, and liquidity injection.

Imagine first the case of takeovers where each initial shareholder has a share of α_i. After acquiring the firm, the raider could exert an effort $e \in \mathbb{R}_+$ to improve the asset value from 1 to e, which incurs a quadratic cost e^2. Given the holdout profile h, the raider has a fraction $1 - h^\top \alpha$, and he optimally chooses the effort to maximize his payoff from his equity shares, i.e.,

$$\max_e (1 - h^\top \alpha)e - e^2. \tag{24}$$

The optimal effort and the corresponding asset value is $v(h) = e^* = 1 - h^\top \alpha$, a decreasing function of h as I assumed earlier.

Imagine a debt restructuring case where each creditor A_i holds a debt with a face value of D_i. There is an underlying asset whose value e follows a distribution G, independent of the capital structure. There is a chance for the firm to file bankruptcy, which destroys a fraction $1 - \lambda$ of the asset value, but the firm can obtain a fraction β of the remaining asset value. So the firm files if and only if $\lambda\beta e \geq e - h^\top D \iff e \leq (1 - \lambda\beta)^{-1} h^\top D$. The expected value before the underlying asset value realization is thus

$$v(h) = \mathbb{E}[e] - \int_0^{(1-\lambda\beta)^{-1} h^\top \boldsymbol{D}} \lambda v \mathrm{d}G(v). \tag{25}$$

This function is more complicated and non-linear but is also a decreasing function of h.

In a DIP financing scenario, the firm offers securities to existing creditors in exchange for liquidity injection. Let l_i be the liquidity the A_i injects into the firm and $\iota = (l_1, \cdots, l_N)$; then the asset value would be $v(h) = v(\mathbf{0}) + (\mathbf{1} - h)^\top \iota$ which is a linear decreasing function of h.

Binding Voting Mechanisms One crucial restriction the model assumes is that no agents are subject to a binding decision made by the majority or supermajority, which is the essence of the holdout problems. This, in many ways, reflects the reality: Typically, such non-consensual decisions are illegal in the US legal environment as they violate the Trust Indenture Actio 316 (b). For takeovers, even though such binding decisions can be made by the board or via the shareholders' voting, the dissenting shareholders still have the option to litigate against the board in violation of their fiduciary duty. In sovereign debt markets, the use of the collective action clauses does not always solve the issue. It is not obvious whether such provisions are desirable as they might infringe on the rights of some minorities when there is substantial heterogeneity among the agents. The sovereign world started with a two-limbed procedure: only allowing binding decisions within each class of the bonds, and they failed to address the holdout issues. A sweeping one-limbed aggregation mechanism could help to facilitate cramming down the dissenting shareholders but faces a bigger risk of being abused. For example, the Pacman strategy and redesignation[59] has been used to achieve a coercive restructuring in practice.

7.2 Discussion of Renegotiation Protocol

Sequential Renegotiation An alternative way to model multilateral bargaining is to specify a sequential protocol. There are multiple ways to specify an extensive game in which bargaining or renegotiation occurs sequentially: i) Shaked's unanimity game, where players propose in order, and any player can veto. The problem with this is that it has many perfect equilibria, and any feasible agreement can be implemented; ii) Legislative Bargaining models where proposers are randomly selected and a binding decision can be confirmed by a less-than-unanimous consent. This approach is plagued with impossibility results like the Condorcet paradox and that the majority core can be empty (Eraslan and Evdokimov, 2019); iii) The exit games considered in Lensberg (1988) where any agent satisfied with his share can leave the bargaining table. This approach requires the consistency axiom I employed in this paper. Krishna and Serrano (1996) showed that the equivalence between Nash's axiomatic solution and Rubinstein's alternating bargaining model extends to the multilateral case given this consistency axiom.

Given the empirical observations that in the holdout problems, there is usually a

[59]See https://theemergingfrontier.com/home/re-designing-pacman and the article on Financial Times https://www.ft.com/content/2b523aa2-402e-4060-8461-969a2132c483.

single entity with the exclusive right to propose and the theoretical consideration that dynamic games either cannot provide a sharp prediction or are equivalent to a static axiomatic one, I adopt the static approach with a possible dynamic game embedded in the credibility condition for simplicity.

Regenotiation-Proofness The most commonly used notion is the two-sided renegotiation-proofness: That is, the principal cannot propose an alternative contract that Pareto dominates the current one, i.e., nobody objects to the alternative offer, and some agent or the principal is strictly better off under this new hypothetical offer. But such a requirement would be too strong as it can be difficult to achieve in reality for various reasons. For example, i) the holdouts typically are tough to handle, and they usually are not negotiated away so the size of the pie can not be increased, and ii) some laws may prohibit preferential treatment of the holdouts. E.g., in takeovers, the *best-price rule*, or sometimes called *all-holders rule* or *Rule 14D-10*.[60] Therefore, I confine the alternative proposals to the contracts that are incentive compatible with the deviation profile, i.e., that the tendering agents still have an incentive to tender under the potential alternative proposal, and the holdouts are not enticed to tender.

Put differently, similar to Hart and Tirole (1988) I am implicitly assuming that the principal can unilaterally renege on the proposed offer whenever any agent deviates, and no agent can hold her accountable. Otherwise, the principal can threaten to give the entire firm to a tendering agent, who would block any alternative offer. Consequentially, the full-value extraction in Proposition 2 would be credible. Readers can also view the protocol in the model as if the principal called off the entire deal and re-proposed an new deal to the tendering agents so that the old proposal does not constrain her.

Renegotiation with Tendering Agents In the potential renegotiation and the formal definition of credibility in Section 4, the renegotiation protocol I laid out on possible punishments via "dilution" is effectively a renegotiation with the tendering agents instead of with the holdouts. It's meant to capture the principal's lack of commitment to the punishment.

[60]This is Code of Federal Regulations §240.14d-10, which can be traced back to the 1968 Williams Act Betton et al. (2008). See `https://www.law.cornell.edu/cfr/text/17/240.14d-10`. SEC also provides a detailed discussion of this rule and possible exemptions in *17 CFR PARTS 200 and 240*. See `https://www.sec.gov/rules/final/2006/34-54684.pdf`

Empirically, holdouts are usually not easily renegotiated away and they extract significant value from sticking to their initial contracts. As mentioned above, holdouts in Greek debt restructuring are paid in full. In *Elliott Associates, L.P. v. Banco de la Nacion and The Republic of Peru*, the holdout creditor purchased bonds with a total face value of 21 million for 11 million and received 58 million in settlement for the principal and accrued interests. Moreover, renegotiation with the holdouts could be illegal. In applications like takeovers, providing additional compensation to the holdouts would violate the best-price rule (Exchange Act Rule 10d-10, see 17 CFR § 240.14d-10 - Equal treatment of security holders.).

Will renegotiation with the holdouts alter the outcome? Not without new information: The principal will offer the same; otherwise, she would have already offered it initially. Indeed, how credibly the principal can punish the holdout is determined by the renegotiation with the tendering agents, not with the holdout. Thus, whether we allow for an explicit renegotiation with the holdout would not alter the outcome.

7.3 Discussions of Empirical Relevance

Relevance of the Holdout Problems Despite many attempts to solve the holdout problems at the institutional level, they remain of first-order concern in all aspects of the economy. In the sovereign bond restructuring case, the IMF proposed adding Collective Action Clauses (CACs) to the new issuance. It has been proven effective in solving the holdout problems within series but not across series (Gelpern and Heller, 2016; Fang et al., 2021). Also, there is a bulk of existing sovereign debts without it. *Squeeze-out* procedures are adopted for takeovers in both the US and EU, which allow the acquirer to gain the full stake of the target when she obtains a majority stake, thus "squeezing out" the holdouts. But the legitimacy has been contested and the holdout can resort to legal remedies such as *"action of avoidance"* and *"price fairness"*.[61] Similarly, the once-popular two-tier tender offer[62] also received great legal challenges. Moreover, the possibility of litigation also restores the incentive to hold out. In urban development,

[61] More discussions in Müller and Panunzi (2004), Broere and Christmann (2021) and Burkart and Lee (2022).

[62] A two-tier tender offer typically offers a high price to purchase shares until the raider obtains a controlling stake and purchases the remaining shares at a lower price. A similar practice is a partial tender offer where the raider only buys a fraction of outstanding shares. Both create a coercive force for the shareholders to tender. The main form of tender offers now is any-and-all, where the bidder promises to buy any shares of the target firm.

eminent domain, which allows the government to expropriate private property for public use, plays a major role in solving holdout problems but is still controversial and incites a constitutional debate related to the Takings Clauses of the Fifth Amendment, whether a private party can benefit from the infringement of property rights after the Supreme Court extended its use to private companies in *Kelo v. New London* . In other jurisdictions, for example, in Colombia, where the legal system follows a civil law tradition, Holland (2022) documented strong property rights protection worsens the holdout problems and curbs city development. In land acquisition for oil drilling, the "*rule of capture*" allows the oil drilling companies to acquire the land adjacent to the holdout block and utilize the oil extracted from a common pool, weakening the bargaining power of the holdout and strengthening the tendering land owners. Yet, the adoption of these legal theories varies across states. For example, in Texas, the land owner has a *possessory interest* in the substances beneath the land. In *Geo Viking, Inc. v. Tex-Lee Operating CO*, the Supreme Court of Texas has ruled a fracture across the property line, as a result of *fracking*, a subsurface trespass. Therefore, a better understanding of the holdout problem and its private solutions would still have first-order relevance in the current state.

Empirical Relevance of Limited Commitment The key assumption, limited commitment, is reflected in a multitude of empirical evidence, e.g., in Pitchford and Wright (2012). It is well-documented that sovereigns lack the commitment to debt repayment, new debt issuance, and, in particular, to the negotiated outcome due to both the doctrine of sovereign immunity and the lack of a statutory regime. For example, Argentina filed with the SEC not to pay anything to the holdout creditors in 2004 and passed the Lock Law not to reopen a new exchange offer in 2005. Yet, Congress suspended the Lock Law in 2009, and the government offered a new exchange offer in 2010. In the Greek debt crisis, Greece opted to pay 435 million euros ($552 million) to the holdout creditors in full in order not to trigger the cross-default clauses and be dragged into litigation, even though it announced in the earlier exchange offer that the holdout would not get anything. Meanwhile, the majority (97%) of the tendering creditors only received cents on the euro.[63]

[63] See https://www.reuters.com/article/us-greece-bond/in-about-face-greece-pays-bond-swap-holdouts-idUSBRE84E0MY20120515

Legality of Certain Solutions One may wonder if the solutions I mentioned earlier, e.g., the unanimity rule and the consent-payment-like contracts in Proposition 2, are feasible in the current legal environment. Right now, there do not seem to be any laws prohibiting the use of unanimity. In takeovers, typically, the acceptance of the tendered shares is "contingent on the delivery of a certain number of shares" (Cohen, 1990, p.116), which can be set to 100%.[64] Indeed, it's already suggested in the optimal threshold result in Holmström and Nalebuff (1992). In addition, despite that the bidder is obliged to complete the deal (Afsharipour, 2010), the raider could nonetheless include a *bidder termination provision*[65] which gives the raider a real option to terminate the transaction at a fee to implement the unanimity rule.

In the Extreme Gouging result, the principal needs to pay the tendering agents a lot when someone holds out. One practical concern is that it would be considered "fraudulent conveyance" when the firm pays certain creditors too much to avoid paying other known creditors (See 11 U.S. Code § 548). But this only applies i) when there is an imminent bankruptcy and ii) if the payments exceed the face value of the liabilities, not market value. Since in bankruptcy, the firm's assets are not enough to pay off all creditors in full; it is also unlikely to exceed the total debt of the tendering creditors when one holds out. It is generally not a concern in practice for distressed exchange offers. Moreover, this notion is only defined for debt, not other contracts.

Another concern is whether such offers violate certain covenants, such as the pari-passu clause and fair-dealing/good-faith provisions. Pari-passu clauses are unlikely to be violated as the offers the principal proposed here *is* symmetric: The allocation is only asymmetric after some creditors reject the offer, which is the case for any other offers. Traditionally, the clause is also interpreted in a very narrow sense: Ratable payment, prior to an innovative reading by the Brussels Court of Appeal in *Elliott Associates, L.P. v. Banco de la Nacion* that prevented Chase Manhattan from facilitating the interest payment of Peru's Brady bond. Typically, in a sophisticated court like the

[64] Grossman and Hart (1980) argues the absence of unanimity is due to the sleepy investor problem. We are not particularly concerned with the issue of inability to find all the agents as most takeover offers are widely publicized (Cohen, 1990) and in other cases, for example, in sovereign debt restructuring, the holdouts are usually big well-known players, such as hedge funds (known as vulture funds), e.g., Elliot Investment Management in the sovereign debt restructuring of Argentina, Peru, Panama; Oppenheimer, Franklin, and Aurelius Capital Management in Puerto Rico's debt crisis; Dart Management in the sovereign debt crisis of Brazil, Argentina, and Greece.

[65] The bidder also has a fiduciary termination right, which allows the raider to terminate when itself receives a takeover offer, and a regulatory termination trigger when it fails to pass the antitrust review, both without recourse.

New York court, the judge would interpret any arrangement consistent with the text of the contracts as good faith, even when it looks exploitative to outsiders.

Dilutability of Existing Securities It's also implicitly assumed in the baseline model that all the existing securities are dilutable, e.g., via senior debt. One might argue this is not feasible when the existing contracts are secured by collateral (e.g., secured corporate debt), or when there's no *de jure* seniority structure, for example, in sovereign debt. For the former, secured debt can sometimes be diluted in bankruptcy through *priming lien*, typically in Debtor-In-Possession (DIP) financing to raise new liquidity under Section 364(d). It's a lien on the pre-petition collateral that is senior to all existing liens, and the DIP lenders would be paid ahead of other creditors secured by the same collateral. Moreover, the firm in bankruptcy is also allowed to use *roll-up provisions* to draw the DIP financing to repay some of the creditors' (usually DIP lenders') pre-petition indebtedness, converting these debts to post-petition supersenior debt.[66] For the latter, despite the lack of a formal bankruptcy regime, sovereign debts issued under foreign law sometimes have priority under the judge's discretion. In the *Elliot Management vs. Argentina*, the Southern District of New York court judge Thomas Griesa issued an injunction preventing the Bank of New York Melon from forwarding the payment to the restructured creditors before paying the holdouts. This injunction would also prevent payment to the creditors or the underwriter in case of any new borrowing, creating a *de facto* seniority of the holdout's debt. Currently, New York is considering a bill to rule with sovereign,[67] which effectively lowers the seniority of the holdouts' debt. Even absent foreign law, Bolton and Jeanne (2009) pointed out the possibility of diluting debts that are easy to restructure, such as bank loans, with debt difficult to restructure, e.g., bonds. And Schlegl et al. (2019) finds sovereigns implement a *de facto* seniority by selectively defaulting on certain creditors.

[66] *Up-tier exchanges* and *drop-down transactions* are also similar tools commonly used in DIP financing to gain priority.

[67] The Assembly Bill A2970 can be found here `https://www.nysenate.gov/legislation/bills/2023/A2970`, and it has received a strong rebuttal from Credit Roundtable, ICMA, IIF, ICI, ACLI, LICONY.

8 Conclusion

Economic transactions that take the form of exchange offers are plagued with holdout problems, a phenomenon in which the incentive to free-ride on other agents' participation impedes efficient actions. The holdout problem is pervasive in all aspects of the economy, like takeover, debt restructuring, etc.

Despite being studied for over four decades, it has not been widely acknowledged that a meaningful discussion of the holdout problems requires relaxing both the contracting space and the commitment assumption. It is generally understood that the lack of commitment makes solving the holdout problem harder, but the extent to which it exacerbates the problem remains underexplored. Similarly, studying the general mechanism without allowing for limited commitment was unfruitful: The holdout problem is too easy to solve when commitment is not a problem.

The paper looks into the role of commitment in the holdout problems and uncovers two effects: First, it will interact with the dilution sensitivity of the initial set of contracts and determine the credibility of the dilution mechanism and, in turn, the optimal exchange offer; second, the commitment has a non-monotone effect through the renegotiation channel.

With full commitment, the holdout problem can be easily solved using a contingent contract that requires unanimous consent; with limited commitment, the effectiveness of contingency is undermined: When the securities in place are equity-like, not fully sensitive to dilutions, contingent securities cannot do any better than a non-contingent contract like cash. The model explains why senior debts, so dominantly used in debt restructuring, are not seen in the takeover.

The non-monotonicity of the commitment has been presented in the literature in a repeated setting but the paper points out that the same force also emerges in static multilateral bargaining with potentially sequential deviations. And following the intuition in this renegotiation channel, it is not so clear giving more property rights protection to the current claim holders would help them since such protection can be weaponized against another claim holder, making the principal more committed to the dilution.

Despite the progress, many remain to be uncovered. The paper shied away from all issues of private information and an emerging concern in the sovereign debt market is the lack of transparency between the debtor and non-Paris club creditors. A dynamic multilateral bargaining with private information will inevitably require solving an *interim informed principal* problem and that is a challenge for the future work.

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A Proofs for Section 3 (Optimal Exchange Offer with Full Commitment)

Proof of Proposition 1. We need to show that there's no SPNE in which the principal can find an non-contingent cash transfer $t_i(h_i)$ and every agent accepts. Suppose such an equilibrium exists, for A_i not to hold out, it must be that

$$t_i(0) \geq t_i(1) + R_i^O(v(e_i), e_i) \tag{26}$$

and since lowest possible payment when A_i holds out is $t_i(1) = 0$, the principal has to pay at least $t_i(0)_{\min} := R_i^O(v(e_i), e_i)$. But under Assumption A1, the principal has a negative profit after the value improvement and she is better off not initiating the transaction. So this cannot be an equilibrium. □

Proof of Proposition 2. Consider the following offer: If $\xi(h) = \mathcal{N}$, $R_i(v(h), h) = \frac{\varepsilon}{N}$. If instead $\xi(h) = \emptyset$, we let $R_j(v(h), h) = v(h)\mathbb{1}_{\{j=j^*\}}$ where $\underline{j} := \min\{i \in \mathcal{N} : v(\mathbf{1} - e_i) > 0\}$ and $j^* := \min\{i \in \mathcal{N} : i \neq \underline{j}\}$. If $\xi(h)$ is neither $\emptyset$ nor $\mathcal{N}$, we let $R_j(v(h), h) = \frac{v(h)}{|\xi(h)|}\mathbb{1}_{\{j \in \xi(h)\}}$.

Clearly, $h = \mathbf{0}$ is an equilibrium: If $A_i$deviates to 1, his payoff will be reduced to 0 from ε/N, so no one wants to deviate. To see why $h = \mathbf{0}$ is unique, first, let's check $\mathbf{1}$ is not an equilibrium: only A_{j^*} gets the full project value $v(\mathbf{1})$ while others get nothing. It's strictly profitable for agent $A_{\underline{j}}$ to accept since this deviation would result in an increase of his payoff by $v(\mathbf{1} - e_{\underline{j}}) > 0$. Second, let's consider an action profile h such that $\xi(h) \neq \mathcal{N}$, then for any agent $i \notin \xi(h)$, he gets 0 in the action profile while deviating to $h_i = 0$ would give him a positive payoff $\frac{v(h_{-i},0)}{N - |\xi(h_{-i},0)|} > 0$. □

B Proofs for Section 4 (Optimal Exchange Offer with Limited Commitment)

Proof of Lemma 1. Let $J(h|\mathcal{R})$ be attained at $\hat{R}$ with $\hat{x} = x(\hat{R}) \equiv \sum_{i \in \xi(h)} \hat{R}_i(v(h), h)$. We have $g(v(h)) = 0 \leq \delta J(h|\mathcal{R})$ and $g(\hat{x}) = J(h|\mathcal{R}) \geq \delta J(h|\mathcal{R})$. So by continuity of g, a solution to the equation $g(x) = \delta J(h|\mathcal{R})$ must exist. In addition, the pre-image of a continuous function $g^{-1}(\delta J(h|\mathcal{R}))$ is a closed set, and it is also bounded in $[\underline{x}(h), v(h)]$, so it is a compact set and the supremum can be attained.

By Rademacher's theorem, Lipschitz continuity of $h \cdot R^O(\cdot, h)$ implies that g is absolutely continuous and differentiable almost everywhere, and we take the left first-order derivatives $g'(x) = -1 + S(v(h) - x; h) \leq 0 \quad a.e.$, so the function g is weakly decreasing. Therefore, $\mathcal{X}$ must be a closed and connected set, which can only be a closed interval or a singleton. The optimality implies $g'(x) = 0$ and thus $S(v(h) - x; h) = 1$ almost everywhere. And since $S(\cdot; h)$ is a derivative, which cannot have jump or removable discontinuity by Darboux's theorem, and is a constant almost everywhere, it must be a constant everywhere.

Again, the continuity of g implies $g^{-1}([\delta J(h|\mathcal{R}), +\infty))$ is a closed set bounded in $[\underline{x}(h), v(h)]$ so the supremum can be attained. And since g is weakly decreasing, the supremum is attained when $g(x) = \delta J(h|\mathcal{R})$. Moreover, the equation $g(x) = \delta g(\hat{x})$ can be rewritten as

$$\int_{\hat{x}}^{x} g'(s)\mathrm{d}s + (1 - \delta)g(\hat{x}) = 0 \tag{27}$$

Using the fact that $g(\hat{x}) = J(h|R)$ and $g'(s) = -S_0(s; h)$ we have

$$(x - \hat{x})S_0([v(h) - x, v(h) - \hat{x}]; h) = (1 - \delta)J(h|\mathcal{R}). \tag{28}$$

And taking $x = \bar{x}(h; \mathcal{R})$ we have the characterization in the lemma.

Lastly, when $\delta = 1$, the RHS $(1 - \delta)J(h|\mathcal{R})$ is zero so at least one of the two terms on the LHS is zero. In particular, if $\bar{x} > \underline{x}$, $S([v(h) - \bar{x}, v(h) - \underline{x}]; h) = 1$ and since $S(v(h) - x; h) \leq 1$ by 1-Lipschitzness and thus it must be identically 1 everywhere, (a derivative cannot be less than 1 on a set of measure zero) which implies $\bar{x}(h; \mathcal{R}) \leq \inf\{x \geq \underline{x}(h; \mathcal{R}) : S(v(h) - x; h) < 1\}$. But it cannot be strictly smaller since we could otherwise increase the dilution, so it has to be equality.

□

Proof of Lemma 2. I first construct an incentive compatible contract $\tilde{R}$ that delivers a payoff of $v(e_i) - R_i^O(v(e_i), e_i)$ to the principal. The construction is similar to that in

Proposition 2. Let $\tilde{R}_i(v(h), h) = 0$ for all h and for $j \neq i$,

$$\tilde{R}_j(v(h), h) = \begin{cases} \varepsilon/N & \text{if } h = e_i \\ 0 & \text{if } h = \mathbf{1} \text{ or } \mathbf{0} \text{ or } (h \neq e_i, \mathbf{1}, \mathbf{0} \text{ and } j \notin \xi(h)) \\ \frac{v(h)-\varepsilon}{|\xi(h)|} & \text{if } h \neq e_i, \mathbf{1}, \mathbf{0} \text{ and } j \in \xi(h) \end{cases}. \tag{29}$$

I will now show that with this proposal, for sufficiently small $\varepsilon \geq 0$, e_i is an equilibrium, and when $\varepsilon > 0$ and $R_i^O(\cdot, h)$ has a strictly positive right derivative at 0 for all h, the equilibrium is unique.

- For agent A_i, as long as $h \neq e_i$,$\mathbf{0}$ or $\mathbf{1}$, the total payment to the tendering agents is $v(h) - \varepsilon$ tendering results in a payoff of 0 while holding out yields a payoff of $R_i^O(\varepsilon, h)$, so holding out is strictly better if $\varepsilon > 0$ and $R_i^O(\cdot, h)$ has a strictly positive payoff. When everyone else holds out, holding out yields a payoff of $R_i^O(v(\mathbf{1}), \mathbf{1})$ while tendering gives him nothing.
- For any other agent A_j, non-tendering gives a payoff of zero, and tendering gives a payoff of either ε/N if everyone else other than A_i tenders, or $\frac{v(h)-\varepsilon}{|\xi(h)|}$ otherwise, which is positive for sufficiently small $\varepsilon > 0$.

Thus, we proved $R^{O|R}$ is incentive compatible with e_i.

For any arbitrary contract $\hat{R} \in \mathcal{I}(e_i)$, let $x(e_i; \hat{R}) = \sum_{k \in \xi(e_i)} R_k(v(e_i), e_i)$ be the payment to the tendering agents and thus the total payment is $x(e_i; \hat{R}) + R_i^O(v(e_i) - x(e_i; \hat{R}), e_i)$.

Suppose the principal wants to find another contract $\hat{R}$ to minimize the total payment. Under Assumption A2, $R_i^O(\cdot, e_i)$ is weakly increasing and 1-Lipschitz, by Lemma 1, the solution to the minimization problem above is obtained at $x = 0$, which is achieved by $\tilde{R}$ when $\varepsilon = 0$. And the principal obtains a payoff of cannot obtain a higher payoff than $v(e_i) - R_i^O(v(e_i), e_i)$. □

Proof of Proposition 3. To prove this result, I first show that when $S_i(v(e_i); e_i) < 1$ for all$i$, the contract R is strongly credible if and only if the off-path punishment at e_i is $x(e_i) := \sum_{j \neq i} R_j(v(e_i), e_i) = 0$. Then, I calculate the value function of the principal and show that it equals the value function when offering cash. Lastly, I show that the principal can do strictly better when the condition $S_i(v(e_i); e_i) < 1$. is violated.

First, from Lemma 2, we know that at the deviation profile e_i the principal was able to obtain $v(e_i) - R_i^O(v(e_i), e_i)$ using an incentive compatible contract. Therefore, the credibility constraint at e_i is

$$v(e_i) - \sum_{k \neq i} R_k(v(e_i), e_i) - R_i^O \left(v(e_i) - \sum_{k \neq i} R_k(v(e_i), e_i), e_i \right) \geq \delta \left[v(e_i) - R_i^O(v(e_i), e_i) \right] \quad (30)$$

Rearranging the terms, we obtain

$$x(e_i; R) + R_i^O(v(e_i) - x(e_i; R), e_i) \leq (1 - \delta) v(e_i) + \delta R_i^O(v(e_i), e_i) \quad (31)$$

where $x(e_i; R) = \sum_{k \neq i} R_k(v(e_i), e_i)$. When $\delta = 1$, using Lemma 1, the unique solution is $x(e_i; R) = 0$ when the first partial derivative $R_i^O(v(e_i), e_i)$ is strictly smaller than 1 at $v(e_i)$. Since any punishment would be renegotiated away and the holdout would be paid $R_i^O(v(e_i), e_i)$, in order to persuade the agent to tender, the principal has to pay at least this much to A_i, leaving at most $v(\mathbf{0}) - \sum_{i=1}^N R_i^O(v(e_i), e_i)$ to the principal, which is equivalent to offering cash. This is lower than c under Assumption A1; therefore, the restructuring plan is infeasible. □

Proof of Proposition 4. To prove this, we first show that the maximum credible punishment is $\bar{x}^\delta(e_i) = (1 - \delta)(v(e_i) - D_i)$. This is obtained by finding the maximum x such that

$$x + \min\{v(e_i) - x, D_i\} \leq v(e_i) - \delta \left[v(e_i) - \min\{v(e_i), D_i\} \right] \quad (32)$$

When $v(e_i) \leq D_i$, the RHS is simplified to $v(e_i)$, while the LHS is always smaller than $v(e_i)$ as $x + \min\{v(e_i) - x, D_i\} \leq \min\{v(e_i), D_i + x\} \leq v(e_i)$. So the maximum punishment is $\bar{x}^\delta(e_i) = v(e_i)$. The holdout A_i doesn't get paid anything.

When $v(e_i) > D_i$, the LHS ranges from D_i to $v(e_i)$ while the RHS $(1 - \delta) v(e_i) + \delta D_i$ is a value strictly in between. So the maximum possible value is given by $\bar{x}^\delta(e_i) = (1 - \delta)(v(e_i) - D_i)$. And the holdout is paid $\min\{v(e_i) - \bar{x}^\delta(e_i), D_i\} = D_i$.

Thus, at $h = \mathbf{0}$, the principal has to pay D_i to any agent A_i such that $D_i < v(e_i)$ since he could otherwise hold out and get paid in full; but nothing to other agents. This confirms the value function of the principal in the proposition. □

Proof of Proposition 5. To prove this, I calculate the principal's value function ignoring the sunk cost.

First, note that the maximum credible dilution at e_i is $x(e_i; R) = (1-\delta)v(e_i)$. This is obtained by substituting the functional form $R_i^O(x, e_i) = \alpha_i x$ into Equation (31), which becomes $x(e_i; R) + \alpha_i(v(e_i) - x(e_i; R)) \le (1-\delta)v(e_i) + \delta\alpha_i v(e_i)$. Rearranging the terms shows that the maximum punishment that can be imposed on A_i is $x(e_i) = (1-\delta)v(e_i)$.

Therefore, the principal has to pay at least $\alpha_i(v(e_i) - x(e_i)) = \alpha_i\delta v(e_i)$ on path to A_i. The firm's value function is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^N \delta\alpha_i v(e_i)$ which is decreasing in δ. □

Proof of Proposition 6. I first prove the maximum punishment $\bar{x}^\delta(e_i)$, given by finding the largest x subject to the inequality

$$x + R_i^O(v(e_i) - x, e_i) \le v(e_i) - \delta(v(e_i) - R_i^O(v(e_i), e_i)), \tag{33}$$

is decreasing in δ for any e_i. I prove this auxiliary statement by contradiction. Suppose there exists $\delta_1 < \delta_2$ and $\bar{x}^{\delta_1}(e_i) < \bar{x}^{\delta_2}(e_i)$ for some e_i. Then we have

$$\bar{x}^{\delta_2}(e_i)+R_i^O(v(e_i)-\bar{x}^{\delta_2}(e_i), e_i) \le v(e_i)-\delta_2(v(e_i)-R_i^O(v(e_i), e_i)) < v(e_i)-\delta_1(v(e_i)-R_i^O(v(e_i), e_i)) \tag{34}$$

where the first inequality is given by the definition of $\bar{x}^{\delta_2}(e_i)$ and the second is by $\delta_2 > \delta_1$. Thus $\bar{x}^{\delta_2}(e_i)$ is a feasible value of x when $\delta = \delta_1$ in Equation (33). This contradicts the optimality of $\bar{x}^{\delta_1}(e_i)$! Thus, it must be $\bar{x}^{\delta_1}(e_i) \ge \bar{x}^{\delta_2}(e_i)$.

The principal's value function $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^N R_i^O(v(e_i) - \bar{x}^\delta(e_i), e_i)$ is increasing in $\bar{x}^\delta(e_i)$ for each e_i since $R_i^O(\cdot, e_i)$ is increasing for each e_i. Combining these two facts, we conclude that $J(\mathbf{0})$ is weakly decreasing in δ for any R^O.

□

Proof of Proposition 7. **Overview** To tackle this problem, we decompose the problem into two sub-problems. First, for each h, we assign a number $J(h)$,[68] and define the sets of contracts that are i) IC at each h and ii) allow the principal to guarantee a payoff of at least $J(\hat{h})$ for all $\hat{h}$:

$$C^\delta(h|J) := \left\{R \in \mathcal{I}(h) : J(\hat{h}|R) \ge \delta J(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\right\} \tag{SP1}$$

The set of contract $C^\delta(\cdot|J)$ is no longer recursively defined, so we can easily see that the set is unique, despite that it might be empty for some values of J. Indeed, we will show

[68] So, with some abuse of notations, J is both an operator and a vector here.

in the proof that the $C^{\delta}(h|J)$ is non-empty if $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$ for $\delta > 0$.

Second, for any sets of contracts $\mathcal{R}$ available at h, we define an upper bound of the value attainable by the principal using contracts within $\mathcal{R}$ to be

$$J(h|\mathcal{R}) := \sup_{\tilde{R} \in \mathcal{R}(h)} J(h|\tilde{R}) \tag{SP2}$$

We follow the convention and define the supremum to be $-\infty$ if the set $\mathcal{R}(h)$ is empty. The supremum need not be attainable if the contract space $\mathcal{R}$ is not compact or the objective function is not continuous in $\tilde{R}$. But regardless, we have the following

Lemma 4 (Fixed Point)**.** *Let J^* be the vector that solves the fixed-point equation*

$$J^*(h) = J(h|C^{\delta}(h|J^*)) \quad \forall h \in H, \tag{35}$$

then $C^{\delta}(\cdot|J^*)$ satisfies the definition of credible contracts in Definition 6. On the other hand, for any credible contracts C^{δ} defined in Definition 6, whenever it exists, the value function $J(h|C^{\delta})$, as defined in Equation (SP2) solves the fixed-point equation (35).

The proof is largely standard: It formalizes the idea that the recursive definition can be characterized by a fixed-point equation. Here, we look at the fixed point of the value function instead of the sets to circumvent technical issues with the mapping between sets of contracts. This approach is very similar to the classic dynamic contracting problems where the dynamic contract problem is reduced to a static one given the continuation value. The main difference is that here, the recursion is over the action space instead of time, so there is no linear order of dependence. Here, the value functions of two different action profiles can mutually depend on each other, which brings up the issue of existence and uniqueness. The next result says it is not a concern.

Proof. I first prove that $C^{\delta}(\cdot|J^*)$ satisfies the definition of credible contracts. For any contract $R \in C^{\delta}(h|J^*)$, the IC at h is satisfied automatically, so we only need to check that at any deviation node $\hat{h} \in \mathcal{B}(h)$, it dominates any contract $\tilde{R} \in C^{\delta}(\hat{h}|J^*)$. From the definition of J^* and thus $C^{\delta}(h|J^*)$, we know that for any $R \in C^{\delta}(h|J^*)$, we have

$J(\hat{h}|R) \geq \delta J^*(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)$ and that

$$J^*(\hat{h}) = J(\hat{h}|C^\delta(\hat{h}|J^*)) = \sup_{\tilde{R} \in C^\delta(\hat{h}|J^*)} J(\hat{h}|\tilde{R}). \tag{36}$$

Passing the inequality from the supremum to each contract in $C^\delta(\hat{h}|J^*)$, we arrive at

$$J(\hat{h}|R) \geq \delta J(\hat{h}|\tilde{R}) \quad \forall \tilde{R} \in C^\delta(\hat{h}|J^*) \quad \forall \hat{h} \in \mathcal{B}(h) \tag{37}$$

which proves that $C^\delta(\cdot|J^*)$ is a set of credible contracts.

Now I prove the other direction by showing that $J(h|C^\delta)$ as a vector solves the fixed-point Equation (35), i.e., $J(h|C^\delta) = J(h|C^\delta(h|J(h|C^\delta)))$. For any h, by definition of C^δ and $J(\cdot|\cdot)$, we have $J(h|C^\delta) = \sup_{\tilde{R} \in C^\delta(h)} J(h|\tilde{R})$ and that $C^\delta(h) = \left\{R \in \mathcal{I}(h) : J(\hat{h}|R) \geq \delta J(\hat{h}|\tilde{R}) \quad \forall \tilde{R} \in C^\delta(\hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h)\right\}$. Substitute in the definition of $C^\delta(\hat{h})$ and passing inequality to the supremum, we can write

$$C^\delta(h) = \left\{R \in \mathcal{I}(h) : J(\hat{h}|R) \geq \delta J(\hat{h}|C^\delta) \quad \forall \hat{h} \in \mathcal{B}(h)\right\} = C^\delta(h|J(h|C^\delta)). \tag{38}$$

To see this, suppose instead $\exists \hat{h} : J(\hat{h}|R) < \delta J(\hat{h}|C^\delta)$, then by definition of sup, there exists a $\tilde{R}$ such that $J(\hat{h}|R) < \delta J(\hat{h}|\tilde{R})$, contradicting the definition of $C^\delta(h)$. Finally, applying the J operator on the identity $C^\delta(h) = C^\delta(h|J(h|C^\delta))$, we get $J(h|C^\delta) = J(h|C^\delta(h|J(h|C^\delta)))$. Thus, we established the equivalence of the recursive Definition 6 and the fixed-point characterization (35). □

The key step in the proof is that the constraint the credibility puts is asymmetric for agents who deviate from holdout to tendering and for those who deviate from tendering to holdout. In the former case, to deter tendering, we must reduce the payoff from tendering for the deviating agent. This can be easily achieved by reducing his payoff from tendering to 0. Doing so would not affect the credibility constraint as it weakly reduces the total payoffs to all agents under the 1-Lipschitz condition, which is weakly beneficial for the principal. However, to discourage an agent from holding out, the principal must try to minimize his payoff off-path. However, there is a limit to what the principal can achieve by imposing externalities on him. In other words, the principal can only punish deviating agents by granting higher payoff to other tendering agents, but doing so would weakly lower the principal's payoff. There will be no renegotiation as long as it's still below the principal's value function at the deviation profile. So, the maximum punishment the principal can credibly impose on the deviator on the deviation node is the one that makes her payoff equivalent to her value function at the deviation node.

This asymmetry in constraints reveals an asymmetric inter-dependence of the value functions that the value of $J(h|C^{\delta}(h|J))$ only depends on the values of $J(\hat{h}|J)$ for the profiles $\hat{h}$ where there are more deviating agents than h, i.e., $\xi(\hat{h}) \subset \xi(h)$. Thus, we can prove the existence by constructing a vector J^* that solves the fixed-point equation (35) in finite steps. We start from an arbitrary vector J^0 in the feasible space (specified in the proof) and calculate the value function on the action profile $\mathbf{1}$ on which everyone holds out. It turns out that, as expected, the value function $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$ is independent of the choice of J^0. Then we replace the value of $J(\mathbf{1})$ by $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$, and use that vector, renamed J^1, for the next iteration, i.e., calculating the value function $J(h|C^{\delta}(h|J^1))$ on the action profiles where exactly one agent tenders. Again, it turns out the value function is independent of the initial choice J^0: it only depends on the value $J(\mathbf{1}|C^{\delta}(\mathbf{1}|J^0))$. We update the vector and continue the process by calculating the value functions on all the profiles where one more agent tenders. This process ends after we calculate the value function on the node $\mathbf{0}$ on which everyone tenders and set the vector J^{N+1} to be J^*. Finally, we conclude that the vector found J^* is indeed the solution to the fixed-point equation by noticing $J^*(h) = J(h|C^{\delta}(h|J^{k+1})) = J(h|C^{\delta}(h|J^*))$ for any h such that $|\xi(h)| = k$.

The uniqueness can be obtained by noticing that in the construction above, the fixed point found is independent of the choice of the initial J^0. In the proof, I give a more formal proof by contradiction, showing that there's no other solution than the one found using the procedure above.

Solving for fixed-point To show a fixed point J^* exists and is unique, I first prove that the set $C^{\delta}(h|J)$ is non-empty for all $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$; then I display the asymmetry mentioned by solving the problem SP2 over the sets $C^{\delta}(h|J)$ for any vector $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, i.e., I want to calculate $J(h|C^{\delta}(h|J))$.

Non-emptiness of $C^\delta(h|J)$ I first show that $C^\delta(h|J)$ is non-empty for any $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$. To do so, I only need to give one example of a contract, and an natural one would be this "no-additional-punishment contract".

- At h, the principal pays to whoever holds out 0 in the new contracts (note the holdout will not have new contracts), and to whoever tenders what he would otherwise obtain if he were to hold out and no punishement is imposed, i.e., $R_i(v(h), h) = R_i^O(v(h + e_i), h)\mathbb{1}_{i\in\xi(h)}$. The IC, the first constraint in the definition (Equation SP1), is clearly satisfied.
- At $\hat{h} \in \mathcal{B}(h)$, the principal pays nothing to the tendering agents and any arbitrary amount, e.g., 0, to those who hold out in the new contract, which they don't accept. Then the total payout to all agents is $0 + \hat{h} \cdot R^O(v(\hat{h}) - 0, \hat{h})$ which is no larger than $v(\hat{h}) - \delta J(\hat{h})$ by the range of the value of $J(\hat{h})$. So the second constraint in the definition (Equation SP1) is also satisfied.
- It takes any arbitrary values on any other action profiles.

Since at least one contract exists in $C^\delta(h|J)$ when $J \in \prod_h [0, \delta^{-1}(v(h) - \delta h \cdot R^O(v(h), h))]$, it's non-empty.

Now, I prove another auxiliary lemma that would be used in the main proof.

Lemma 5. *Let $f(\cdot)$ and $g(\cdot)$ be two weakly increasing 1-Lipschitz functions and so is their sum. Given three constants $a \geq b \geq 0$ and $c > 0$, the solution to the problem*

$$\inf_{x\in[b,a]} g(a - x) \quad \textit{subject to} \quad g(a - x) + f(a - x) + x \leq c \tag{39}$$

exists if and only of $f(a - b) + g(a - b) \leq c$ and one such solution is given by

$$\bar{x} := \max\{x \in [b, a] : g(a - x) + f(a - x) + x = c\}. \tag{40}$$

Proof. Invoking Lemma 1, the fact that $f(\cdot)+g(\cdot)$ is 1-Lipschitz implies that $g(a-x)+f(a-x)+x$ is a weakly increasing function and its minimum can always be attained at $x = b$, so the feasible set is non-empty if and only if $f(a-b)+g(a-b) \leq c$. Moreover, the continuity of $f(\cdot)$ and $g(\cdot)$ also implies the feasible set $\{x \in [b, a], g(a - x) + f(a - x) + x \leq b\}$ is compact so the infimum can be attained whenever it is non-empty. Since $g(a - x)$ is a weakly decreasing function of x, its minimum can be achieved at the largest x in which the constraint is satisfied. Since $g(a-x)+f(a-x)+x$ is a weakly increasing, an obvious one is simply $\bar{x} = \max\{x \in [b,a] : g(a-x)+f(a-x)+x = c\}$. □

Asymmetry in ICs We want to show the value of the vector J only affects the credibility constraints at the deviation node $\hat{h}$, which in turn affects the IC constraint through the off-path threat $u_i(1-h_i|h_{-i},R)$. To be more specific, let's say, at h, the agent A_j deviates to $\hat{h} = (h_{-j}, 1-h_j)$, which includes two cases:

- Agent A_j deviates from 1 to 0, i.e., $h_j = 1$ and $\hat{h}_j = 0$: This is the easy case as P only needs to make a zero offer to A_j. The IC to make sure agent A_j holds out is

$$u_j(h_j = 1|h_{-j}, R) \equiv R_j^O\left(v(h) - \sum_{i\in\xi(h)} R_i(v(h),h), h\right) \geq R_j(v(\hat{h}),\hat{h}). \tag{41}$$

  We can set the RHS to 0 but we need to check that the credibility constraint at $\hat{h}$ will not be vioalted. The credibility constraint can be written as

$$x(\hat{h}) + \sum_{i\notin\xi(\hat{h})} R_i^O\left(v(\hat{h}) - x(\hat{h}), \hat{h}\right) \leq v(\hat{h}) - \delta J(\hat{h}) \tag{42}$$

  where $x(\hat{h}) = \sum_{k\in\xi(h)} R_k(v(\hat{h}),\hat{h}) + R_j(v(\hat{h})$. Note here I used the fact that $\xi(\hat{h}) = \xi(h)\coprod\{j\}$ and consequently $\{j\}\coprod \xi(\hat{h})^c \coprod \xi(h) = \mathcal{N}$, which allows me to write the total payoff on the left-hand side to all agents in three parts. Under the 1-Lipschitz conditon A2, the minimum of the LHS is achieved by setting $x(\hat{h}) = 0$[69] and the credibility constraint is satisfied as long as $J(\hat{h}) \leq \delta^{-1}(v(\hat{h}) - \delta\hat{h}\cdot R^O(v(\hat{h}),\hat{h})$.

- Agent A_j deviates from 0 to 1, i.e., $h_j = 0$ and $\hat{h}_j = 1$. The on-path IC for agent j is

$$u_j(h_j = 0|h_{-j}, R) \equiv R_j(v(h),h) \geq R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}), \hat{h}\right) \tag{43}$$

  Again, the problem can be relaxed if we can make $R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}), \hat{h}\right)$

[69]Note R need not be IC at $\hat{h}$ as it is a deviation profile.

smaller, if unimpeded by the credibility constraint at $\hat{h}$, which now is

$$\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}) + R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right) \tag{44}$$

$$+ \sum_{k\notin\xi(h)} R_k^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right) \leq v(\hat{h}) - \delta J(\hat{h}) \tag{45}$$

having used the fact that $\{j\} \coprod \xi(\hat{h}) \coprod \xi(h)^c = \mathcal{N}$. And again, the left-hand side could be minimized by setting $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$ to zero without affecting other constraints under 1-Lipschitz condition using Lemma 1. So the condition for the existence of the solution is again $\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h}\cdot R^O(v(\hat{h}),\hat{h})$.

However, setting $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$ to zero,[70] despite of minimizing the total payoff to $\{j\} \coprod \xi(h)^c$, doesn't necessarily minimize $R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right)$ as the value of it is $R_j^O(v(\hat{h}),\hat{h})$ instead of zero. I could further increase the value of $\sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h})$, the value of the LHS might also increase until the constraint is binding, *without additional constraints*. Using Lemma 5, we know that $R_j^O\left(v(\hat{h}) - \sum_{i\in\xi(\hat{h})} R_i(v(\hat{h}),\hat{h}),\hat{h}\right)$ is minimized at

$$\bar{x}^\delta(J(\hat{h});\hat{h}) := \max\left\{x \in [0, v(\hat{h})] : \hat{h}\cdot R^O\left(v(\hat{h}) - x,\hat{h}\right) + x = v(\hat{h}) - \delta J(\hat{h})\right\}, \tag{46}$$

using the fact $\{j\} \coprod \xi(h)^c = \xi(\hat{h})^c$. Note the solution exists because $0 + \hat{h}\cdot R^O\left(v(\hat{h}),\hat{h}\right) = v(\hat{h}) - J(\hat{h}) \leq v(\hat{v}) - \delta J(\hat{h}) \leq v(\hat{h}) + \hat{h}\cdot R^O\left(0,\hat{h}\right)$ and the LHS is a continuous function of x . The maximum is attainable because the zeros of a Lipschitz function on a closed interval constitute a compact set.

Summary of existence and uniqueness In summary, the condition for a credible to exist is that for any deviation $\hat{h} \in \mathcal{B}(h)$, the highest value $J(\hat{h})$ that can be alternatively obtained using a credible contract at $\hat{h}$ is smaller than the difference between the asset

[70] Note, I do not require R to be IC at $\hat{h}$ so it can be set to 0. The alternative contract $\tilde{R}$ that can be proposed needs to be IC, but it's captured in the $J(\hat{h})$.

value $v(\hat{h})$ and the collective holdout payout $\hat{h} \cdot R^O(v(\hat{h}), \hat{h})$, i.e.,

$$\delta J(\hat{h}) \leq v(\hat{h}) - \hat{h} \cdot R^O(v(\hat{h}), \hat{h}) \quad \forall \hat{h} \in \mathcal{B}(h). \tag{47}$$

Moreover, the analysis above shows that, for any $J \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$, the value $J(h|C^\delta(h|J))$ depends only on $J(h + e_i)$ for some $i \in \xi(h)$, and recursively on any $h' \geq h$. On the contrary, the value $J(h - e_j)$ for some $j \notin \xi(h)$ does not affect the value of $J(h|C^\delta(h|J))$ and recursively so does any h' not in the upper contour set of h: $\{h' : h' \geq h\}$.

Construction of the fixed point: The discussion above allows us to calculate the J^* via the following procedure for $\delta > 0$. In particular, we want to emphasize that we are not calculating $J(h|C^\delta(h|J))$ for a specific J.

1. First we decompose $H = \{0, 1\}^N$ into $N+1$ disjoint sets $H^k = \{h : \xi(h) = k\}$ for $k = 0, ..., N$ on which exactly k agents tender.
2. We calculate the $J(h|C^\delta(h|J^0))$ on $H^0 = \{\mathbb{1}\}$ for any fixed $J^0 \in \prod_h [0, \delta^{-1}(v(h) - h \cdot R^O(v(h), h))]$. At $h = \mathbb{1}$, none of the credibility constraints matter since all hold out, and the ICs are simply the non-negativity constraints $R_i^O(1|\mathbb{1}_{-i}, \mathbb{1}) = R_i^O(v(\mathbb{1}), \mathbb{1}) \geq 0 \quad \forall i \in \mathcal{N}$. So we can calculate the value function

$$J^*(\mathbb{1}) := J(\mathbb{1}|C^\delta(\mathbb{1}|J^0)) = v(\mathbb{1}) - \mathbb{1} \cdot R^O(v(\mathbb{1}, \mathbb{1})) \tag{48}$$

and the maximum credible punishment at $\mathbb{1}$

$$\bar{x}^\delta(\mathbb{1}) := \bar{x}^\delta(J^*(\mathbb{1}), \mathbb{1}) = v(\mathbb{1}) - \delta J^*(\mathbb{1}) = \delta \mathbb{1} \cdot R^O(v(\mathbb{1}), \mathbb{1}) + (1 - \delta) v(\mathbb{1}). \tag{49}$$

Then, we update our J^0 to J^1 as follows

$$J^1(h) = \begin{cases} J^0(h) & \text{if } h \notin H^0 \\ J^*(h) & \text{if } h \in H^0 \end{cases} \tag{50}$$

$k + 3$. Now we carry out the calculation by induction: Suppose $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ are defined on all H^κ for $\kappa = 0, 1, \ldots, k$, and J^{k+1} is also defined. We solve for $J^*(\cdot)$, $\bar{x}^\delta(\cdot)$ defined on all H^{k+1} by solving for $J(h|C^\delta(h|J^{k+1}))$ and update J^{k+1} to J^{k+2}. For any $h \in H^{k+1}$, the relevant IC constraints are

$$R_i(v(h), h) \geq R_i^O(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i) \quad \forall i \in \xi(h). \tag{51}$$

The RHS is known as $h + e_i \in H^k$. We re-iterate the principal's problem following the simplification above:

$$\max_R v(h) - \sum_{i \in \xi(h)} R_i(v(h), h) - \sum_{j \notin \xi(h)} R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i(v(h), h), h\right). \tag{52}$$

Again, under the assumption that $h \cdot R^O(\cdot, h)$ is 1-Lipschitz, the objective is weakly decreasing in each on-path payoff $R_j(v(h), h)$ for each $j \in \xi(h)$. And we have

$$\begin{aligned} J^*(h) \equiv J(h|C^\delta(h|J^{k+1})) = v(h) - \sum_{i \in \xi(h)} R_i^O\left(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i\right) - \\ \sum_{j \notin \xi(h)} R_j^O\left(v(h) - \sum_{i \in \xi(h)} R_i^O\left(v(h + e_i) - \bar{x}^\delta(h + e_i), h + e_i\right), h\right). \end{aligned} \tag{53}$$

To calculate the maximum credible punishment at h, we find the largest solution to the equation $h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)$. By Lemma 1, the maximum solution exists and is unique, and we calculate

$$\bar{x}^\delta(h) = \max\{x \in [0, v(h)] : h \cdot R^O(v(h) - x, h) + x = v(h) - \delta J^*(h)\}. \tag{54}$$

We also update J^{k+1} to J^{k+2} as follows

$$J^{k+2}(h) = \begin{cases} J^{k+1}(h) & \text{if } h \notin H^{k+1} \\ J^*(h) & \text{if } h \in H^{k+1} \end{cases} \tag{55}$$

$N + 3$. Finally, after calculating J^* for $k = N - 1$, we obtain $J^* = J^{N+1}$, and we need to verify that it satisfies $J^*(h) = J(h|C^\delta(h|J^*))$. This could be easily done by observing that $J^*(h) = J(h|C^\delta(h|J^{k+1})) = J(h|C^\delta(h|J^*))$ for any $h \in H^k$ for $k = 0, 1, ..., N - 1$.

Finally, the uniqueness should be obvious from the iteration. Notice that the J^* we calculated in the procedure is independent of the initial choice J^0. But the readers may wonder if there's a fixed point not found through the procedures above. To alleviate this concern, suppose there exist two fixed-points J and $\tilde{J}$ such that $J \neq \tilde{J}$ and $J(h) = J(h|C^\delta(h|J))$ (resp. $\tilde{J}(h) = J(h|C^\delta(h|\tilde{J}))$) for any h. Since $J(1|C^\delta(1|J))$ doesn't depend on J, it must be that $J(1) = J(1|C^\delta(1|J)) = J(1|C^\delta(1|\tilde{J})) = \tilde{J}(1)$. Then there must be an h such that $J(h) \neq \tilde{J}(h)$. Let $\underline{k} = \min\{k \geq 1 : \exists h \in H^k : J(h) \neq \tilde{J}(h)\}$. Then on all the action profiles $h \in H^{\underline{k}-1}$, $J(h) = \tilde{J}(h)$, then we would have for all $h \in H^{\underline{k}}$, $J(h) = J(h|C^\delta(h|J)) = J(h|C^\delta(h|\tilde{J})) = \tilde{J}(h)$, contradicting the definition of $\underline{k}$. Thus, the fixed-point equation (35) has a unique solution.

□

Proof of Proposition 8. The "only if" part is derived in the proof of Proposition 7, and the "if" part is by uniqueness. □

Proof of Lemma 3. We first calculate the initial condition at $h = 1$. Since credibility constraint matters at 1, the principal obtains her highest value by paying every agent his holdout payoff $J(1) = v(1) - 1 \cdot R^O(v(1), 1)$. To solve for $\bar{x}^\delta(1)$, I solve the equation $x + 1 \cdot R^O(v(1) - x, 1) = (1-\delta)v(1) + \delta 1 \cdot R^O(v(1), 1)$ which, in the equity case, after rearranging, can be written as $(1 - \langle 1, \alpha\rangle)x = (1-\delta)(1 - \langle 1, \alpha\rangle)v(1)$.

If $\langle 1, \alpha\rangle \neq 1$, the only solution is $\bar{x}^\delta(1) = (1-\delta)v(1) = 0$ using the normalization $v(1) = 0$. If instead $\langle 1, \alpha\rangle = 1$, the equation is reduced to an identity that always holds regardless of the choice of x. Thus, the largest possible solution is $\bar{x}^\delta(1) = v(1) = 0$.

Now, I show the iterative relation. When $\bar{x}^\delta(h + e_i)$ is known, I can write the value function at h as

$$J^*(h) = v(h) - (1 - \langle h, \alpha\rangle) \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^\delta(h + e_i)\right) - \langle h, \alpha\rangle v(h) \tag{56}$$

Then in order to find $\bar{x}^\delta(h)$, we solve the equation $\langle h, \alpha\rangle(v(h) - x) + x = v(h) - \delta J^*(h)$. Substitute in $J^*(h)$, use the fact $\langle h, \alpha\rangle \neq 1$ for all $h \neq 1$, and we obtain

$$\bar{x}^\delta(h) = (1-\delta)v(h) + \delta \sum_{i \in \xi(h)} \alpha_i \left(v(h + e_i) - \bar{x}^\delta(h + e_i)\right) \tag{57}$$

This concludes the proof of the lemma. □

Proof of Proposition 9. To prove this result, we need to show that i) the initial condition is satisfied and that ii) the equation (18) is satisfied when we plug the equation (19) in. The initial condition is very easy to verify: at 1, there are no tendering agents, so the RHS is non-existent.

Before plugging, we want to state several basic facts about the set of permutations. By definition, $\xi(h) = \xi(h+e_i) \cup \{i\}$, and thus $|\xi(h+e_i)| = |\xi(h)| - 1$. Moreover, consider two sets of permutations $\Sigma(\xi(h+e_i))$ and $\Sigma(\xi(h))$. It's easy to see that $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))|$ but conditional on the kth element being i, the subset $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ is isomorphic to $\Sigma(\xi(h+e_i))$. Moreover, the disjoint union of them is isomorphic to $\Sigma(h)$. That is,

$$\coprod_{i\in\xi(h)} \Sigma(\xi(h+e_i)) \cong \coprod_{i\in\xi(h)} \{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\} \cong \Sigma(\xi(h)) \quad \forall\, k = 1, \ldots, |\xi(h)| \tag{58}$$

Now we plug the solution in Equation (19) into the recursive Equation (18), the right hand side of the Equation (18) is $(1-\delta)v(h) + \delta \sum_{i\in\xi(h)} \alpha_i(v(h+e_i) - \bar{x}(h+e_i))$. The second term is

$$\begin{aligned}
&\delta \sum_{i\in\xi(h)} \alpha_i(v(h+e_i) - \bar{x}(h+e_i)) \\
=&\delta \sum_{i\in\xi(h)} \alpha_i \left(\delta v(h+e_i) - \sum_{k=1}^{|\xi(h+e_i)|} \frac{(-\delta)^{k+1}}{(|\xi(h+e_i)|-k)!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h + e_i + \sum_{s=1}^{k} e_{\sigma(s)}\right)\right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h + e_{\sigma(1)}\right) \\
&+ \sum_{i\in\xi(h)} \sum_{k'=2}^{|\xi(h)|} \frac{(-\delta)^{k'+1}}{(|\xi(h)|-k')!} \sum_{\sigma\in\Sigma(\xi(h+e_i))} \left(\alpha_i \prod_{s=1}^{k'-1} \alpha_{\sigma(s)}\right) v\left(h + e_i + \sum_{s=1}^{k'-1} e_{\sigma(s)}\right) \\
=&\frac{\delta^2}{(|\xi(h)|-1)!} \sum_{\sigma\in\Sigma(\xi(h))} \alpha_{\sigma(1)} v\left(h + e_{\sigma(1)}\right) \\
&+ \sum_{k=2}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{i\in\xi(h)} \sum_{\sigma\in\Sigma(\xi(h)):\sigma(k)=i} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)}\right)
\end{aligned}$$

$$= \sum_{k=1}^{|\xi(h)|} \frac{(-\delta)^{k+1}}{(|\xi(h)|-k)!} \sum_{\sigma\in\Sigma(\xi(h))} \left(\prod_{s=1}^{k} \alpha_{\sigma(s)}\right) v\left(h + \sum_{s=1}^{k} e_{\sigma(s)}\right)$$

where the first quality is the result with $\bar{x}(h+e_i)$ directly plugged in; the second equality is the separation of the first term and the rest, with the replacement $k' = k+1$. Note, we have the $\frac{1}{(|\xi(h)|-1)!}$ term because $|\Sigma(\xi(h))| = |\xi(h)| \cdot |\Sigma(\xi(h+e_i))| = |\xi(h)| \cdot (\xi(h)-1)!$ so the term is used to offset the repetitive counting. In the third equality, we switch the indicator to k, and change the order of the summation using the isomorphism between $\{\sigma \in \Sigma(\xi(h)) : \sigma(k) = i\}$ and $\Sigma(\xi(h+e_i))$. The last line combines the two parts, using the isomorphism in equation (58).

At $h = \mathbf{0}$, the maximum punishment is also $\bar{x}^\delta(\mathbf{0}) = v(\mathbf{0}) - \delta J(\mathbf{0})$. Substituting it into $J(\mathbf{0}) = \delta^{-1}(v(\mathbf{0}) - \bar{x}^\delta(\mathbf{0}))$ and using $\xi(\mathbf{0}) = \mathcal{N}$ and $|\xi(\mathbf{0})| = N$, we arrive at the expression in the proposition. □

C Proofs for Section 5 (Property Rights)

Proof of Proposition 10. When A_i deviates, the principal could promise to give the entire asset to other tendering agents, and the holdout A_i still enjoys a value of π_i by retaining his property. Thus, to convince A_i to tender, he must be paid π_i on path. Therefore, the value at $\mathbf{0}$ is the asset value minus the sum of property values. □

Proof of Example 5.1. Since the asset value $v(\mathbf{1}) = 0$, when all three agents hold out, they get nothing more than their property value, so to convince one of them, say A_i, to tender, the principal only needs to pay him π_i, and the principal obtains a value

$$J(\mathbf{1}-e_i) = v(\mathbf{1}-e_i) - \pi_i - \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - \pi_i, \mathbf{1}-e_i) \tag{59}$$

Solving for the maximum x such that $x + \sum_{j\neq i} R_j^O(v(\mathbf{1}-e_i) - x, \mathbf{1}-e_i) \leq J(\mathbf{1}-e_i)$ yields $\bar{x}(\mathbf{1}-e_i) = \pi_i$ given the parametric assumption on the slopes of R_j^O.

Now consider the holdout profile e_i. The principal obtains a value $J(e_i) = v(e_i) - \underline{x}(e_i) - R_i^O(v(e_i) - \underline{x}(e_i), e_i)$ where $\underline{x}(e_i) = \sum_{j\neq i}\left[R_j^O(v(e_i+e_j) - \pi_k, e_i+e_j) + \pi_j\right]$ for $k \neq i, j$.

Again, solving for the maximum x such $x + R_i^O(v(e_i) - x, e_i) \leq v(e_i) - J(e_i)$ yields $\bar{x}(e_i) = \underline{x}(e_i)$. Taking derivatives with respect to π_j gives $\frac{\mathrm{d}\bar{x}(e_i)}{\mathrm{d}\pi_j} = 1 - \frac{\partial}{\partial v} R_k^O(v(e_i+e_k) - \pi_j, e_i+e_k)$

The principal's value at $h = \mathbf{0}$ is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{3} \left[R_i^O(v(e_i) - X(e_i), e_i) + \pi_i\right]$. Taking the derivative with respect to π_i gives

$$\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} = -1 + \sum_{j \neq i} \frac{\partial}{\partial v} R_j^O(v(e_j) - \bar{x}(e_j), e_j) \left[1 - \frac{\partial}{\partial v} R_k^O(v(e_j + e_k) - \pi_i, e_j + e_k)\right] \tag{60}$$

Given the parameters in the proposition, we have $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_1} = -1 + \alpha_2(1-\alpha_3) + \beta_3(1-\beta_2) = \frac{13}{50} > 0$ as $\bar{x}(e_2) = 1.1$ and $\bar{x}(e_3) = 0.806$. □

Proof of Proposition 11. We first show that the maximum punishment satisfies the recursion

$$\bar{x}(h) = \sum_{i \in \xi(h)} \left[\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i\right] \tag{61}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$. This is because given $\bar{x}(h + e_i)$, at h, each tendering agent A_i could have otherwise obtained a value of $\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i$ were he to hold out. Thus, the value function of the principal is $J(h) = v(h) - \underline{x}(h) - \langle h, \alpha\rangle(v(h) - \underline{x}(h))$ where $\underline{x}(h) = \sum_{i \in \xi(h)} \left[\alpha_i(v(h + e_i) - \bar{x}(h + e_i)) + \pi_i\right]$. And solving for the maximum x such that $x + \langle h, \alpha\rangle(v(h) - x) \leq v(h) - J(h)$ yields $(1 - \langle h, \alpha\rangle)x \leq \langle h, \alpha\rangle \underline{x}(e_i)$, which gives

$$\bar{x}(h) = \begin{cases} v(h) & \text{if } h = \mathbf{1} \\ \underline{x}(h) & \text{otherwise} \end{cases} \tag{62}$$

From the recursive relation of $\bar{x}$, we obtain

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = \mathbb{1}_{\{i \in \xi(h)\}} - \sum_{j \in \xi(h)} \alpha_j \frac{\mathrm{d}\bar{x}(h + e_j)}{\mathrm{d}\pi_i} \tag{63}$$

with the initial condition $\frac{\mathrm{d}\bar{x}(\mathbf{1})}{\mathrm{d}\pi_i} = 0$ since $\bar{x}(\mathbf{1}) = 0$. To solve $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i}$, we establish two lemmata:

Lemma 6. *For any h and any i such that $i \notin \xi(h)$, $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 0$.*

Proof. I prove the lemma by induction. For any h such that $|\xi(h)| = 0$, i.e., $h = \mathbf{1}$, we have the obvious case $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 0$.

Now I show that if the statement is true for any h such that $i \notin \xi(h)$ and $|\xi(h)| = n$, it is also true for any h such that $i \notin \xi(h)$ and $|\xi(h)| = n + 1$. First notice that if $i \notin \xi(h)$, then for any $j \in \xi(h)$, $j \notin \xi(h+e_j)$. And $|\xi(h+e_j)| = |\xi(h)| - 1$. Then, we have

$$\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = -\sum_{j\in\xi(h)} \alpha_j \frac{\mathrm{d}\bar{x}(h+e_j)}{\mathrm{d}\pi_i} = 0 \tag{64}$$

where the first equality holds because $i \notin \xi(h)$ and the second holds by induction hypothesis. □

Lemma 7. *For any h and any i such that $i \in \xi(h)$, $0 < \frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} \leq 1$*

Proof. I prove the lemma by induction. For any h such that $|\xi(h)| = 1$, i.e., $h = \mathbf{1} - e_i$, we have the obvious case $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 1$.

Now I show that if the statement is true for any h such that $i \in \xi(h)$ and $|\xi(h)| = n$, it is also true for any h such that $i \in \xi(h)$ and $|\xi(h)| = n+1$. First notice that if $i \in \xi(h)$, then for any $j \in \xi(h) : j \neq i$, $j \in \xi(h+e_j)$. And $|\xi(h+e_j)| = |\xi(h)| - 1$. Thus, the recursive relation could be written as $\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} = 1 - \sum_{j\in\xi(h):j\neq i} \alpha_j \frac{\mathrm{d}\bar{x}(h+e_j)}{\mathrm{d}\pi_i}$ since $\frac{\mathrm{d}\bar{x}(h+e_i)}{\mathrm{d}\pi_i}$ is zero. Since by induction hypothesis, each $\frac{\mathrm{d}\bar{x}(h+e_j)}{\mathrm{d}\pi_i}$ is in $(0,1]$, we have $0 < \frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i} < 1$ since $\sum_{j\in\xi(h):j\neq i} \alpha_i < 1$. Thus, it holds for all h such that $i \in \xi(h)$. □

Using $\frac{\mathrm{d}J(h)}{\mathrm{d}\pi_i} = -(1 - \langle h, \alpha\rangle)\frac{\mathrm{d}\bar{x}(h)}{\mathrm{d}\pi_i}$ I obtain $\frac{\mathrm{d}J(\mathbf{0})}{\mathrm{d}\pi_i} = -\frac{\mathrm{d}\bar{x}(\mathbf{0})}{\mathrm{d}\pi_i} \in [-1, 0)$ Thus, a higher property rights protection always undermines restructuring when the initial set of contracts are equities.

□

Proof of Example 5.2. We first show that the maximum punishment satisfies the recursion

$$\bar{x}(h) = \sum_{i\in\xi(h)} [\alpha_i(v(h+e_i) - \bar{x}(h+e_i)) + \pi_i] \tag{65}$$

with the initial condition $\bar{x}(\mathbf{1}) = 0$. This is because given $\bar{x}(h+e_i)$, at h, each tendering agent A_i could have otherwise obtained a value of $\alpha_i(v(h+e_i) - \bar{x}(h+e_i)) + \pi_i$ were he to hold out. Thus the value function of the principal is $J(h) = (1 - \langle h, \alpha\rangle)(v(h) - \underline{x}(h))$ where $\underline{x}(h) = \sum_{i\in\xi(h)} [\alpha_i(v(h+e_i) - \bar{x}(h+e_i)) + \pi_i]$. And solving for the maximum x

from $x + \langle h, \alpha \rangle (v(h) - x) \leq v(h) - J(h)$ yields

$$\bar{x}(h) = \begin{cases} v(h) & \text{if } h = \mathbf{1} \\ \underline{x}(h) & \text{otherwise} \end{cases} \tag{66}$$

Using this recursion with the parameters specified, we obtain $\bar{x}(\mathbf{1} - e_i) = \pi_i \quad \forall i$ and $\bar{x}(e_i) = \sum_{j \neq i} \left[\alpha_j (v(e_i + e_j) - \pi_k) + \pi_j \right] \quad \forall k \neq i, j \quad \forall i$. The value function of the principal is

$$J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{3} \alpha_i v(e_i) + \sum_{i=1}^{3} \sum_{j \neq i} \alpha_i \alpha_j v(e_i + e_j) - \sum_{i=1}^{3} \left(1 - \sum_{j \neq i} \alpha_i (1 - \alpha_k) \right) \pi_i \tag{67}$$

Taking partial derivatives yields the expression in the proposition.

Without loss of generality, look at the coefficient of π_1. Even if I ignore the constraint $\langle \mathbf{1}, \alpha \rangle = 1$, the coefficient $1 - \alpha_2 - \alpha_3 + 2\alpha_2\alpha_3$ is minimized at $\alpha_2 = \alpha_3 = 1/2$ with a minimum value of $1/2$. Thus, all coefficients of π_i are positive. □

Proof of Proposition 12. Consider the deviation profile e_i, let $X(e_i)$ be the total payments to the tendering creditors according to one of the optimal δ-credible contracts, which could be a function of $\{\pi_i\}_i$

Then, the principal's value at e_i is $J(e_i) = v(e_i) - X(e_i) - \min\{D_i, v(e_i) - X(e_i)\}$ and the maximum punishment is the largest x such that $x + \min\{D_i, v(e_i) - x\} \leq v(e_i) - J(e_i)$ which yields

$$\bar{x}(e_i) = \begin{cases} v(e_i) & v(e_i) - X(e_i) \leq D_i \\ (1 - \delta)(v(e_i) - D_i) + \delta X(e_i) & v(e_i) - X(e_i) \geq D_i \end{cases} \tag{68}$$

Then, the principal's value is $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{N} \left[D_i \mathbb{1}_{\{v(e_i) \geq X(e_i) + D_i\}} + \pi_i \right]$ because whenever $v(e_i) - X(e_i) \leq D_i$, $\bar{x}(e_i) = v(e_i)$ and thus $\min\{D_i, v(e_i) - \bar{x}(e_i)\} = 0$; In contrast, when $v(e_i) - X(e_i) > D_i$, we have $v(e_i) - \bar{x}(e_i) = \delta(v(e_i) - X(e_i) - D_i) + D_i > D_i$ so $\min\{D_i, v(e_i) - \bar{x}(e_i)\} = D_i$. In either case, the payment to each tendering agent is independent of the renegotiation off-path. Thus $\frac{\partial J(\mathbf{0})}{\partial \pi_i} = -1 \quad \forall i$, which implies a locally small increase in property rights protection always hinders restructuring. □

Proof of Proposition 13. At every e_i, the principal only needs to compensate A_j at most π_j for him to tender so the principal's value is $J(e_i) = v(e_i) - \pi_j - \min\{v(e_i) - \pi_j, D \cdot e_i\}$. Solviong $x + \min\{v(e_i) - x, D \cdot e_i\} \leq v(e_i) - J(e_i)$, we get the maximum credible dilution

$$\bar{x}(e_i) = \begin{cases} v(e_i) & \text{if } v(e_i) \leq \pi_j + D \cdot e_i \\ \pi_j & \text{otherwise.} \end{cases} \tag{69}$$

The principal's value at $\mathbf{0}$ is then $J(\mathbf{0}) = v(\mathbf{0}) - \sum_{i=1}^{2} \left[D_i \mathbb{1}_{\{v(e_i) \geq \pi_j + D_i\}} + \pi_i \right]$. When $\pi_j \in (1/2, 1)$, given that $D_i = 1$ and $v(e_i) = 2$, we have $v(e_i) > \pi_j + D_i$; In contrast, when $\pi_j \in (1, 3/2)$, we have $v(e_i) \leq \pi_j + D_i$, so the change in the principal's value is $D_i - \Delta\pi_j > 0$ since $\Delta\pi_j < 3/2 - 1/2 = 1$. □