SYSTEMIC RISK IN FINANCIAL NETWORKS REVISITED

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FACTS

Banks' gross debts bigger than net

E.g. HSBC's net position $\big|£24\text{B} - £21.5\text{B}\big| \approx 10\%$ gross

Thought to habor systemic risk $\implies$ Policy makers advocate netting out

Supported by networks models (e.g. Acemoglu–Ozdaglar–Tahbaz-Salehi 15)

Based on one-period debt capturing overnight debts (e.g. repos)

Much interbank debt longer maturity

Germany: Average mat. more than year; frac. overnight less than 10%

QUESTIONS

Do long-term debt networks harbor same systemic risks as short-?

Do the same network structures lead risks to propagate?

Do gross debts serve function that could be undermined by netting out?

THIS PAPER

Model of N banks connected in network of long-term debts

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Assumption: $y > \ell > \theta y$

LIMITED PLEDGEABILITY

B$_i$'s Balance Sheet

Assets	Liabilities
long-term	short-term
investments	liq. shock
	equity

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y	ℓ
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LIMITED PLEDGEABILITY

B_i's Balance Sheet

Assets	Liabilities
y	ℓ
debt from B_j	debt to B_j
	equity

RESULTS

High indebtedness and connectedness sources of value and stability

- Zero net long-term positions have positive NPV
- Embed option to dilute with new debt $\implies$ liquidity insurance
- Contingent transfers via plain debt

"Exponential networks" implement optimal transfers for any shocks

RESULTS MATTER FOR POLICY

Policies that help with short-term debt backfire with long-term debt

Decreasing indebtedness/connectedness can decrease efficiency

MODEL

MODEL OVERVIEW

Two dates: Date 1 and Date 2; no discounting; universal risk neutrality

N banks: Assets y at Date 2 and risk of liquidity shock $\ell < y$ at Date 1

Interbank network: Network of long-term debts $\mathbf{F} = [F_{i \to j}]_{ij}$ (due at Date 2)

Friction: Limited pledgeability: Only $\theta y < \ell$ pledgeable

BANKS $B_1, ..., B_N$ AND BALANCE SHEETS

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<u>Assumption</u>: New debt senior (e.g. repo) $\implies$ $F_{i\rightrightarrows}$ diluted

REPAYMENTS

Denote B_i's equilibrium repayment to B_j by $R_{i\to j}$

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NB: Liquidation inefficient (destroys $(1-\theta)y$), default alone is not (transfer)

EQUILIBRIUM

A payment equilibrium is a repayment profile $[R_{i\to j}]_{ij}$ for each $(\sigma_i)_i$ s.t.

Repayments are sequentially rational

Repayments are paid pro rata: $\dfrac{R_{i\to j}}{R_{i\rightrightarrows}} = \dfrac{F_{i\to j}}{F_{i\rightrightarrows}}$

TIMELINE/SUMMARY

Date 1: Shocks realized; banks raise new liq.; banks liquidated/continue

Date 2: Assets y realized; banks repay or default

DEFINITION: EFFICIENCY

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A network more efficient than another if fewer banks liquidated $\forall(\sigma_i)_i$

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Benchmark isomorphic to AOT mutatis mutandis

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Say B_i and B_j have offsetting debts $\alpha F_{i \to j} = \alpha F_{j \to i} = \alpha F$ and B_i shocked

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But claims more than fully encumbered by liabilities created on RHS

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Formalized via “harmonic distance” (captures direct and indirect links)

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Neighbors' neighbors provide them liquidity...

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Neighbors' neighbors provide them liquidity...

Overall: Not-shocked near shocked pay out so much that can't meet shocks

BM3: CONNECTEDNESS

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Intuition: Liquidations propagate through network per default radius (BM2)

RESULTS

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B_i can dilute liability αF

<u>Overall</u>: High long-term debt creates claims on the LHS of balance sheet

Claims not encumbered by liabilities created on RHS (can be diluted)

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B_i's Balance Sheet

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long-term	short-term
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B_i's Balance Sheet

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SHOCKED BANK B$_i$ NEEDS LIQUIDITY

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
αF	αF
	equity

B$_i$ RAISES CASH VIA NEW DEBT AGAINST y & α

B$_i$'s Balance Sheet

Assets	Liabilities
y	ℓ
αF	αF
	equity

$\rightarrow$

B$_i$'s Balance Sheet

Assets	Liabilities
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αF	αF
cash	new debt
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DILUTES B_j

B_i's Balance Sheet

Assets	Liabilities
y	ℓ
αF	αF
	equity

$\rightarrow$

B_i's Balance Sheet

Assets	Liabilities
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	equity

B_j NOT WORSE OFF EX ANTE

Gross debts mean B_j diluted when B_i is shocked

But B_j can also dilute B_i when it is shocked

Gross debt implement transfer from not-shocked to shocked bank

Coinsurance via option to dilute

PRACTICAL IMPLEMENTATION

Banks hold gross long-term dilutable debts

- E.g. interbank loans/bonds
- Rationalizes why long-maturity

Banks dilute with short-term senior debt

- Rationalizes e.g. super-seniority for repos

Explains large interbank positions (quarter of balance sheets)

DILUTION COMPLEMENTS DEFAULT

Banks use the option to default to implement contingencies

- Implements transfer from not-shocked to shocked at Date 2
- Allen–Gale 98, Dubey–Geanakoplos–Shubik 88, and Zame 93

But default not enough here

- Need dilution to prevent liquidation at Date 1

Like defaultable debt, dilutable debt can be good

- Implements transfers before maturity

R2: SALVATION RADIUS

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Overall: Banks near not-shocked banks dilute so much that meet shocks

R3: CONNECTEDNESS INCREASES EFFICIENCY

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Increasing connectedness increases efficiency

Formalized using "bottleneck parameter"/"delta connectedness"

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Intuition: Liquidity propagates through network per salvation radius (R2)

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

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Answer: No!

TO SUM UP: LT DEBT NETWORKS UNLIKE ST

Indebtedness and connectedness sources of efficiency

Reason: Option to dilute gross debts provides insurance

Question: Do high indebtedness and connectedness suffice for efficiency?

Answer: No! Complete network (fully connected) inefficient no matter debt

R4: COMPLETE NETWORK INEFFICIENT

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Let S be number of shocked banks and suppose $S\ell > N\theta y$

If $\mathbf{F}$ is complete ($F_{i\to j} \equiv F$) then all shocked banks are liquidated

R4: COMPLETE INEFFICIENT: PROOF (SKETCH)

Complete $\implies$ each not-shocked bank pays at most $\dfrac{\theta y}{S}$ to each shocked

$\implies$ shocked liquidated given $\ell - \theta y > R_{i\leftarrow} \geq (N-S)\dfrac{\theta y}{S}$ or $S\ell > N\theta y$

R4: COMPLETE INEFFICIENT: INTUITION

Complete network delivers all shocked banks same net payment

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Question: How much better can we do?

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A network is constrained efficient if L is minimized for each $(\sigma_i)_i$ s.t.

$$(S - L)(\ell - \theta y) \leq (N - S)\theta y$$

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I.e. liq. provided to shocked not-liquidated $\leq$ available from not-shocked

PRINCIPLES OF EFFICIENCY

Planner should allocate liquidity to save largest number of shocked banks:

(i) Not-shocked banks pay out all liquidity (θy)

(ii) Allocate none to liquidated banks (so all used to save shocked)

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NB: Ordering by indices arbitrary, can consider permutation

R5: EXPONENTIAL NETWORKS ARE CONSTRAINED EFFICIENT

R5: EXP. NETWORKS CONSTRAINED EFFICIENT

Let $\mathbf{F}$ be an exponential network with base s small enough

For α large enough, $\alpha\mathbf{F}$ is generically constrained efficient

R5: EXP. NETWORKS EFF.: PROOF (SKETCH)

Echoes principles of efficiency

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NB: "Almost" is enough except in non-generic cases (also manageable)

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(iii) Banks can be heterogeneous

$\Longrightarrow$ Exp. network imperfect as ranking ind. of state (but not that bad)

CONCLUSION

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Off-setting long-term debts provide insurance

- Indebtedness and connectedness sources of efficiency
- Contrary to conclusions based on short-term debt

Indebtedness and connectedness implement efficiency if network exponential

- Minimize number of liquidations no matter realization of shocks
- “Robust but never fragile”

SYSTEMIC RISK IN FINANCIAL NETWORKS REVISITED

APPENDIX

NON-CONTINGENT LIQUIDTY

If transfer ℓ to all banks at Date 0 at rate $R = \dfrac{L - \pi\theta y}{(1-\pi)L}$

All banks meet their shocks

Shocked banks repay θy, not-shocked banks repay RL

Outside lender breaks even

Works, but requires outside liquidity $NL \gg ML$ at Date 0

OUTSIDE CREDIT LINES

Extend credit line to all banks to borrow ℓ at Date 1 at rate ϵ

For non-contingent repayment $F = \dfrac{L - \pi\theta y}{1 - \pi}$

Shocked banks draw down, not shocked banks don't

Outside lender breaks even

Works, but requires commitment from outside lender